Unit 75.1
e hoch x und co. DK, red problem, sandbox
Basis

$$\frac{1}{e^x} = \begin{vmatrix} x^0 & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \frac{1}{6!} & \dashrightarrow \\ x^1 & \frac{1}{0!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \dashrightarrow \\ x^2 & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \dashrightarrow \\ x^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \dashrightarrow \\ x^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{1!} & \frac{1}{2!} & \dashrightarrow \\ x^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{1!} & \dashrightarrow \\ x^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix} \quad (1.1)$$

$$= \sum_{i=0}^{\infty} \frac{-1^i}{i!} x^i \quad (1.2)$$

$$= [1; -\frac{x+1}{x}, 1,1, -\frac{x+3}{x}, 1,1, -\frac{x+5}{x}, 1,1, -\frac{x+7}{x}, 1,1, \dots] \quad (1.3)$$

$$= [1; -\frac{1}{x}, -2, \frac{3}{x}, 4, -\frac{5}{x}, -6, \frac{7}{x}, 8, -\frac{9}{x}, -10, \dots] \quad (1.4)$$

Aus typographischen Gründen ist in vorstehender und

in folgenden Determinanten das Element 0 durch $\frac{0}{1!}$,

das Element 1 durch $0! = \frac{1}{0!} = \frac{2^0}{0!} = x^0 = \ldots$dargestellt.

$$\frac{x}{e^x-1} = \begin{vmatrix} x^0 & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \frac{1}{6!} & \frac{1}{7!} & \cdots\rightarrow \\ x^1 & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \frac{1}{6!} & \cdots\rightarrow \\ x^2 & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \cdots\rightarrow \\ x^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \cdots\rightarrow \\ x^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \cdots\rightarrow \\ x^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \cdots\rightarrow \\ x^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix} \qquad (2.1)$$

$$= \sum_{i=0}^{\infty} \frac{B_i}{i!} x^i \qquad (2.2)$$

B_i - Bernoullische Zahl [1]

$$B_0 = 1, B_1 = -\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0, B_4 = -\frac{1}{30},$$
$$B_5 = 0, B_6 = \frac{1}{42}, B_7 = 0, B_8 = -\frac{1}{30}, B_9 = 0, \ldots$$

(2.3)

[1] Weisstein, Eric W. "Bernoulli Number." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/BernoulliNumber.html

$$\tanh x = \frac{e^{2x}-1}{e^{2x}+1} = -\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ x^1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ x^3 & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ x^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ x^7 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ x^9 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ x^{11} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= \sum_{i=0}^{\infty} 2^{2i}(2^{2i}-1)\frac{B_{2i}}{(2i)!}x^{2i-1}$$

$$= \cfrac{x}{1+\cfrac{x^2}{3+\cfrac{x^2}{5+\cfrac{x^2}{7+\cfrac{x^2}{9+\cfrac{x^2}{11+\cdots}}}}}}$$

$$= [0; \frac{1}{x}, \frac{3}{x}, \frac{5}{x}, \frac{7}{x}, \frac{9}{x}, \frac{11}{x}, \ldots] \qquad (3.1\text{-}4)$$

$$\tanh x = -j\tanh jx = \begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ -x^1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ x^3 & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ -x^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ x^7 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ -x^9 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ x^{11} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= \sum_{i=0}^{\infty}(-1)^{i-1}2^{2i}(2^{2i}-1)\frac{B_{2i}}{(2i)!}x^{2i-1}$$

$$= \cfrac{x}{1-\cfrac{x^2}{3-\cfrac{x^2}{5-\cfrac{x^2}{7-\cfrac{x^2}{9-\cfrac{x^2}{11-\cdots}}}}}}$$

$$= [0; \frac{1}{x}, \frac{-3}{x}, \frac{5}{x}, \frac{-7}{x}, \frac{9}{x}, \frac{-11}{x}, \ldots] \qquad (4.1\text{-}4)$$

$$\tanh^2 x = \frac{\frac{1}{2}(1-\cosh 2x)}{\frac{1}{2}(1+\cosh 2x)}$$

$$= -\begin{vmatrix} 0 & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \frac{2^7}{8!} & \frac{2^9}{10!} & \frac{2^{11}}{12!} & \dashrightarrow \\ x^2 & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \frac{2^7}{8!} & \frac{2^9}{10!} & \dashrightarrow \\ x^4 & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \frac{2^7}{8!} & \dashrightarrow \\ x^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \dashrightarrow \\ x^8 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \dashrightarrow \\ x^{10} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \dashrightarrow \\ x^{12} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix} \quad (5)$$

$$\tan^2 x = \frac{\frac{1}{2}(1-\cos 2x)}{\frac{1}{2}(1+\cos 2x)}$$

$$= \begin{vmatrix} 0 & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \frac{2^7}{8!} & \frac{2^9}{10!} & \frac{2^{11}}{12!} & \dashrightarrow \\ -x^2 & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \frac{2^7}{8!} & \frac{2^9}{10!} & \dashrightarrow \\ x^4 & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \frac{2^7}{8!} & \dashrightarrow \\ -x^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \frac{2^5}{6!} & \dashrightarrow \\ x^8 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \frac{2^3}{4!} & \dashrightarrow \\ -x^{10} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \frac{2^1}{2!} & \dashrightarrow \\ x^{12} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2^0}{0!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix} \tag{6}$$

$$\frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \begin{vmatrix} x^0 & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \frac{1}{12!} & \dashrightarrow \\ x^2 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \dashrightarrow \\ x^4 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \dashrightarrow \\ x^6 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \dashrightarrow \\ x^8 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \dashrightarrow \\ x^{10} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \dashrightarrow \\ x^{12} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= \sum_{i=0}^{\infty} \frac{E_{2i}}{(2i)!} x^{2i} \tag{7.1,2}$$

