Unit n°81

Arithmetisches, geometrisches und harmonisches Mittel von Primzahlnachbarn

Arithmetisches, geometrisches und harmonisches Mittel von Primzahlnachbarn

$$a(x,y)=\frac{x+y}{2}\geq g(x,y)=\sqrt{xy}\geq h(x,y)=\frac{2xy}{x+y}$$

Stellen Sie die Werteverläufe von

** $a(p_{i+1},p_i)-g(p_{i+1},p_i)$

** $g(p_{i+1},p_i)-h(p_{i+1},p_i)$

** $\dfrac{a(p_{i+1},p_i)-g(p_{i+1},p_i)}{g(p_{i+1},p_i)-h(p_{i+1},p_i)}$

über $i=1,2,3,4,\ldots$ graphisch dar.

$p_1=2,\ p_2=3,\ p_3=5,\ p_4=7,\ p_5=11,\ \ldots$ prime sequence

Diskutieren Sie ihre Ergebnisse unter Einbeziehung von [1,2,3].

References

[1] Weisstein, Eric W. "Andrica's Conjecture." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/AndricasConjecture.html

[2] https://en.wikipedia.org/wiki/Andrica‘s_conjecture

[3] https://de.wikipedia.org/wiki/Vermutung_von_Andrica



Photo von Jonathan Stoschek

Erste Ergebnisse von Dagmar Schönfeld

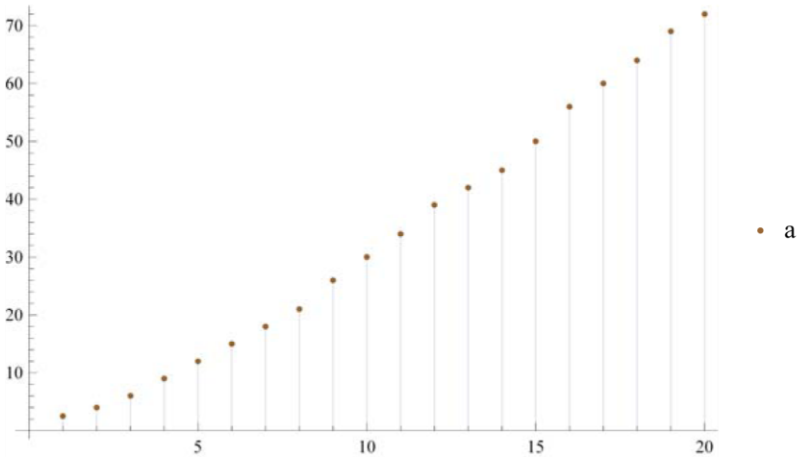
In[141]:=
```
anz = 20; a = g = h = Table[0, {i, anz}]; werte = Table[0, {i, anz}, {j, 6}];
Table[{a[[i]] = werte[[i, 1]] = (Prime[i + 1] + Prime[i]) / 2;
   g[[i]] = werte[[i, 2]] = Sqrt[Prime[i + 1] * Prime[i]];
   h[[i]] = werte[[i, 3]] = 2 * Prime[i + 1] * Prime[i] / (Prime[i + 1] + Prime[i]);
   werte[[i, 4]] = werte[[i, 1]] - werte[[i, 2]];
   werte[[i, 5]] = werte[[i, 2]] - werte[[i, 3]];
   werte[[i, 6]] = (werte[[i, 1]] - werte[[i, 2]]) / (werte[[i, 2]] - werte[[i, 3]]);
  }, {i, 1, anz}];
Print[Table[a[[i]], {i, 1, 10}]];
Print[Table[g[[i]], {i, 1, 10}]]; Print[Table[h[[i]], {i, 1, 10}]];
abild = ListPlot[a, Filling → Axis, PlotLegends → {"a"}, PlotStyle → {Brown}]
gbild = ListPlot[g, Filling → Axis, PlotLegends → {"g"}, PlotStyle → {Orange}]
hbild = ListPlot[h, Filling → Axis, PlotLegends → {"h"}, PlotStyle → {Cyan}]
(*Show[{abild,gbild,hbild}]*)
ListPlot[Table[Table[werte[[i, j]], {i, anz}], {j, 6}], Filling → Axis,
 PlotLegends → {"a", "g", "h", "a-g", "g-h", "(a-g)/(g-h)"},
 PlotStyle → {Brown, Orange, Cyan, Green, Red, Blue}]
ListPlot[Table[Table[werte[[i, j]], {i, anz}], {j, 4, 6}], Filling → Axis,
 PlotLegends → {"a-g", "g-h", "(a-g)/(g-h)"}, PlotStyle → {Green, Red, Blue}]
(*ListPlot[a,g,h,Filling→Axis,Frame→True]*)
```

