Unit 7, special 1
Continued fraction to common fraction, difference quotient, linear convergence accelerator

$$a_0 + \cfrac{b_1}{a_1 + \cfrac{b_2}{a_2 + \cfrac{b_3}{a_3 + \ddots + \cfrac{b_n}{a_n}}}} = a_0 + \frac{b_1|}{|a_1} + \frac{b_2|}{|a_2} + \frac{b_3|}{|a_3} + .. + \frac{b_n|}{|a_n} \Rightarrow \frac{A(n)}{B(n)}$$

$$a_i, b_i, A(n), B(n) \in \mathrm{N}$$

2 by 2 matrix chain product representation

$$\begin{bmatrix} \cdot & B(n) \\ \cdot & A(n) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & a_0 \end{bmatrix} \begin{bmatrix} 0 & b_1 \\ 1 & a_1 \end{bmatrix} \begin{bmatrix} 0 & b_2 \\ 1 & a_2 \end{bmatrix} \begin{bmatrix} 0 & b_3 \\ 1 & a_3 \end{bmatrix} .. \begin{bmatrix} 0 & b_n \\ 1 & a_n \end{bmatrix}$$

$$\begin{bmatrix} A(n) & B(n) \\ \cdot & \cdot \end{bmatrix} = \begin{bmatrix} a_n & b_n \\ 1 & 0 \end{bmatrix} .. \begin{bmatrix} a_3 & b_3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_2 & b_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 & 1 \\ 1 & 0 \end{bmatrix}$$

Classical determinant notation:

$$\begin{array}{|cccccc|}
a_0 & b_1 & 0 & 0 & 0 & 0 \\
-1 & a_1 & b_2 & 0 & 0 & 0 \\
0 & -1 & a_2 & b_3 & 0 & 0 \\
0 & 0 & -1 & a_3 & \ddots & 0 \\
0 & 0 & 0 & -1 & \ddots & b_n \\
0 & 0 & 0 & 0 & -1 & a_n
\end{array}
=
\begin{array}{|cccccc|}
a_n & b_n & 0 & 0 & 0 & 0 \\
-1 & a_{n-1} & b_{n-1} & 0 & 0 & 0 \\
0 & -1 & \ddots & \ddots & 0 & 0 \\
0 & 0 & -1 & a_2 & b_2 & 0 \\
0 & 0 & 0 & -1 & a_1 & b_1 \\
0 & 0 & 0 & 0 & -1 & a_0
\end{array}
\qquad (14.1.2$$

Cartesian dynamic determinant notation over (R; +, ·, 0, 1),
only column [1] and [2] navigation

	6	0 → a	0 → b	0	b_4	a_4	0
	5	0	0	b_3	a_3	0	0
	4	0	b_2	a_2	0	0	0
↑f=							
only	3	b_1	a_1	0	0	0	0
col[1]							
col[2]	2	a_0	0	0	0	0	0
	1	0	0	0	0	0	0
	•	1	2	3	4	5	6

n = 4

Example 2

$$1+\frac{1|}{|1}+\frac{1|}{|6}+\frac{1|}{|8} \Rightarrow \frac{A(3)}{B(3)}=\frac{106}{57}$$

$$\left[\begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 1 \\ \hline \end{array}\right]\left[\begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 1 \\ \hline \end{array}\right]\left[\begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 6 \\ \hline \end{array}\right]\left[\begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 8 \\ \hline \end{array}\right]=\left[\begin{array}{|c|c|}\hline 7 & B(3)=57 \\ \hline 13 & A(3)=106 \\ \hline \end{array}\right]$$

$$\left[\begin{array}{c|c} 8 & 1 \\ \hline 1 & 0 \end{array}\right]\left[\begin{array}{c|c} 6 & 1 \\ \hline 1 & 0 \end{array}\right]\left[\begin{array}{c|c} 1 & 1 \\ \hline 1 & 0 \end{array}\right]\left[\begin{array}{c|c} 1 & 1 \\ \hline 1 & 0 \end{array}\right]=\left[\begin{array}{c|c} A(3)=106 & B(3)=57 \\ \hline 13 & 7 \end{array}\right]=(\underline{m}^{-1})^{T}$$