E_i - Eulersche Zahl [2]

$$E_0 = 1, E_1 = 0\,, E_2 = -1\,, E_3 = 0\,, E_4 = 5\,,$$
$$E_5 = 0\,, E_6 = -61\,, E_7 = 0\,, E_8 = 1385\,, E_9 = 0\,, \ldots$$

[2] Weisstein, Eric W. "Euler Number." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/EulerNumber.html

$$\frac{I_1(2x)}{I_0(2x)} = -\begin{vmatrix}
0 & \frac{1}{0!1!} & \frac{1}{1!2!} & \frac{1}{2!3!} & \frac{1}{3!4!} & \frac{1}{4!5!} & \frac{1}{5!6!} & \cdots\to \\
x^1 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \frac{1}{5!5!} & \cdots\to \\
x^3 & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \cdots\to \\
x^5 & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \cdots\to \\
x^7 & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \cdots\to \\
x^9 & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \cdots\to \\
x^{11} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \cdots\to \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow
\end{vmatrix}$$

$$= \cfrac{x}{1+\cfrac{x^2}{2+\cfrac{x^2}{3+\cfrac{x^2}{4+\cfrac{x^2}{5+\cfrac{x^2}{6+\cfrac{x^2}{7+\cfrac{x^2}{8+\cdots}}}}}}}}$$

$$= \left[0, \frac{1}{x}, \frac{2}{x}, \frac{3}{x}, \frac{4}{x}, \frac{5}{x}, \frac{6}{x}, \frac{7}{x}, \frac{8}{x}, \ldots\right] \qquad (8.1\text{-}3)$$

$$\frac{J_1(2x)}{J_0(2x)} = \begin{vmatrix} 0 & \frac{1}{0!1!} & \frac{1}{1!2!} & \frac{1}{2!3!} & \frac{1}{3!4!} & \frac{1}{4!5!} & \frac{1}{5!6!} & \cdots\to \\ -x^1 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \frac{1}{5!5!} & \cdots\to \\ x^3 & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \cdots\to \\ -x^5 & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \cdots\to \\ x^7 & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \cdots\to \\ -x^9 & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \frac{1}{1!1!} & \cdots\to \\ x^{11} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{0}{1!1!} & \frac{1}{0!0!} & \cdots\to \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= \cfrac{x}{1-\cfrac{x^2}{2-\cfrac{x^2}{3-\cfrac{x^2}{4-\cfrac{x^2}{5-\cfrac{x^2}{6-\cfrac{x^2}{7-\cfrac{x^2}{8+\cdots}}}}}}}}$$

$$= \left[0, \frac{1}{x}, \frac{-2}{x}, \frac{3}{x}, \frac{-4}{x}, \frac{5}{x}, \frac{-6}{x}, \frac{7}{x}, \frac{-8}{x}, \ldots\right] \tag{9.1-3}$$

[3] http://mathworld.wolfram.com/BesselFunctionoftheFirstKind.html
[4] http://mathworld.wolfram.com/ModifiedBesselFunctionoftheFirstKind.html
[5] https://keisan.casio.com/exec/system/1180573470

$$\frac{1}{\ln(1+x)} = \begin{vmatrix} x^{-1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \dashrightarrow \\ -x^{0} & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \dashrightarrow \\ x^{1} & \frac{0}{1} & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \dashrightarrow \\ -x^{2} & \frac{0}{1} & \frac{0}{1} & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \dashrightarrow \\ x^{3} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \dashrightarrow \\ -x^{4} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{1} & \frac{1}{2} & \dashrightarrow \\ x^{5} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{1} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= \left[\frac{1}{x}; \frac{2}{1}, \frac{3}{x}, \frac{2}{2}, \frac{5}{x}, \frac{2}{3}, \frac{7}{x}, \frac{2}{4}, \frac{9}{x}, \frac{2}{5}, \dots\right] \quad (10.1,2)$$

Zum Vergleich

$$\frac{1}{x}\left(\sqrt{x^2+1}-1\right) = \left[0; \frac{2}{x}, \frac{2}{x}, \frac{2}{x}, \frac{2}{x}, \frac{2}{x}, \frac{2}{x}, \dots\right] \quad (11.1)$$

$$\sqrt{x+1} = 1 + \cfrac{x}{2 + \cfrac{x}{2 + \cfrac{x}{2 + \cfrac{x}{2 + \dots}}}} \quad (11.2)$$

$$\#?(x) = 1 + \cfrac{x}{2 - \cfrac{x}{2 + \cfrac{x}{2 - \cfrac{x}{2 + \cfrac{x}{2 - \cfrac{x}{2 + \cfrac{x}{2 - \cfrac{x}{2+\cdots}}}}}}}} \quad (11.3)$$

$$\#\#?(x) = 1 + \cfrac{2}{x + \cfrac{2}{x + \cfrac{2}{x + \cfrac{2}{x + \ldots}}}} \qquad (11.4)$$

Problem

**** Welche endlichen Formelausdrücke (vgl. linke Seite der Gln (1) bis (9)) werden durch folgende infinite Determinanten dargestellt?**

$$?(x) = \begin{vmatrix} 0!\,x^0 & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \frac{1}{6!} & \frac{1}{7!} & \cdots\rightarrow \\ 1!\,x^1 & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \frac{1}{6!} & \cdots\rightarrow \\ 2!\,x^2 & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \frac{1}{5!} & \cdots\rightarrow \\ 3!\,x^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \frac{1}{4!} & \cdots\rightarrow \\ 4!\,x^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \cdots\rightarrow \\ 5!\,x^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \frac{1}{2!} & \cdots\rightarrow \\ 6!\,x^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{1!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix} \qquad (12.1)$$

$$= \sum_{i=0}^{\infty} B_i\, x^i \qquad (12.2)$$

$$??(x) = \begin{vmatrix} x^0 & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \frac{2!}{8!} & \dashrightarrow \\ x^1 & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \dashrightarrow \\ x^2 & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \dashrightarrow \\ x^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \dashrightarrow \\ x^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \dashrightarrow \\ x^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \dashrightarrow \\ x^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix} \qquad (14.1)$$

