Unit 88
Variations on the continued fraction expansion
Variation 1
Feedback1 continued fraction expansion

Continued fraction expansion $z \rightarrow scf(z)$
Input: number $z \in \mathbb{R}_+$, limiter $k \in \mathbb{N}$
$x:=z$
$i:=0$
$a_0:=int(x)$
$x:=(x-int(x))^{-1}$
$i:=i+1$
$a_i:=int(x) \rightarrow$
$x=int(x)$ or $i=k$
$n:=i$
Output: n, $scf(z)=[a_0;a_1,a_2,a_3,a_4,\ldots,a_n]$

Feedback1 continued fraction expansion $z \rightarrow f1scf(z)$
Input: number $z \in \mathbb{R}_+$, limiter $k \in \mathbb{N}$
$x:=z$
$i:=0$
$c_0:=int(x)$
$x:=(x-int(x))^{-1}*int(x)$
$i:=i+1$
$c_i:=int(x) \rightarrow$
$x=int(x)$ or $i=k$
$n:=i$
Output: n, $f1scf(z)=\{c_0;c_1,c_2,c_3,c_4,\ldots,c_n\}$

Example 1
$z=e=2.7182818284\ldots$
$scf(e)=[2;1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$
$f1scf(e)=\{2;2,2,3,4,5,6,7,8,9,\ldots\}$

Example 2

$z=\Phi=\frac{1}{2}(\sqrt{5}+1)=1.618033989\ldots$
$scf(\Phi)=[1;1,1,1,1,1,1,1,\ldots]$
$f1scf(\Phi)=\{1;1,1,1,1,1,1,1,\ldots\}$

Example 3
$z=\sqrt{e}=1.648721270700128146848650787814163571653776100710 1\ldots$
$scf(\sqrt{e})=[1;1,1,1,5,1,1,9,1,1,13,1,1,17,1,1,\ldots]$
$f1scf(\sqrt{e})=\{1;1,1,1,5,9,17,109,260,2909,3072,3310,3678,6715,35175,37269,\ldots\}$

Example 4
$z=e^2=7.389056098930650227230427460575007813180315570551 8\ldots$
$scf(e^2)=[7;2,1,1,3,18,5,1,1,6,30,8,1,1,9.42,11,1,1,\ldots]$
$f1scf(e^2)=\{7;17,17,128,484,498,784,1316,1327,1416,2501,69664,79556,\ldots\}$

Example 5
$z=\pi$
$scf(\pi)=[3;7,15,1,292,1,1,1,2,1,3,1,14,2,\ldots]$
$f1scf(\pi)=\{3;21,111,113,158,160,211,216,525,1634,1721,7063,\ldots\}$

Example 6
$z=\sqrt{2}$
$scf(\sqrt{2})=[1;2,2,2,2,2,2,2,\ldots]$
$f1scf(\sqrt{2})=\{2;4,4,4,4,4,4,4,\ldots\}$

Example 7
$z=\sqrt{3}$
$scf(\sqrt{3})=[1;1,2,1,2,1,2,1,2,\ldots]$
$f1scf(\sqrt{3})=\{1;1,2,2,2,2,2,2,\ldots\}$

Example 8
$z=\sqrt{5}$
$scf(\sqrt{5})=[2;4,4,4,4,4,4,4,\ldots]$
$f1scf(\sqrt{5})=\{2;8,16,16,16,16,16,16,\ldots\}$

Example 9
$z=\sqrt{11}$

scf($\sqrt{11}$)=[3;3,6,3,6,3,6,3,6,…]
f1scf($\sqrt{11}$)={3;9,18,18,18,18,18,…}

Example 10
z=C_1=0.697774657964…
scf(C_1)=[0;1,2,3,4,5,6,7,8,9,…]
f1scf$\left(\frac{1}{C_1}\right)$ ={1;1*2,2*3,3*4,4*5,5*6,6*7,7*8,…}

Example 11
z=1.31303528549933130363616124693084783291201394124 05…
scf(z)=[1;3,5,7,9,11,13,15,17,19,21,23,25,…]
f1scf(z)={1;1*3,3*5,5*7,7*9,9*11,11*13,…}

Example 12
z=1.24328847839971564408249654539442949923120026119 74…
scf(z)=[1^2;2^2,3^2,4^2,5^2,6^2,7^2,8^2,9^2,10^2,…]
f1scf(z)={1^1;$(1*2)^2$,$(2*3)^2$,$(3*4)^2$,$(4*5)^2$,$(5*6)^2$,$(6*7)^2$,…}

Example 13
z = $\frac{e-2}{3-e}$ = 2.549646778303844882226392684797695600486425283354 0 …
scf(z)=[2;1,1,4,1,1,6,1,1,8,1,1,10,1,1,…]
f1scf(z)={2;3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,…}

Example 14
z = $\frac{2e-5}{3-e}$ = 1.549646778303844882226392684797695600486425283354 0
scf(z)=[1;1,1,4,1,1,6,1,1,8,1,1,10,1,1,…]
f1scf(z)={1;1,1,4,7,14,14,14,15,49,496,515,1421,1453,1870,3462,8255,10593,16090,41616, …}

Example 15
z = $\frac{2e-5}{3e-8}$ =2.819350243598079891783487492068884035948021618584 5…
scf(z)=[2;1,4,1,1,6,1,1,8,1,1,10,1,1,…]
f1scf(z)={ 2;2,4,7,14,14,14,15,49,496,515,1421,1453,1870,3462,8255,10593,16090,41616, …}

Example 16
z = $\frac{5e-13}{3e-8}$ =3.819350243598079891783487492068884035948021618584 5…
scf(z)=[3;1,4,1,1,6,1,1,8,1,1,10,1,1,…]

f1scf(z)={3;3,4,7,14,14,14,15,49,496,515,1421,1453,1870,3462,8255,10593,16090,41616,…}

Problems:

** Add more examples.

** Discuss and systematize all these examples.

** Discuss f1scf-applications.

** Add inv.algs scf(z) → z and f1scf(z) → z (see Units 7, 26, 30.1, 67; Deb 18).

** Add inv.dets scf(z) → z and f1scf(z) → z (see Unit 7) .

** Connections between z, scf(z), and f1scf(z)

$\frac{z}{z+1} = [0;1,a_0,a_1,a_2,a_3,a_4,a_5,\ldots] =$

$$\frac{1|}{|1} + \frac{1|}{|a_0} + \frac{1|}{|a_1} + \frac{1|}{|a_2} + \frac{1|}{|a_3} + \frac{1|}{|a_4} + \frac{1|}{|a_5} + \ldots$$

$$= \frac{c_0|}{|c_0} + \frac{c_1|}{|c_1} + \frac{c_2|}{|c_2} + \frac{c_3|}{|c_3} + \frac{c_4|}{|c_4} + \frac{c_5|}{|c_5} + \frac{c_6|}{|c_6} + \ldots$$

Compare the convergence of the a_i and the c_i continued fractions.

Find more.

** Come up with other scf loop body variations, f.e.

$$x := (x - \mathrm{int}(x))^{-1} * \left\{ \begin{array}{c} 1 \\ \text{math. const.} \\ x \\ \mathrm{int}(x) \\ x^p \\ ? \end{array} \right\},$$

$$x := (x - \mathrm{int}(x))^{-2}, \ldots$$

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