Example 3 e – experiment, **2 by 2 matrix tuning**

$$e=2+\frac{1|}{|1}+\frac{1|}{|2}+\frac{1|}{|1}+\frac{1|}{|1}+\frac{1|}{|4}+\frac{1|}{|1}+\frac{1|}{|1}+\frac{1|}{|6}+\frac{1|}{|1}+\frac{1|}{|1}+\frac{1|}{|8}+\frac{1|}{|1}+..$$

$$\left[\begin{array}{c|c} & B(\infty) \\ \hline & A(\infty) \end{array}\right]=\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 2 \end{array}\right]\underbrace{\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 1 \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 2 \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 1 \end{array}\right]}_{\downarrow}\underbrace{\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 1 \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 4 \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 1 \end{array}\right]}_{\downarrow}\ldots$$

$$=\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 2 \end{array}\right]\qquad\left[\begin{array}{c|c} 2 & 3 \\ \hline 3 & 4 \end{array}\right]\qquad\left[\begin{array}{c|c} 4 & 5 \\ \hline 5 & 6 \end{array}\right]$$

$$=\prod_{i=0}^{\infty}\left[\begin{array}{c|c} 2i & 2i+1 \\ \hline 2i+1 & 2i+2 \end{array}\right]$$

Example 4 How strange!

$$\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+..=\frac{3-e}{e-2}$$

$$\left[\begin{array}{c|c} & B(\infty) \\ \hline & A(\infty) \end{array}\right]=\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 2 \end{array}\right]\left[\begin{array}{c|c} 0 & 2 \\ \hline 1 & 3 \end{array}\right]\left[\begin{array}{c|c} 0 & 3 \\ \hline 1 & 4 \end{array}\right]\left[\begin{array}{c|c} 0 & 4 \\ \hline 1 & 5 \end{array}\right]..$$

$$\frac{2|}{|1}+\frac{3|}{|2}+\frac{4|}{|3}+\frac{5|}{|4}+\frac{6|}{|5}+\frac{7|}{|6}+..=1\ !$$

$$\left[\begin{array}{c|c} & B(\infty) \\ \hline & A(\infty) \end{array}\right]=\left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]\left[\begin{array}{c|c} 0 & 2 \\ \hline 1 & 1 \end{array}\right]\left[\begin{array}{c|c} 0 & 3 \\ \hline 1 & 2 \end{array}\right]\left[\begin{array}{c|c} 0 & 4 \\ \hline 1 & 3 \end{array}\right]\left[\begin{array}{c|c} 0 & 5 \\ \hline 1 & 4 \end{array}\right]..$$

Example 5

$$e^{k}\Rightarrow\left[\begin{array}{c|c} & B(\infty) \\ \hline & A(\infty) \end{array}\right]=\prod_{i=0}^{\infty}\left[\begin{array}{c|c} 2i+1-k & 2i+1 \\ \hline 2i+1 & 2i+1+k \end{array}\right]$$

$$\sqrt[k]{e} \Rightarrow \left[\begin{array}{c|c} & B(\infty) \\ \hline & A(\infty) \end{array}\right] = \prod_{i=0}^{\infty} \left[\begin{array}{c|c} k(2i+1)-1 & k(2i+1) \\ \hline k(2i+1) & k(2i+1)+1 \end{array}\right]$$

Find similar compact 2by2 matrix chain product and determinant representations for the difference quotient

$$\mathrm{d}\frac{A}{B}(n) = \frac{\frac{A}{B}(n) - \frac{A}{B}(n+1)}{\frac{A}{B}(n+1) - \frac{A}{B}(n+2)}$$

of $\frac{A}{B}(n)$.

And then, in the case of linear convergence (see Deb 203, p. 7) for the convergence accelerator

$$\frac{A}{B}acc(n) = \frac{\mathrm{d}}{\mathrm{d}-1}\frac{A}{B}(n+1) - \frac{1}{\mathrm{d}-1}\frac{A}{B}(n) \; ;$$

$$\mathrm{d} = \lim_{\mathrm{n}\to\infty} \mathrm{d}\frac{A}{B}(n) \; .$$

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