$$= \sum_{i=0}^{\infty} \frac{C_i}{i!} x^i \qquad (14.2)$$

$$C_0 = 1, C_1 = -\frac{1}{3}, C_2 = \frac{1}{18}, C_3 = \frac{1}{90},$$

$$C_4 = -\frac{1}{270}, \ldots \qquad (14.3)$$

** **Geben Sie für ?(x) und ??(x) Graphen an und Führen Sie zugehörige Kurvendiskussionen durch.**

** **Stellen Sie ?(x) und ??(x) durch infinite einfache und verzweigte Kettenbrüche dar..**

** Diskutieren Sie die Dezimalwerte von ?(x) und ??(x) für sehr kleine x, z.B. $x=10^{-6}$.

$?(10^{-6}) = 0.9999995000001666666 7\ldots$

$??(10^{-6}) = 0.9999996666666944444 5\ldots$

Lösungsschritte für ??(x)

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

$$??(1) = \begin{vmatrix} 1^0 & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \frac{2!}{8!} & \dashrightarrow \\ 1^1 & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \dashrightarrow \\ 1^2 & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \dashrightarrow \\ 1^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \dashrightarrow \\ 1^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \dashrightarrow \\ 1^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \dashrightarrow \\ 1^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= 0.6961055955886664071882764392399082641869891926576435617956622835416397852308054633458552936338064994\ldots$$

$$= \frac{1}{2}[1;\mathbf{2},1,1,\mathbf{4},1,1,\mathbf{6},1,1,\mathbf{8},1,1,\mathbf{10},1,1,\mathbf{12},1,1,\ldots$$

$$= [0;1,2,3,\mathbf{2},3,1,\mathbf{2},1,3,\mathbf{4},3,1,\mathbf{4},1,3,\mathbf{6},3,1,\mathbf{6},1,\ldots]$$

$$= \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{2}{4+\cfrac{3}{5+\cfrac{4}{6+\cfrac{5}{7+\cfrac{6}{8+\cdots}}}}}}}}$$

$$= \frac{1}{2(e-2)} \qquad (15.1\text{-}6)$$

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$$??(2) = \begin{vmatrix} 2^0 & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \frac{2!}{8!} & \dashrightarrow \\ 2^1 & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \dashrightarrow \\ 2^2 & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \dashrightarrow \\ 2^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \dashrightarrow \\ 2^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \dashrightarrow \\ 2^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \dashrightarrow \\ 2^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$= 0.45567884185560536945385445910261039347454921847785111458486869662560545791603936108743886034868549 56$

$= [0; 2,5,7,9,11,13,15,17,19,21,23,25,27,\ldots]$

$$= \frac{2}{e^2 - 3} \qquad (16.1\text{-}4)$$

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$$??(-1) = \frac{e}{2}\,,\; ??(-2) = \frac{2}{1+e^{-2}} \qquad (17.1,2)$$

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Versuchen Sie, basierend auf den Gln (1) bis (8) und dem nachfolgenden Sandkasten nun aus den Gln (14) bis (17) einen allgemeinen endlichen Formelausdruck für ??(x) zu extrahieren.

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Im Umfeld

$$\tanh 1 = \frac{e^2-1}{e^2+1} = -\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ 1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= [0;1,3,5,7,9,11,13,15,17,19,21,23,25,27,\ldots]$$

(18)

$$\tan 1 = \begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ -1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ -1 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ -1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= [1;1,1,3,1,5,1,7,1,9,1,11,1,13,1,\ldots]$$
$$= [0;1,-3,5,-7,9,-11,13,-15,-17,19,\ldots] \qquad (19)$$

$$\tanh\frac{1}{2} = \frac{e-1}{e+1} = [0;2,6,10,14,18,22,26,30,34,\ldots]$$

$$= \frac{1}{2}[0;1,12,5,28,9,44,13,60,17,\ldots] \qquad (20)$$

$$\tan\frac{1}{2} = [0;1,1,4,1,8,1,12,1,16,1,20.1,24,1,\ldots] \qquad (21)$$

$$\tanh\frac{1}{4} = \frac{\sqrt{e}-1}{\sqrt{e}+1} = [0;4,12,20,28,36,44,52,60,68,\ldots] \qquad (22)$$

$$\tan\frac{1}{4} = [0;3,1,10,1,18,1,26,1,34,1,42.1,50,1,\ldots] \qquad (23)$$

$$\tanh x = \frac{e^{2x}-1}{e^{2x}+1} = -\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ x^1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ x^3 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ x^5 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ x^7 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ x^9 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ x^{11} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \searrow \end{vmatrix}$$

$$= \cfrac{x}{1+\cfrac{x^2}{3+\cfrac{x^2}{5+\cfrac{x^2}{7+\cfrac{x^2}{9+\cfrac{x^2}{11+\cdots}}}}}}$$

$$= [0; \frac{1}{x}, \frac{3}{x}, \frac{5}{x}, \frac{7}{x}, \frac{9}{x}, \frac{11}{x}, \ldots] \qquad (24.1\text{-}3)$$

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Sandkasten.ak.er

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$\frac{1}{e} = [0;2,1,\mathbf{2},1,1,\mathbf{4},1,1,\mathbf{6},1,1,\mathbf{8},1,1,\mathbf{10},1,\ldots]$

$$= \cfrac{1}{2+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cdots}}}}}}}} \qquad (25.1,2)$$

$\frac{1}{2e} = [0;5,2,3,\mathbf{2},3,1,\mathbf{2},1,3,\mathbf{4},3,1,\mathbf{4},1,3,\mathbf{6},3,1,\mathbf{6},1,3,\ldots]$

(26)

