Unit n°ABC

Up and down

ABC.1 Launch pad

ABC.1.1

$$1+\frac{1|}{|2}+\frac{1|}{|3}+\frac{1|}{|4}+\frac{1|}{|5}+\frac{1|}{|6}+\frac{1|}{|7}+\ldots=$$

$$[1;2,3,4,5,6,7,\ldots]=$$

$$\frac{1}{2}[2;1,6,2,10,3,14,4,18,5,\ldots]=$$

$$\frac{I_0(2)}{I_1(2)}=1.433127426722311758317183455775991820431512767905980523443428636394309183254172900136503726435786115\ldots$$

ABC.1.2

$$1+\frac{2|}{|2}+\frac{3|}{|3}+\frac{4|}{|4}+\frac{5|}{|5}+\frac{6|}{|6}+\frac{7|}{|7}+\ldots=$$

$$[1;1,2,1,1,4,1,1,6,1,1,8,1,1,\ldots]=$$

$$\frac{1}{2}[3;2,3,2,3,1,2,1,3,4,3,1,4,1,3,6,3,1,6,1,\ldots]=$$

$$2[0;1,6,10,14,18,22,26,30,\ldots]=$$

$$e-1 = 1.71828182845904523536028747135266249$$
$$77572470936999595749669676277240766303535475945713821785251664 27\ldots$$

ABC.1.3

$$1+\frac{2|}{|3}+\frac{4|}{|5}+\frac{6|}{|7}+\frac{8|}{|9}+\frac{10|}{|11}+\frac{12|}{|13}+\ldots=$$

$$[1;1,1,5,1,1,9,1,1,13,1,1,17,\ldots]=$$

$$\frac{1}{2}[3;12,20,28,36,44,52,60,\ldots]=$$

$$2[0;1,3,2,1,3,4,1,3,6,1,3,8,1,3,10,1,3,\ldots]=$$

$$\frac{1}{\sqrt{e}-1} = 1.54149408253679828413110344447251$$
$$46383404592368418821094741369566375426391433148070718257240850077 42\ldots$$

ABC.1.4

$$\frac{2|}{|4}+\frac{3|}{|5}+\frac{4|}{|6}+\frac{5|}{|7}+\frac{6|}{|8}+\frac{7|}{|9}+\ldots=$$

1.441755879289969382381855225990535922831
233824708904326494956992790470419752501 4
54090430507152957809…=
[1; 2, 3, 1, 3, 1, 4, 2, 1, 1, 1, 2, 4, 1, 1, 13, 1, 1, 3,
3, 2, 2, 2, 1, 14, 59, 52, 1, 1, 21, 1, 2, 1, 2, 1, 3, 5, 2,
1, 4, 1, 9, 1, 1, 1, 1, 4, 7, 160, 1, 65, 2, 4, 1, 1, 10, 2,
5, 1, 34, 5, 6, 1, 1, 1, 2, 1, 6, 1, 1, 1, 1, 13,
5, 2, 2, 1, 2, 1, 2, 2, 2, 6, 2, 2, 2, 1, 1, 33, 1, 3, 1,
1, 1, 1, 1, 7, 4, 2, 1, 1, 1, …]
Find compact formula expressions.

ABC.1.5

A string of pearls

$$\begin{matrix} & 2\to 3 & & 6\to 7 & \\ \underline{1} & \swarrow & \nearrow & \swarrow & \nearrow \ldots \end{matrix}$$

$$\begin{matrix} & 4\to 5 & & 8\to 9 & \\ 1+\frac{2|}{|4} & + & \frac{3|}{|5} & + & \frac{6|}{|8} & + & \frac{7|}{|9} & + & \frac{10|}{|12} & + & \frac{11|}{|13} & + & \frac{14|}{|16} & + & \frac{15|}{|17} & +\ldots= \end{matrix}$$

1.441755879... = ? formula expressions?

ABC.2 Extensions

ABC.2.1

$$f1(x) = 1^x + \frac{1|}{|2^x} + \frac{1|}{|3^x} + \frac{1|}{|4^x} + \frac{1|}{|5^x} + \frac{1|}{|6^x} + \frac{1|}{|7^x} + \ldots =$$

$$[1^x;2^x,3^x,4^x,5^x,6^x,7^x,\ldots]$$

Display the graph of the function f1(x) for

** domain(f1) = codomain(f1) = $\mathbb{R}$

** domain(f1) = codomain(f1) = $\mathbb{C}$.

Find a continued fraction formula representation of the first derivative $\frac{df1(x)}{dx}$ of f1(x).

Special case

$$f1(\frac{1}{2}) = 1 + \frac{1|}{|\sqrt{2}} + \frac{1|}{|\sqrt{3}} + \frac{1|}{|\sqrt{4}} + \frac{1|}{|\sqrt{5}} + \frac{1|}{|\sqrt{6}} + \frac{1|}{|\sqrt{7}} + \ldots =$$

1.532182507609544002671245752212695076213
412951904740008582272670980753543440932653
711028190028373549…= ?formula expressions?

ABC.2.2

$$f2(x) = 1^x + \frac{2^x|}{|2^x} + \frac{3^x|}{|3^x} + \frac{4^x|}{|4^x} + \frac{5^x|}{|5^x} + \frac{6^x|}{|6^x} + \frac{7^x|}{|7^x} + \ldots$$

Display the graph of the function f2(x) for

** domain(f2) = codomain(f2) = $\mathbb{R}$

** domain(f2) = codomain(f2) = $\mathbb{C}$.

Find a continued fraction formula representation of the first derivative $\frac{df2(x)}{dx}$ of f2(x).

Special case

$$f2(\frac{1}{2}) = 1+\frac{\sqrt{2}}{\left|\sqrt{2}\right|}+\frac{\sqrt{3}}{\left|\sqrt{3}\right|}+\frac{\sqrt{4}}{\left|\sqrt{4}\right|}+\frac{\sqrt{5}}{\left|\sqrt{5}\right|}+\frac{\sqrt{6}}{\left|\sqrt{6}\right|}+\frac{\sqrt{7}}{\left|\sqrt{7}\right|}+\ldots =$$

1.667646230761443849391842331013276823243
207793883911632410746074092164792672537 8
19038667657323226642…=?formula expressions?

ABC.2.3

Compare the courses of the functions f1(x) and f2(x) with the course of the Riemann Zeta function

$$\zeta(x) = \frac{1}{1^x}+\frac{1}{2^x}+\frac{1}{3^x}+\frac{1}{4^x}+\frac{1}{5^x}+\frac{1}{6^x}+\frac{1}{7^x}+\ldots \quad [1,2]$$

in $\measuredangle$ and in $\coprod$, esp. in connection with the Riemann hypothesis [3].

References

[1] Sondow, Jonathan and Weisstein, Eric W. "Riemann Zeta Function." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/RiemannZetaFunction.html
[2] https://en.wikipedia.org/wiki/Riemann_zeta_function
https://de.wikipedia.org/wiki/Riemannsche_%CE%B6-Funktion
[3] https://en.wikipedia.org/wiki/Riemann_hypothesis