$\left\{\frac{5}{2}, 4, 6, 9, 12, 15, 18, 21, 26, 30\right\}$

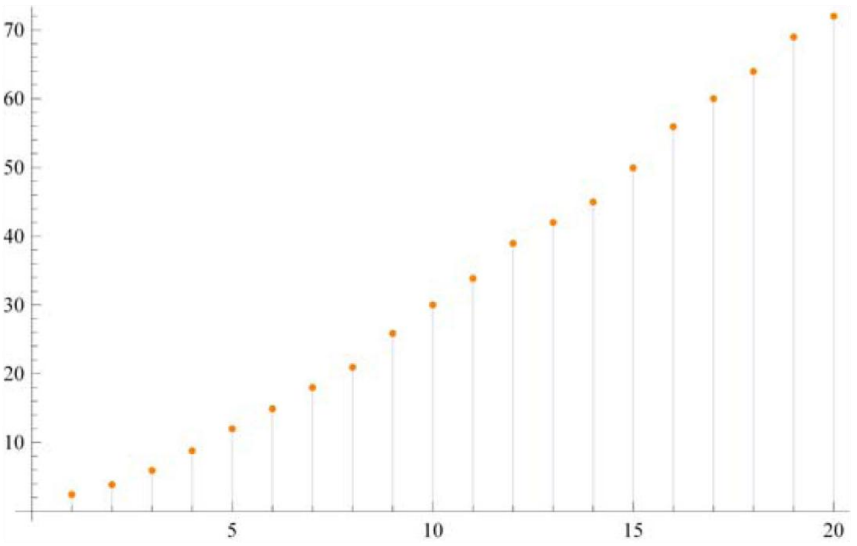
$\left\{\sqrt{6}, \sqrt{15}, \sqrt{35}, \sqrt{77}, \sqrt{143}, \sqrt{221}, \sqrt{323}, \sqrt{437}, \sqrt{667}, \sqrt{899}\right\}$

$\left\{\frac{12}{5}, \frac{15}{4}, \frac{35}{6}, \frac{77}{9}, \frac{143}{12}, \frac{221}{15}, \frac{323}{18}, \frac{437}{21}, \frac{667}{26}, \frac{899}{30}\right\}$