$\frac{2e-5}{6-2e} = [0; 1,3, \mathbf{2}, 3,1, \mathbf{2}, 1,3, \mathbf{4}, 3,1, \mathbf{4}, 1,3, \mathbf{6}, 3,1, \mathbf{6}, 1,3, \ldots.]$

(27)

$\frac{2}{e} = [0;1,2,\mathbf{1,3,1,1,1,3,3,3,1,3,1,3},5,\ldots]$

$= [0;\ 1,\ 2,\ \mathbf{1},\ 3,\ 1,\ \mathbf{1},\ 1,\ 3,\ \mathbf{3},\ 3,\ 1,\ \mathbf{3},\ 1,\ 3,\ \mathbf{5},\ 3,\ 1,\ \mathbf{5},\ 1,\ 3,\ \mathbf{7},\ 3,\ 1,\ \mathbf{7},\ 1,\ 3,\ \mathbf{9},\ 3,\ 1,$
$\mathbf{9},\ 1,\ 3,\ \mathbf{11},\ 3,\ 1,\ \mathbf{11},\ 1,\ 3,\ \mathbf{13},\ 3,\ 1,\ \mathbf{13},\ 1,\ 3,\ \mathbf{15},\ 3,\ 1,\ \mathbf{15},\ 1,\ 3,\ \mathbf{17},\ 3,\ 1,$
$\mathbf{17},\ 1,\ 3,\ \mathbf{19},\ 3,\ 1,\ \mathbf{19},\ 1,\ 3,\ \mathbf{21},\ 3,\ 1,\ \mathbf{21},\ 1,\ 3,\ \mathbf{23},\ 3,\ 1,\ \mathbf{23},\ 1,\ 3,\ \mathbf{25},\ 3,\ 1,$
$\mathbf{25},\ 1,\ 3,\ \mathbf{27},\ 3,\ 1,\ \mathbf{27},\ 1,\ 3,\ \mathbf{29},\ 3,\ 1,\ \mathbf{29},\ 1,\ 3,\ \mathbf{31},\ 3,\ 1,\ \mathbf{31},\ 1,\ 3,\ \mathbf{33},\ \ldots]$

(28)

$\frac{1}{e-1} = [0;1,1,\mathbf{2},1,1,\mathbf{4},1,1,\mathbf{6},1,1,\mathbf{8},1,1,\mathbf{10},1,\ldots]$

$$= \frac{1}{2}[1;6,10,14,18,22,26,30,\ldots]$$

$$= \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cdots}}}}}}}} \qquad (29.1\text{-}3)$$

$$\frac{3-e}{e-2} = [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,\ldots]$$

$$= 2[0;5,10,14,18,22,26,30,34,\ldots]$$

$$= \frac{1}{2}[0,\mathbf{1,3,1,1,1,3,3,3,1,3,1,3},5,\ldots?]$$

$$= \frac{1}{2}[\mathbf{0; 1, 3, 1, 1, 1, 3, 3, 3, 1, 3, 1, 3,} 5, 3, 1, 5, 1, 3, 7, 3, 1, 7, 1, 3, 9, 3, 1, 9, 1, 3, 11,$$
3, 1, 11, 1, 3, 13, 3, 1, 13, 1, 3, 15, 3, 1, 15, 1, 3, 17, 3, 1, 17, 1, 3, 19, 3, 1, 19, 1, 3, 21,
3, 1, 21, 1, 3, 23, 3, 1, 23, 1, 3, 25, 3, 1, 25, 1, 3, 27, 3, 1, 27, 1, 3, 29, 3, 1, 29, 1, 3, 31,
3, 1, 31, 1, 3, 33, 3, 1, …]DS

$$= \cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cdots}}}}}} \qquad (30.1\text{-}4)$$

$$\frac{1}{e-2} = [1;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,\ldots]$$

$$= 2[0;1,2,3,\mathbf{2},3,1,\mathbf{2},1.3,\mathbf{4},3,1,\mathbf{4},1,3,\ldots]$$

$$= \frac{1}{2}[2,\mathbf{1,3,1,1,1,3,3,3,1,3,1,3},5,\ldots?]$$

$$= \frac{1}{2}[2; \mathbf{1, 3, 1, 1, 1, 3, 3, 3, 1, 3, 1, 3,} 5, 3, 1, 5, 1, 3, 7, 3, 1, 7,$$
1, 3, 9, 3, 1, 9, 1, 3, 11, 3, 1, 11, 1, 3, 13, 3, 1, 13, 1, 3, 15,
3, 1, 15, 1, 3, 17, 3, 1, 17, 1, 3, 19, 3, 1, 19, 1, 3, 21, 3, 1, 21,
1, 3, 23, 3, 1, 23, 1, 3, 25, 3, 1, 25, 1, 3, 27, 3, 1, 27, 1, 3, 29,
3, 1, 29, 1, 3, 31, 3, 1, 31, 1, 3, 33, 3, 1, …]DS

$$= 1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cdots}}}}}} \qquad (31.1\text{-}5)$$

Frage: Ist die Teilsequenz „**1,3,1,1,1,3,3,3,1,3,1,3**“ in einem scf ein „e-Marker“ (vgl. „Bernoulli-Marker“ 691)? Finden Sie weitere Beispiele. Diskussion!

$$\frac{1}{\sqrt{e}-1} = [1;1,1,\mathbf{5},1,1,\mathbf{9},1,1,\mathbf{13},1,1,\mathbf{17},1,1,\mathbf{21},1,1,\ldots]$$

$$= \frac{1}{2}[3;12,20,28,36,44,52,60,\ldots]$$

$$= 2[\mathbf{0};1,3,\mathbf{2},1,3,\mathbf{4},1,3,\mathbf{6},1,3,\mathbf{8},1,3,\mathbf{10},\ldots]$$

$$= 1+\cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cfrac{12}{13+\cfrac{14}{15+\cfrac{16}{17+\cdots}}}}}}}} \qquad (32.1\text{-}4)$$

$$\frac{2-\sqrt{e}}{\sqrt{e}-1} = [0;1,1,5,1,9,1,13,1,1,17,1,1,21,1,1,\ldots]$$

$$= \frac{1}{2}[1;12,20,28,36,44,52,60,\ldots]$$

$$= 2[\mathbf{0};3,1,\mathbf{2},3,1,\mathbf{4},3,1,\mathbf{6},3,1,\mathbf{8},3,1,\mathbf{10},\ldots]$$

$$= \cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cfrac{12}{13+\cdots}}}}}} \qquad (33.1\text{-}4)$$

