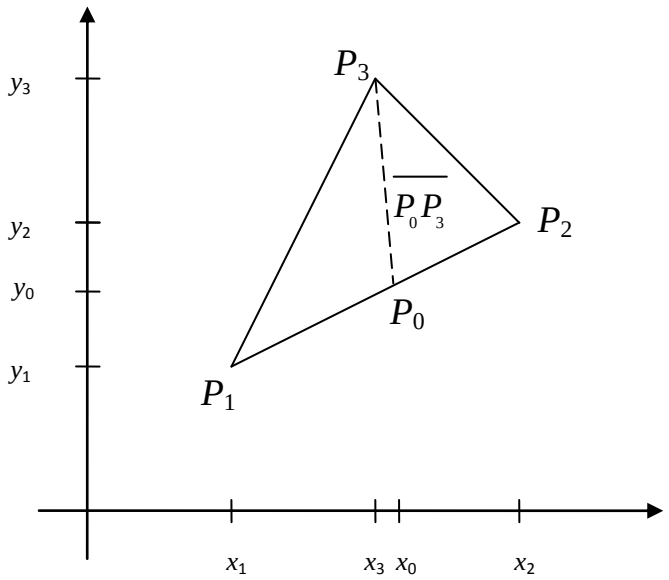
# Unit n°84

## Divide et impera




Given is a planar triangle $\Delta P_1P_2P_3$ by the Cartesian coordinates of its vertices $P_i = (x_i, y_i)$ ; $i = 1(1)3$ .

The line segment $\overline{P_0P_3}$ divides $\Delta P_1P_2P_3$ into two parts, $\Delta P_1P_0P_3$ and $\Delta P_0P_2P_3$, the quotient $q = \dfrac{area\Delta P_1P_0P_3}{area\Delta P_0P_2P_3}$ is given.

**1 Find the coordinates of $P_0 = (x_0, y_0)$ as a function of the $x_i, y_i$, and $q$ taking a simplest and shortest way.

**2 Find $q$ as a function of the $x_i$ and $y_i$ for the special cases:

$\overline{P_0P_3}$ is
* median,
* angle bisector,
* altitude.

## Guidance to **1

Two unknowns $x_0$ and $y_0$ $\rightarrow$ two equations

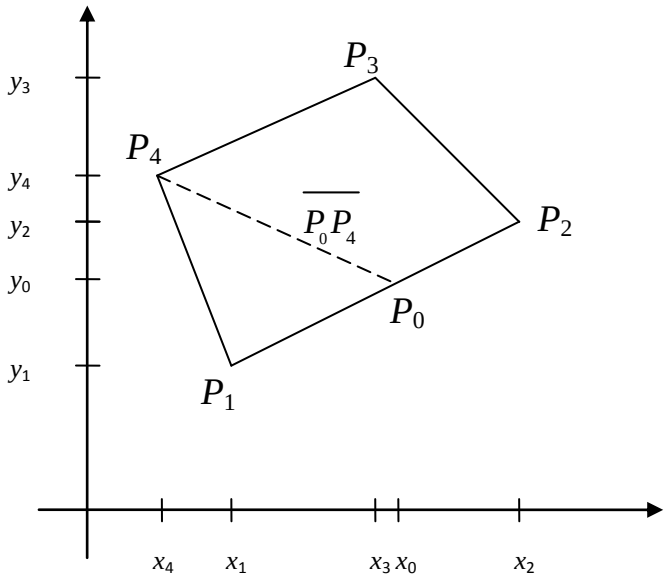
$$\begin{vmatrix} x_1 & x_0 & x_2 \\ y_1 & y_0 & y_2 \end{vmatrix} = 0 \ (P_1,\ P_0 \text{ and } P_2 \text{ are collinear})$$

and $$\begin{vmatrix} x_1 & x_0 & x_3 \\ y_1 & y_0 & y_3 \end{vmatrix} = q \begin{vmatrix} x_0 & x_2 & x_3 \\ y_0 & y_2 & y_3 \end{vmatrix}$$

## Solution **1

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \begin{pmatrix} \dfrac{1}{1+q} \\ \dfrac{q}{1+q} \end{pmatrix}$$

## Extension




Given is a planar quadrangle $\square P_1P_2P_3P_4$ by the Cartesian coordinates of its vertices $P_i = (x_i, y_i)$ ; $i = 1(1)4$ .

The line segment $\overline{P_0P_4}$ divides $\square P_1P_2P_3P_4$ into two parts, $\Delta P_1P_0P_4$ and $\square P_0P_2P_3P_4$, the quotient $q = \frac{area\Delta P_1P_0P_4}{area\square P_0P_2P_3P_4}$ is given.

Find the coordinates of $P_0 = (x_0, y_0)$ taking a simplest and shortest way.

More

## References


[1] https://en.wikipedia.org/wiki/Triangle