Out[143]=

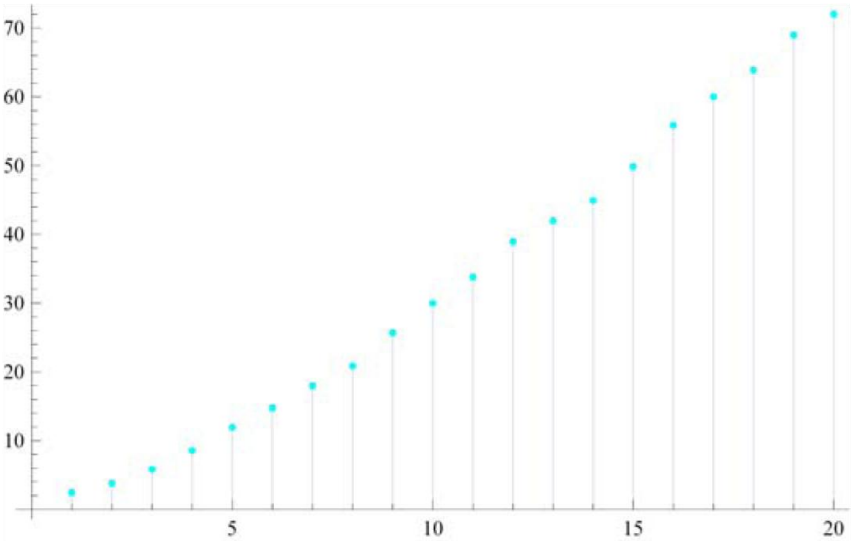


Out[144]=



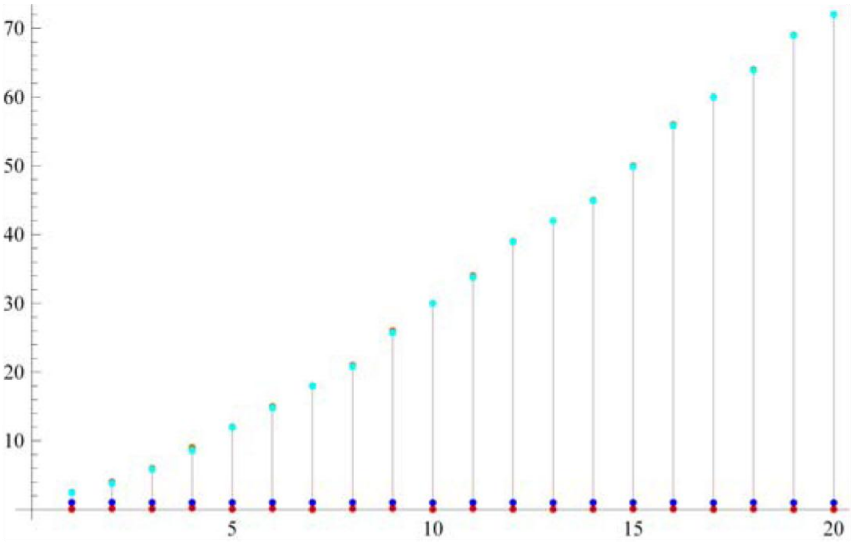
g

Out[145]=



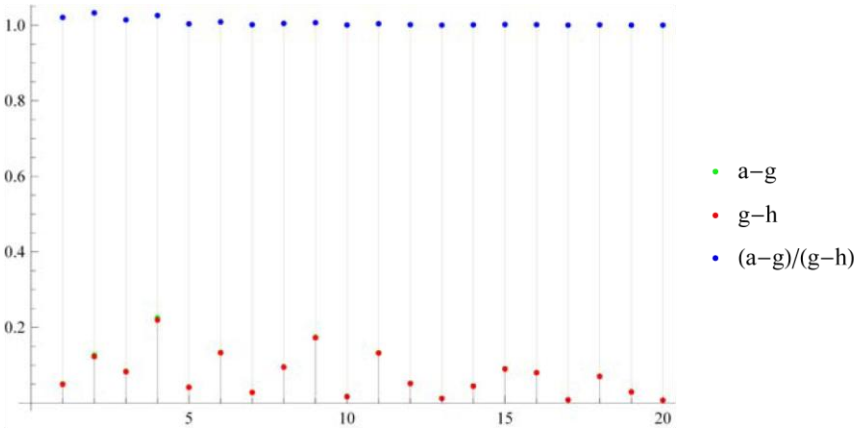
h

Out[146]=



a
g
h
a−g
g−h
(a−g)/(g−h)