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$$\tanh 1 = \frac{e^2-1}{e^2+1} = -\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ 1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= [0;1,3,5,7,9,11,13,15,17,19,21,23,25,27,\ldots]$$

$$= \frac{1}{2}[1;1,1,10,3,1,1,4,22,6,1,1,7,34,9,1,1,\ldots] \quad (18)$$

$$\tan 1 = \begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ -1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ -1 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ -1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= [1;1,1,3,1,5,1,7,1,9,1,11,1,13,1,\ldots]$$

$$= [0;1,-3,5,-7,9,-11,13,-15,-17,19,\ldots] \qquad (19)$$

$$\tanh\frac{1}{2} = \frac{e-1}{e+1} = [0;2,6,10,14,18,22,26,30,34,\ldots]$$

$$= \frac{1}{2}[0;1,12,5,28,9,44,13,60,17,\ldots]$$

$$= 2[0;4,3,20,7,36,11,52,15,68,19,\ldots] \qquad (20)$$

$$\tan\frac{1}{2} = [0;1,1,4,1,8,1,12,1,16,1,20.1,24,1,\ldots] \qquad (21)$$

$$\tanh\frac{1}{4} = \frac{\sqrt{e}-1}{\sqrt{e}+1} = [0;4,12,20,28,36,44,52,60,68,\ldots] \qquad (22)$$

$$\tan\frac{1}{4} = [0;3,1,10,1,18,1,26,1,34,1,42.1,50,1,\ldots] \qquad (23)$$

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Verallgemeinerungen

$$\frac{1}{\sqrt[n]{e}-1} = [n\text{-}1;1,1,3n\text{-}1,1,1,5n\text{-}1,1,1,7n\text{-}1,1,1,\ldots]$$

$$= \frac{1}{2}[2n;6n,10n,14n,18n,22n,26n,30n,\ldots] \qquad (34)$$

$$\tanh\frac{1}{2n} = \frac{\sqrt[n]{e}+1}{\sqrt[n]{e}-1}$$

$$= [0;1,1,3n\text{-}1,1,1,5n\text{-}1,1,1,7n\text{-}1,1,1,\ldots]$$

$$= \frac{1}{2}[0;2n,6n,10n,14n,18n,22n,26n,30n,\ldots]$$

$$= 2\,[1;\, n,12n,5n,28n,9n,44n,13n,60n,\ldots] \qquad (35)$$

$$[0;1,1,a,1,1,a+b,1,1,a+2b,1,1,a+3b,1,1,a+4b,1,1,\ldots] = \frac{1}{2}[1;2(a+1),2(a+b+1),2(a+2b+1),2(a+3b+1),2(a+4b+1),\ldots]$$

(50.1)

$$f(n) = \cfrac{1}{1+\cfrac{n+1}{n+1+\cfrac{2n+1}{2n+1+\cfrac{3n+1}{3n+1+\cfrac{4n+1}{4n+1+\cfrac{5n+1}{5n+1+\cfrac{6n+1}{6n+1+\cdots}}}}}}}} =$$

$$[0;1,1,n+1,1,1,3n+1,1,1,5n+1,1,1,7n+1,1,1,\ldots] =$$

$$\frac{1}{2}[1;2n+4,6n+4,10n+4,14n+4,18n+4,\ldots] \qquad (50.2.1)$$

Untersuchen Sie den Verlauf von f(n): Graph, spezielle Werte.

$$f(0) = \frac{1}{2}(\sqrt{5}\,-1);\ f(1) = \frac{1}{e-1}\ ;\ f(\pm\infty) = \frac{1}{2}\ ;\ \ldots \qquad (50.2.2)$$

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Kettenbruchdarstellungen von 1 und 2

$$\cfrac{2}{1+\cfrac{4}{3+\cfrac{6}{5+\cfrac{8}{7+\cfrac{10}{9+\cfrac{12}{11+\cfrac{14}{13+\cfrac{16}{15+\cdots}}}}}}}} = \cfrac{3}{2+\cfrac{5}{4+\cfrac{7}{6+\cfrac{9}{8+\cfrac{11}{10+\cfrac{13}{12+\cfrac{15}{14+\cfrac{17}{16+\cdots}}}}}}}}$$

$$= \cfrac{2}{1+\cfrac{3}{2+\cfrac{4}{3+\cfrac{5}{4+\cfrac{6}{5+\cfrac{7}{6+\cfrac{8}{7+\cfrac{9}{8+\cdots}}}}}}}} = 1 \qquad (36)$$

$$\cfrac{+1}{-1+\cfrac{3}{1+\cfrac{5}{3+\cfrac{7}{5+\cfrac{9}{7+\cfrac{11}{9+\cdots}}}}}} + \cfrac{-1}{+1+\cfrac{1}{3+\cfrac{3}{5+\cfrac{5}{7+\cfrac{7}{9+\cfrac{9}{11+\cdots}}}}}} = 2 \qquad (37)$$

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$$??(1) = \begin{vmatrix} 1^0 & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \frac{2!}{8!} & \dashrightarrow \\ 1^1 & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \frac{2!}{7!} & \dashrightarrow \\ 1^2 & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \frac{2!}{6!} & \dashrightarrow \\ 1^3 & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \frac{2!}{5!} & \dashrightarrow \\ 1^4 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \frac{2!}{4!} & \dashrightarrow \\ 1^5 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \frac{2!}{3!} & \dashrightarrow \\ 1^6 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{2!}{2!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

