Unit Lambda.1
Entanglement of continued fractions

$$\mathbf{g} = \cfrac{b_1}{a_1+\cfrac{b_2}{a_2+\cfrac{b_3}{a_3+\cfrac{b_4}{a_4+\cdots}}}} \qquad (1)$$

$$\mathbf{h} = \cfrac{B_1}{A_1+\cfrac{B_2}{A_2+\cfrac{B_3}{A_3+\cfrac{B_4}{A_4+\cdots}}}} \qquad (2)$$

Entanglement of **g** and **h**

g#h =

$$\cfrac{b_1}{a_1+\cfrac{B_1}{A_1+\cfrac{b_2}{a_2+\cfrac{B_2}{A_2+\cfrac{b_3}{a_3+\cfrac{B_3}{A_3+\cfrac{b_4}{a_4+\cfrac{B_4}{A_4+\cdots}}}}}}}} \qquad (3)$$

Example 1

$$\mathbf{g} = \cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7+\cdots}}}} = [0;1,3,5,7,9,11,13,15,\ldots]$$

$$\mathbf{h} = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cdots}}}} = [0;1,2,3,4,5,6,7,8,\ldots]$$

g#h = [0;1,1,3,2,5,3,7,4,9,5,11,6,13,7,…]
=0.563178619811711385425752903703563553276052254864… = ?

h#g = [0;1,1,2,3,3,5,4,7,5,9,6,11,7,13,…]
= 0.589228208902086890408884740634016321602061704547… = ?

(4.1,2,3,4)

See Unit 999, Lateral 1, equs (7.3) and (8).

Example 2

$$\mathbf{g} = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cdots}}}} = [0;1,2,3,4,5,6,7,8,\ldots]$$

$$\mathbf{h} = \cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{6+\cfrac{1}{8+\cdots}}}} = [0;2,4,6,8,10,12,14,16,\ldots]$$

g#h = [0;1,2,2,4,3,6,4,8,5,10,6,12,7,14,…]
= 0.709984738882673017522949695574456984172821198 14... = ?

(5.1,2,3)

Example 3

$$\mathbf{g} = \frac{1}{e-1} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cdots}}}} = 0.581976706\ldots$$

$$\mathbf{h} = \frac{\pi}{4} = \cfrac{1}{1+\cfrac{1}{3+\cfrac{4}{5+\cfrac{9}{7+\cfrac{16}{9+\cfrac{25}{11+\cfrac{36}{13+\cfrac{49}{15+\cdots}}}}}}}} = 0.785398163\ldots$$

$$\mathbf{g\#h} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{2+\cfrac{1}{3+\cfrac{3}{3+\cfrac{4}{5+\cfrac{4}{4+\cfrac{9}{7+\cfrac{5}{5+\cfrac{16}{9+\cfrac{6}{6+\cfrac{25}{11+\cfrac{7}{7+\cfrac{36}{13+\cfrac{8}{8+\cfrac{49}{15+\cdots}}}}}}}}}}}}}}}$$

$$= ?$$

$$\mathbf{g\#g} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{2+\cfrac{2}{2+\cfrac{3}{3+\cfrac{3}{3+\cfrac{4}{4+\cfrac{4}{4+\cfrac{5}{5+\cfrac{5}{5+\cfrac{6}{6+\cfrac{6}{6+\cfrac{7}{7+\cfrac{7}{7+\cfrac{8}{8+\cfrac{8}{8+\cdots}}}}}}}}}}}}}}}$$

$$= 0.634549847\ldots$$

$$= \quad [0;1,\mathbf{1},1,\mathbf{2},1,\mathbf{3},1,\mathbf{4},1,\mathbf{5},1,\mathbf{6},1,7,1,\mathbf{8}\ldots]$$

$$= [0;\text{-}1,\mathbf{1},\ \mathbf{\text{-}2},\ \ \mathbf{3},\ \mathbf{\text{-}4},\ \ \mathbf{5},\ \mathbf{\text{-}6},\ \ \mathbf{7},\ \mathbf{\text{-}8},\ldots]$$

$$= \frac{J_0(2)}{J_1(2)-J_0(2)} = \frac{1}{\frac{J_1(2)}{J_0(2)}-1}$$

(6.1,2,3,4)

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$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{2+\cfrac{2}{2+\cfrac{3}{3+\cfrac{3}{3+\cdots+\cfrac{4}{4+\cfrac{4}{4+\cdots+\cfrac{k}{k+\frac{k}{k}}}}}}}}}}} = [0;1,\mathbf{1},1,\mathbf{2},1,\mathbf{3},1,\mathbf{4},\ldots,1,\mathbf{k}]$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{3}{3+\cfrac{3}{3+\cfrac{5}{5+\cfrac{5}{5+\cdots+\cfrac{(2k+1)}{(2k+1)+\frac{(2k+1)}{(2k+1)}}}}}}}} = [0;1,\mathbf{1},1,\mathbf{3},1,\mathbf{5},\ldots,(\mathbf{2k+1})]$$

$$\cfrac{a_1}{a_1+\cfrac{a_1}{a_1+\cfrac{a_2}{a_2+\cfrac{a_2}{a_2+\cfrac{a_3}{a_3+\cfrac{a_3}{a_3+\cdots+\cfrac{a_n}{a_n+\frac{a_n}{a_n)}}}}}}}} =$$

$$[0;1,\mathbf{a_1},1,\mathbf{a_2},1,\mathbf{a_3},\ldots,1,a_n)]$$

(7.1,2,3)