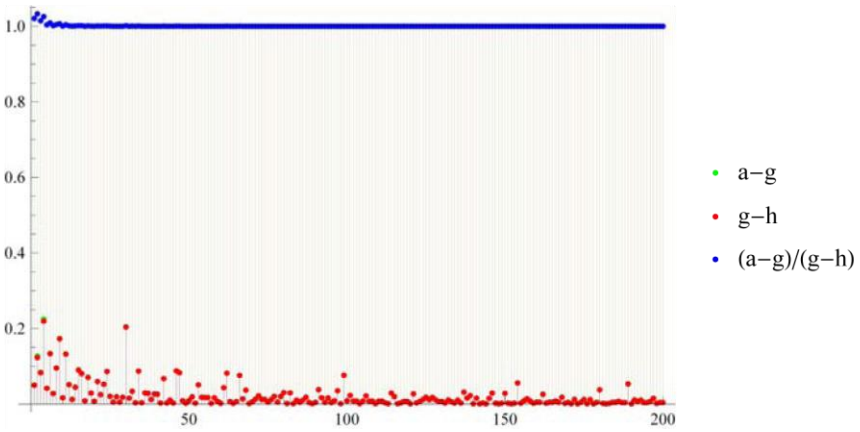
Out[147]=



In[148]:=

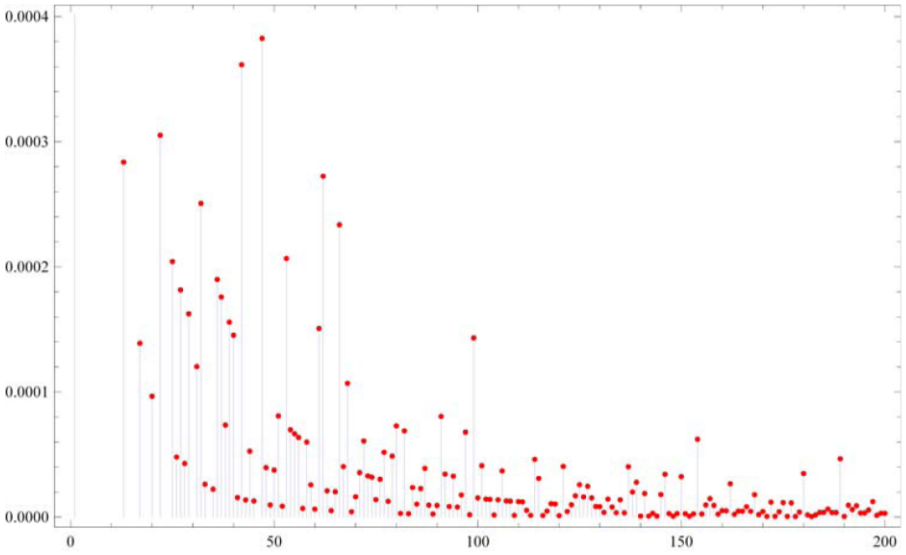
```
anz = 200; a = g = h = Table[0, {i, anz}]; werte = Table[0, {i, anz}, {j, 6}];
Table[{a[[i]] = werte[[i, 1]] = (Prime[i + 1] + Prime[i]) / 2;
   g[[i]] = werte[[i, 2]] = Sqrt[Prime[i + 1] * Prime[i]];
   h[[i]] = werte[[i, 3]] = 2 * Prime[i + 1] * Prime[i] / (Prime[i + 1] + Prime[i]);
   werte[[i, 4]] = werte[[i, 1]] - werte[[i, 2]];
   werte[[i, 5]] = werte[[i, 2]] - werte[[i, 3]];
   werte[[i, 6]] = (werte[[i, 1]] - werte[[i, 2]]) / (werte[[i, 2]] - werte[[i, 3]]);
  }, {i, 1, anz}];
(*Print[Table[a[[i]],{i,1,10}]];
Print[Table[g[[i]],{i,1,10}]];Print[Table[h[[i]],{i,1,10}]];
abild=ListPlot[a,Filling→Axis,PlotLegends→{"a"},PlotStyle→{Brown}]
   gbild=ListPlot[g,Filling→Axis,PlotLegends→{"g"},PlotStyle→{Orange}]
    hbild=ListPlot[h,Filling→Axis,PlotLegends→{"h"},PlotStyle→{Cyan}]
    (*Show[{abild,gbild,hbild}]*)
    ListPlot[Table[Table[werte[[i,j]],{i,anz}],{j,6}],
     Filling→Axis,PlotLegends→{"a","g","h","a-g","g-h","(a-g)/(g-h)"},
     PlotStyle→{Brown,Orange,Cyan,Green,Red,Blue}]*)
ListPlot[Table[Table[werte[[i, j]], {i, anz}], {j, 4, 6}], Filling → Axis,
 PlotLegends → {"a-g", "g-h", "(a-g)/(g-h)"}, PlotStyle → {Green, Red, Blue}]
(*ListPlot[a,g,h,Filling→Axis,Frame→True]*)
```