= 0.69610559558866640718827643923990826418698919265764356179566228354163978523080546334585529363380649 94…

$$= \frac{1}{2}[1;\mathbf{2},1,1,\mathbf{4},1,1,\mathbf{6},1,1,\mathbf{8},1,1,\mathbf{10},1,1,\mathbf{12},1,1,\ldots$$

$$= [0;1,2,3,\mathbf{2},3,1,\mathbf{2},1,3,\mathbf{4},3,1,\mathbf{4},1,3,\mathbf{6},3,1,\mathbf{6},1,\ldots]$$

$$= \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{2}{4+\cfrac{3}{5+\cfrac{4}{6+\cfrac{5}{7+\cfrac{6}{8+\cdots}}}}}}}}$$

$$= \frac{1}{2(e-2)} \qquad (38.1\text{-}5)$$

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

$$??(2) = [0;2,5,7,9,11,13,15,17,19,21,23,25,27,\ldots]$$

= 0.45567884185560536945385445910261039347454921847785111458486 86

$$= \frac{2}{e^{2}-3} \qquad (39.1,2)$$

96625605457916039361087438860348685495 6

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Im Umfeld

$$\tanh 1 = \frac{e^{2}-1}{e^{2}+1} = -\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \dashrightarrow \\ 1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \dashrightarrow \\ 1 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \dashrightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \dashrightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \dashrightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \dashrightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \dashrightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= [0;1,3,5,7,9,11,13,15,17,19,21,23,25,27,\ldots]$$

$$\tan 1 = \begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \frac{1}{11!} & \cdots\rightarrow \\ -1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \frac{1}{10!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \cdots\rightarrow \\ -1 & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \cdots\rightarrow \\ 1 & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \cdots\rightarrow \\ -1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \frac{1}{2!} & \cdots\rightarrow \\ 1 & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{0}{1!} & \frac{1}{0!} & \cdots\rightarrow \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \searrow \end{vmatrix}$$

$$= [1;1,1,3,1,5,1,7,1,9,1,11,1,13,1,\ldots]$$
$$= [0;1,-3,5,-7,9,-11,13,-15,-17,19,\ldots]$$

$$\tanh\frac{1}{2} = \frac{e-1}{e+1} = [0;2,6,10,14,18,22,26,30,34,\ldots]$$

$$= \frac{1}{2}[0;1,12,5,28,9,44,13,60,17,\ldots]$$

$$\tan\frac{1}{2} = [0;1,1,4,1,8,1,12,1,16,1,20.1,24,1,\ldots]$$

$$\tanh\frac{1}{4} = \frac{\sqrt{e}-1}{\sqrt{e}+1} = [0;4,12,20,28,36,44,52,60,68,\ldots]$$

$$\tan\frac{1}{4} = [0;3,1,10,1,18,1,26,1,34,1,42.1,50,1,\ldots]$$

(18-23)

$$\frac{I_1(2)}{I_0(2)} = [0;1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots]$$

(8.4)

$$\frac{J_1(2)}{J_0(2)} = [2;1,1,2,1,3,1,4,1,5,1,6,1,7,1,8,1,9,1,10,1,11,1,\ldots]$$

(9.4)

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Verdoppeln, Halbieren, Vorzeichenwechsel, Eskortieren; predictive coding

Ideeneinzugsgebiete:

[6] M. Pauque. Universal Scaling. Teil 1: Eine Schallwelle strukturiert die Zahlen.
raum&zeit 219/2019, S. 62-67

[7] https://www.spektrum.de/news/sagt-unser-gehirn-die-zukunft-voraus/1613666?utm_source=pocket-newtab

[8] Th. Lin (Ed.). The Prime Number Conspiracy. A Collection of the Best Quanta Math Stories.
MIT Press 2018, ISBN-13: 978-0262536356

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Es e-t so e, wenn Kettenbrüche …

$$e = [1; 0, 1, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1,10, \ldots]$$

$$\frac{1}{e-2} = [1; 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1,10,1, 1,12, \ldots]$$

---

1)

$$\sqrt[2]{e} = [1; 1, 1, 1, 5, 1, 1, 9, 1, 1,13,1, 1,17,1, 1,21, \ldots]$$

↓ ↓ ↓ ↓ ↓ ↓

0+2-1 2+4-1 4+6-1 6+8-1 8+10-1 10+12-1

2)

$$\sqrt[3]{e} = [1; 2, 1, 1, 8, 1, 1,14, 1, 1,20,1, 1,26,1, 1,32, \ldots]$$

↓ ↓ ↓ ↓ ↓ ↓

2*0+2 2*2+4 2*4+6 2*6+8 2*8+10 2*10+12

3)

?? = [1; 1, 1, 1, 3, 1, 1, 5, 1, 1, 7, 1, 1, 9, 1, 1, 11, …]

↓ ↓ ↓ ↓ ↓ ↓

$$\frac{0+2}{2} \quad \frac{2+4}{2} \quad \frac{4+6}{2} \quad \frac{6+8}{2} \quad \frac{8+10}{2} \quad \frac{10+12}{2}$$

4)

$f(a,b) =$

$[0;1,1,a,1,1,a+b,1,1,a+2b,1,1,a+3b,1,1,a+4b,1,1,\ldots] =$

$\frac{1}{2}[1;2(a+1),2(a+b+1),2(a+2b+1),2(a+3b+1),2(a+4b+1),\ldots]$

5)

$f(5,4) = [0;1,1,5,1,1,9,1,1,13,1,1,17,1,1,21,1,1,\ldots]$

$= \frac{1}{2}[1;12,20,28,36,44,52,\ldots]$

$= 2[0;3,1,2,3,1,4,3,1,6,3,1,8,3,1,10,3,1,\ldots]$

$= \frac{2-\sqrt{e}}{\sqrt{e}-1}$

6)

