Unit 83.1
Number theoretic finger exercises to train creative thinking

83.1.1
A prime problem

The polynomial

$p_2(x,v,A) = x^2+vx+A$; $x,A\in\mathbb{N}$, $v\in\{+,-\}$

generates for x = 0(1)a b primes.
c_1 of these have the final digit 1,
c_3 3,
c_7 7,
c_9 9.
Calculate and discuss b(v,A,a) and the frequencies

$\frac{c_1}{b}(v,A,a)$, $\frac{c_3}{b}(v,A,a)$, $\frac{c_7}{b}(v,A,a)$, $\frac{c_9}{b}(v,A,a)$

for representative values of v, A and a.
Find associated approximation formulas for these functions.
Example 1
v = +, A = 81

a	b	c_1	c_3	c_7	c_9

50	13	5	5	3	0
100	22	9	7	6	0
200	38	15	16	7	0
300	51	21	22	8	0
400	64	28	26	10	0
500	79	34	32	13	0
1000	137	54	55	28	0
2000	243	103	92	48	0
10000	946	376	381	189	0
20000	1734	684	701	349	0
50000	3976	1590	1599	787	0
200000	59809	23923	23871	12015	0

Conjecture

$\lim_{a\to\infty} c_1 = c_3 = 2c_7 = 0.4b$, $c_9 = 0$

Proof! (see Unit 11)

Example 2

v = -, A = 41 (Euler's prime generating polynomial)

a	b	c_1	c_3	c_7	c_9
50	46	17	19	10	0
100	86	34	34	18	0
200	156	63	63	30	0
300	211	85	83	43	0
400	270	105	109	56	0

500	326	130	131	65	0
1000	581	234	237	110	0
2000	1021	416	407	198	0
10000	4149	1644	1657	848	0
20000	7620	3031	3033	1556	0
50000	17071	6762	6842	3467	0
200000	59809	23923	23871	12015	0

Conjecture

$\lim_{a\to\infty} c_1 = c_3 = 2c_7 = 0.4b$, $c_9 = 0$

Proof! (see Unit 11)

Example 3

v = -, A = 17 (Euler)

a	b	c_3	c_7	c_9	c_1
50	38	8	16	14	0
100	60	14	23	23	0
200	105	20	47	38	0
300	139	26	62	51	0
400	175	33	76	66	0
500	214	40	93	81	0
1000	365	66	148	151	0
2000	651	121	257	273	0
10000	2627	527	1048	1052	0
20000	4774	950	1910	1914	0
50000	10822	2168	4287	4367	0

200000 37813 7575 15066 15172 0
500000 87052 17374 34741 34937 0
Conjecture
$\lim_{a\to\infty} 2c_3 = c_7 = c_9 = 0.4b\ ,\ c_1 = 0$
Proof! (see Unit 11)

More
v = -, A = 19

a	b	c_1	c_9	c_3	c_7
500000	37297	18599	18698	0	0

Conjecture
$\lim_{a\to\infty} c_1 = c_9 = 0.5b\ ,\ c_3 = c_7 = 0$
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v = -, A = 479

a	b	c_1	c_9	c_3	c_7
500000	74322	37083	37239	0	0

Conjecture
$\lim_{a\to\infty} c_1 = c_9 = 0.5b\ ,\ c_3 = c_7 = 0$
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v = -, A = 23

a	b	c_3	c_9	c_1	c_7
500000	44678	29804	14874	0	0

Conjecture

$$\lim_{a\to\infty} c_3 = \frac{2}{3}b, \quad \lim_{a\to\infty} c_9 = \frac{1}{3}b, \quad c_1 = c_7 = 0$$

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v = -, A = 43

a	b	c_3	c_9	c_1	c_7
500000	25914	17215	8699	0	0

Conjecture

$$\lim_{a\to\infty} c_3 = \frac{2}{3}b, \quad \lim_{a\to\infty} c_9 = \frac{1}{3}b, \quad c_1 = c_7 = 0$$

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Next conjecture:

The last digit of A determines the final values of the frequencies

$$\left(\frac{c_1}{b}(v,A,\infty),\frac{c_3}{b}(v,A,\infty),\frac{c_7}{b}(v,A,\infty),\frac{c_9}{b}(v,A,\infty)\right)$$

$$A = ...1 \quad \rightarrow \quad \left(\frac{2}{5},\frac{2}{5},\frac{1}{5},0\right)$$

$$A = ...3 \quad \rightarrow \quad \left(0,\frac{2}{3},0,\frac{1}{3}\right)$$

$$A = ...7 \quad \rightarrow \quad \left(0,\frac{1}{5},\frac{2}{5},\frac{2}{5}\right)$$

$$A = ...9 \quad \rightarrow \quad \left(\frac{1}{2},0,0,\frac{1}{2}\right)$$

and

$$A = ...5 \quad \rightarrow \quad (?,?,?,?)$$

Check it, if ok → proof; discussion!

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Open is, and waiting for you: b(v,A,a).

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83.1.2

163 curiosities

** Deb 9, S. 2-6

** Unit 15, part 27, S. 3-4

** Unit 15, part 29, S. 3-4

** New: $163 = 10 + \sum_{i=1}^{5} i!$

Lateral:

k	$10 + \sum_{i=1}^{k} i!$
1	11 prime
2	13 p
3	19 p
4	43 p
5	163 p
6	883 p
7	5923 p
8	46243 not p
9	409123 not p
10	4037923 p

Continue and discuss.

References

[1] http://mathworld.wolfram.com/Prime-GeneratingPolynomial.html
[2] http://mathworld.wolfram.com/PrimeFormulas.html
[3] https://en.wikipedia.org/wiki/Formula_for_primes#Prime_formulas_and_polynomial_functions

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