Out[150]=



In[4]:=
```
anz = 200; a = g = h = Table[0, {i, anz}]; werte = Table[0, {i, anz}, {j, 7}];
Table[{a[[i]] = werte[[i, 1]] = (Prime[i + 1] + Prime[i]) / 2;
    g[[i]] = werte[[i, 2]] = √(Prime[i + 1] * Prime[i]) ;
    h[[i]] = werte[[i, 3]] = 2 * Prime[i + 1] * Prime[i] / (Prime[i + 1] + Prime[i]);
    werte[[i, 4]] = werte[[i, 1]] - werte[[i, 2]];
    werte[[i, 5]] = werte[[i, 2]] - werte[[i, 3]];
    werte[[i, 6]] = (werte[[i, 1]] - werte[[i, 2]]) / (werte[[i, 2]] - werte[[i, 3]]);
    werte[[i, 7]] = (werte[[i, 1]] - werte[[i, 2]]) / (werte[[i, 2]] - werte[[i, 3]]) - 1;
  }, {i, 1, anz}];
(*Print[Table[a[[i]],{i,1,10}]];
Print[Table[g[[i]],{i,1,10}]];Print[Table[h[[i]],{i,1,10}]];
abild=ListPlot[a,Filling→Axis,PlotLegends→{"a"},PlotStyle→{Brown}]
   gbild=ListPlot[g,Filling→Axis,PlotLegends→{"g"},PlotStyle→{Orange}]
    hbild=ListPlot[h,Filling→Axis,PlotLegends→{"h"},PlotStyle→{Cyan}]
    (*Show[{abild,gbild,hbild}]*)
    ListPlot[Table[Table[werte[[i,j]],{i,anz}],{j,6}],
     Filling→Axis,PlotLegends→{"a","g","h","a-g","g-h","(a-g)/(g-h)"},
     PlotStyle→{Brown,Orange,Cyan,Green,Red,Blue}]
    ListPlot[Table[Table[werte[[i,j]],{i,anz}],{j,4,6}],Filling→Axis,
     PlotLegends→{"a-g","g-h","(a-g)/(g-h)"},PlotStyle→{Green,Red,Blue}]*)
ListPlot[Table[Table[werte[[i, j]], {i, anz}], {j, 7, 7}], Filling → Axis,
 PlotLegends → {"(a-g)/(g-h)-1"}, PlotStyle → {Red}, Frame → True]
(*ListPlot[a,g,h,Filling→Axis,Frame→True]*)
```

Out[6]=



• (a−g)/(g−h)−1

Nachtrag

Arithmetisches, geometrisches und harmonisches Mittel

$$a(x,y)=\frac{x+y}{2}\geq g(x,y)=\sqrt{xy}\geq h(x,y)=\frac{2xy}{x+y}=\frac{(g(x,y))^2}{a(x,y)}$$

$$\left.\begin{matrix}a(x,y)\\g(x,y)\\h(x,y)\end{matrix}\right\}\text{ ist Lösung } z \text{ der Gleichung } \frac{z-x}{y-z}=\left\{\begin{matrix}\frac{x}{x}\\\frac{x}{z}\\\frac{x}{y}\end{matrix}\right..$$

$$\frac{a-g}{g-h}=\frac{a^2-ag}{ag-g^2}=\frac{\begin{vmatrix}a & a\\g & a\end{vmatrix}}{\begin{vmatrix}a & g\\g & g\end{vmatrix}}=\frac{a}{g}$$