$f(8,6) = [0;1,1,8,1,1,14,1,1,20,1,1,26,1,1,32,1,1,\ldots]$

$= \frac{1}{2}[1;18,30,42,54,66,78,\ldots]$

$= 2[0;3,1,3,1,3,7,3,1,11,1,3,15,3,1,19,1,3,\ldots]$

$= \frac{3-2\sqrt[3]{e}}{\sqrt[3]{e}-1}$

7)

$f(3,4) = [0;1,1,3,1,1,7,1,1,11,1,1,15,1,1\ldots]$

$= ???$

8)

$\sqrt[2k]{e} = [1;2k-1,1,1,6k-1,1,1,10k-1,1,1,14k-1,1,1,\ldots]$

$= \frac{1}{2}[2;???...]$

$= 2[0;???...]$

(40-49)

qqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqq

$$\frac{1}{2+\frac{2}{3+\frac{3}{4+\frac{4}{5+\frac{5}{6+\cdots}}}}} = \frac{3-e}{e-2}\ ;\ \frac{2}{1+\frac{3}{2+\frac{4}{3+\frac{5}{4+\frac{6}{5+\cdots}}}}} = 1 \quad (51.1,2)$$

$$\frac{2}{2+\frac{3}{3+\frac{4}{4+\frac{5}{5+\frac{6}{6+\cdots}}}}} = e - 2\ ;\ 1+\frac{1}{2+\frac{2}{3+\frac{3}{4+\frac{4}{5+\frac{5}{6+\cdots}}}}} = \frac{1}{e-2}$$

(52.1,2)

rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr

$$\frac{[0;(1,3)\mathrm{per}]}{[0:(3,1)\mathrm{per}]} = 3 \qquad (53)$$

$$\frac{[5;(1,1,1,10)\mathrm{per}]}{[\frac{5}{7};(10,1,1,1)\mathrm{per}]} = 7 \qquad (54)$$

ßßß
Triaden

$$\frac{1}{3+\frac{2}{4+\frac{3}{5+\frac{4}{6+\frac{5}{7+\frac{6}{8+\cdots}}}}}} =$$

[0;(3,2,3),(1,2,1),(3,4,3),(1,4,1),(3,6,3),(1,6,1),(3,8,3),(1,8,1),…]

$$= \frac{6-2e}{2e-5} \qquad (55.1\text{-}3)$$

$$\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cdots}}}}}} =$$

$$[0;2,1,\ (1,4,1),(1,6,1),(1,8,1),(1,10,1),(1,12,1),(1,14,1),\ldots]$$

$$= \frac{3-e}{e-2} \qquad (56.1\text{-}3)$$

$$[(0;6,2),1,1,(3,18,5),1\ ,1,(6,30,8),1,1,(9,42,11),1,1,\ldots] = \frac{1}{e^2-1} \qquad (57)$$

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

A collection of scf-„myriapods“, complete it:

$$e^x = [0;\ 1, -\frac{1}{x}, , -2, \frac{3}{x}, 4, -\frac{5}{x}, -6,\ \frac{7}{x}, 8, -\frac{9}{x}, -10, \ldots]$$

$$\ln(x+1) = [0;\ 1, \frac{2}{x}, 3, \frac{2}{2x}, 5, \frac{2}{3x}, 7, \frac{2}{4x}, 9, \frac{2}{5x}, 11, \frac{2}{6x}, 13, \ldots]$$

$$\left.\begin{matrix}\tanh x \\ \tan x\end{matrix}\right\} = [0; \frac{1}{x}, \pm\frac{3}{x}, \frac{5}{x}, \pm\frac{7}{x}, \frac{9}{x}, \pm\frac{11}{x}, \frac{13}{x}, \ldots]$$

$$\frac{I_1(2x)}{I_0(2x)} = [0, \frac{1}{x}, \frac{2}{x}, \frac{3}{x}, \frac{4}{x}, \frac{5}{x}, \frac{6}{x}, \frac{7}{x}, \frac{8}{x}, \ldots]$$

$$\frac{J_1(2x)}{J_0(2x)} = [0, \frac{1}{x}, \frac{-2}{x}, \frac{3}{x}, \frac{-4}{x}, \frac{5}{x}, \frac{-6}{x}, \frac{7}{x}, \frac{-8}{x}, \ldots]$$

$$\frac{1}{x}\left(\sqrt{x^2+1}-1\right)=[0;\frac{2}{x},\frac{2}{x},\frac{2}{x},\frac{2}{x},\frac{2}{x},\frac{2}{x},\ldots]$$

(58-63)

** Wie verändern sich die Verläufe der durch vorstehende scf dargestellten Funktionen, wenn in diesen scf die Folge der natürlichen Zahlen durch die Primzahlfolge ersetzt wird?

** Welche neue Qualität könnte entstehen, wenn
** vorstehende scf in ähnlicher Weise wie die DNA eines Genoms zu (systematisch aus Schlingenstrukturen aufgebauten) Knäueln aufgewickelt würde (vgl. [9]),
** wobei durch „dichte Packung“ zusätzliche lokale numerische „Wechselwirkungen“ und damit neue intrinsische Verknüpfungen entstehen würden?