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$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{3}{3+\cfrac{3}{3+\cfrac{5}{5+\cfrac{5}{5+\cfrac{7}{7+\cfrac{7}{7+\cdots}}}}}}}}} = 0.642092615\ldots$$

$$= [0;1,\mathbf{1},1,\mathbf{3},1,\mathbf{5},1,\mathbf{7},1,\mathbf{9},1,\mathbf{11},\ldots]$$

$$= [\quad \mathbf{1};\ \mathbf{-3},\ \ \mathbf{5},\ \mathbf{-7},\ \ \mathbf{9},\ \mathbf{-11},\ldots]$$

$$= \frac{1}{\tan(1)}$$

(6.5)

Next **h#g** = ?

h#h = ?

g#g#g = $\mathbf{g}^{\#3}$ = ?

Example 4

$$\mathbf{g} = \frac{1}{e-1} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cdots}}}}$$

$$\mathbf{h}(n) = \frac{N(n)}{D(n)} = \cfrac{1}{2+\cfrac{n+2}{3+\cfrac{n+3}{4+\cfrac{n+4}{5+\cfrac{n+5}{6+\cfrac{n+6}{7+\cdots}}}}}}\ ;\ n=2,3,4,5,\ldots$$

$$\begin{pmatrix} N(n+1) \\ D(n+1) \end{pmatrix} = \begin{pmatrix} n & 1 \\ n+2 & n+2 \end{pmatrix} \begin{pmatrix} N(n) \\ D(n) \end{pmatrix};$$

$$n=1,2,3,4,5,\ldots$$

$$\binom{N(1)}{D(1)} = \binom{1}{0}$$

(8.1,2,3,4)

g#h(n) = ? ; n=2,3,4,5,…

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Lateral

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$$\cfrac{2}{1+\cfrac{3}{2+\cfrac{4}{3+\cfrac{5}{4+\cfrac{6}{5+\cfrac{7}{6+\cdots}}}}}} = 1 \qquad (9.1)$$

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$$\cfrac{3}{2+\cfrac{5}{4+\cfrac{7}{6+\cfrac{9}{8+\cfrac{11}{10+\cfrac{13}{12+\cdots}}}}}} = 1 \qquad (9.2)$$

but

$$\cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cfrac{12}{13+\cdots}}}}}} = \frac{2-\sqrt{e}}{\sqrt{e}-1} = 0.541494082\ldots$$

$$= [0;1,1,\mathbf{5},1,1,\mathbf{9},1,1,\mathbf{13},1,1,\ldots]$$

$$= 2*[0;3,1,\mathbf{2},3,1,\mathbf{4},3,1,\mathbf{6},3,1,\mathbf{8},\ldots]$$

$$= \frac{1}{2} * [1; 12,20,28,\ 36,44,\ 52, \ldots]$$

$$= \frac{1}{4} * [2;\ \ 6,40,14,\ 72,22,104, \ldots]$$

$$= \frac{1}{8} * [4;\ \ 3,80,\ \ 7{,}144{,}11{,}208{,}\ldots$$

$$= \frac{1}{2} * [1; 4 * \mathbf{3},\ \ 4 * \mathbf{5}, 4 * \mathbf{7},\ \ 4 * \mathbf{9}, 4 * \mathbf{11},\ \ 4 * \mathbf{13}, \ldots]$$

$$= \frac{1}{4} * [2; 2 * \mathbf{3},\ \ 8 * \mathbf{5}, 2 * \mathbf{7},\ \ 8 * \mathbf{9}, 2 * \mathbf{11},\ \ 8 * \mathbf{13}, \ldots]$$

$$= \frac{1}{8} * [4; 1 * \mathbf{3}, 16 * \mathbf{5}, 1 * \mathbf{7}, 16 * \mathbf{9}, 1 * \mathbf{11}, 16 * \mathbf{13}, \ldots]$$

(9.3)

Check the convergence behaviour of the different continued fraction representations of $\frac{2-\sqrt{e}}{\sqrt{e}-1}$.

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Discuss possible applications

** Coding problem carrier**g**#signal/message**h**
(see Small challenges to go, part 3, Ch 8)

** Analysis problem object**g**#toolbox**h**(n)
(see example 4)

Open

** Properties of the operation #
(ass., kom.. …)

** Inversion of a#b=c

** # chains