Unit n°78

gcd(a,b) → gd(a)

You know the sound of two hands clapping;
tell me, what it the sound of one hand?
Hakuin Ekaku (1686 - 1768)

You know the Euclidean Algorithm

terminating way from (a,b) to the greatest common divisor of a and b, $\gcd(a,b)$, step by step without factorization into primes, reduction from large to small numbers.

$(u,v) := (a,b);\ a,b \in \mathbb{N}$	
repeat	$(u,v) := (v, u(\mathrm{mod}\, v))$
until $v = 0$ is true	
$\gcd(a,b) := u$	

Example

$$(1272,684)\to(684,588)\to(588,96)\to(96,12)\to(12,0)$$
$$\uparrow\ \uparrow \qquad\qquad \downarrow\ \downarrow$$
$$a\quad b \qquad\qquad \gcd(a,b)\quad 0\to stop$$

Find a similar algorithm for calculation of the greatest divisor of $a\in\mathbb{N}$, $\mathrm{gd}(a)$, step by step without factorization into primes.

Lateral

$$\gcd(a,b)=\gcd(b,a) \qquad comm.\ law$$
$$\gcd(a,b,c)=\gcd(\gcd(a,b),c)=\gcd(a,\gcd(b,c)) \qquad assoc.\ law$$

Extend the Euclidean Algorithm for the compact calculation of $\gcd(a,b,c)$ (not via *assoc. law*).

References

[1] https://en.wikipedia.org/wiki/Hakuin_Ekaku
[2] https://en.wikipedia.org/wiki/Euclidean_algorithm