[9] E.L. Aiden. Biologie. Die Entwirrung des Genoms. Spektrum der Wissenschaft 6.19, S. 32-38
https://www.spektrum.de/magazin/die-entwirrung-des-genoms/1640012

** In diesem Abschnitt „A collection of …“ ist ein weiteres Kettenbruchproblem codiert. Finden Sie den hier benutzten Algorithmus und führen Sie die zugehörige Dekodierung durch.

ddddddddddddddddddddddddddddddddddddddddddddddd
Go to Unit 60.
ddddddddddddddddddddddddddddddddddddddddddddddd

scf(e/n)-Strukturen
scf(e/2) =
1D Zeichenketten-Darstellung:

[1; 2, 1, 3, 1, 1, 1, 3, 3, 3, 1, 3, 1, 3, 5, 3, 1, 5, 1, 3, 7, 3, 1, 7, 1, 3, 9, 3, 1, →
9, 1, 3, 11, 3, 1, 11, 1, 3, 13, 3, 1, 13, 1, 3, 15, 3, 1, 15, 1, 3, 17, 3, 1, →
17, 1, 3, 19, 3, 1, 19, 1, 3, 21, 3, 1, 21, 1, 3, 23, 3, 1, 23, 1, 3, 25, 3, 1, →
25, 1, 3, 27, 3, 1, 27, 1, 3, 29, 3, 1, 29, 1, 3, 31, 3, 1, 31, 1, 3, 33, 3, 1, …] =
2D Muster-Darstellung:

[1; 2, →
1, 3, 1, **1**, 1, →
3, **3**, 3, 1, **3**, 1, →
3, **5**, 3, 1, **5**, 1,
3, **7**, 3, 1, **7**, 1,
3, **9**, 3, 1, **9**, 1,
3, **11**, 3, 1, **11**, 1,
3, **13**, 3, 1, **13**, 1,
3, **15**, 3, 1, **15**, 1,
3, **17**, 3, 1, **17**, 1,
3, **19**, 3, 1, **19**, 1,
3, **21**, 3, 1, **21**, 1,
3, **23**, 3, 1, **23**, 1,
3, **25**, 3, 1, **25**, 1,
3, **27**, 3, 1, **27**, 1,
3, **29**, 3, 1, **29**, 1,
3, **31**, 3, 1, **31**, 1
3, **33**, 3, 1, …]

Zum Vergleich:

scf(e) =

[2; 1,
2, 1, 1,
4, 1, 1,
6, 1, 1,
8, 1, 1,
10, 1, 1,

12, 1, 1,
14, 1, 1,
16, 1, 1, …]

scf(e/3) =

[1; 9,
1, 1, **1**, 5, 1, **1**, 1, 1,
26, 1, 1, **3**, 5, 1, **3**, 1, 1,
44, 1, 1, **5**, 5, 1, **5**, 1, 1, 44=26+**18**
62, 1, 1, **7**, 5, 1, **7**, 1, 1, 62=44+**18**
80, 1, 1, **9**, 5, 1, **9**, 1, 1, 80=62+**18**
98, 1, 1, **11**, 5, 1, **11**, 1, 1, ↓
116, 1, 1, **13**, 5, 1, **13**, 1, 1,
134, 1, 1, **15**, 5, 1, **15**, 1, 1,
152, 1, 1, **17**, 5, 1, **17**, 1, 1,
170, 1, 1, **19**, 5, 1, **19**, 1, 1,
188, 1, 1, **21**, 5, 1, **21**, 1, 1,
206, 1, 1, **23**, 5, 1, **23**, 1, 1,
224, 1, 1, **25**, 5, 1, **25**, 1, 1,
242, 1, 1, **27**, 5, 1, **27**, 1, 1,
260, 1, 1, **29**, 5, 1, **29**, 1, 1,
278, 1, 1, 31, 5, 1, **31**, 1, 1,
296, 1, 1, **33**, 5, 1, **33**, 1, 1,
314, 1, 1, **35**, 5, 1, **35**, 1, 1,
332, 1, 1, **37**, 5, 1, **37**, 1, 1,
350, 1, 1, **39**, 5, 1, **39**, 1, 1,
368, 1, 1, **41**, 5, 1, **41**, 1, 1,
386, 1, 1, **43**, 5, 1, **43**, 1, 1,
404, 1, 1, **45**, 5, 1, **45**, 1, 1,
422, 1, 1, 47, 5, 1, 47, 1, 1,
440, 1, 1, …]

scf(e/4) =

[1; 2, 8, 3, 1,
1, 1, **1**, 7, 1, **1**, 2, 1,
1, 1, **2**, 7, 1, **2**, 2, 1,

1, 1, **3**, 7, 1, **3**, 2, 1,
1, 1, **4**, 7, 1, **4**, 2, 1,
1, 1, **5**, 7, 1, **5**, 2, 1,
1, 1, **6**, 7, 1, **6**, 2, 1,
1, 1, **7**, 7, 1, **7**, 2, 1,
1, 1, **8**, 7, 1, **8**, 2, 1,
1, 1, **9**, 7, 1, **9**, 2, 1,
1, 1, **10**, 7, 1, **10**, 2, 1,
1, 1, **11**, 7, 1, **11**, 2, 1,
1, 1, **12**, 7, 1, **12**, 2, 1,
1, 1, **13**, 7, 1, **13**, 2, 1,
1, 1, **14**, 7, 1, **14**, 2, 1,
1, 1, **15**, 7, 1, **15**, 2, 1,
1, 1, **16**, 7, 1, **16**, 2, 1,
1, 1, **17**, 7, 1, **17**, 2, 1,
1, 1, **18**, 7, 1, **18**, 2, 1,
1, 1, **19**, 7, 1, **19**, 2, 1,
1, 1, **20**, 7, 1, **20**, 2, 1,
1, 1, **21**, 7, 1, **21**, 2, 1,
1, 1, **22**, 7, 1, **22**, 2, 1,
1, 1, **23**, 7, 1, **23**, 2, 1,
1, 1, **24**, 7, 1, **24**, 2, 1,
1, 1, **25**, 7, 1, **25**, 2, 1,
1, 1, **26**, 7, 1, **26**, 2, 1,
1, 1, **27**, 7, 1, **27**, 2, 1,
1, 1, **28**, 7, 1, **28**, 2, 1,
1, 1, **29**, 7, 1, **29**, 2, 1,
1, 1, **30**, 7, 1, **30**, 2, 1,
1, 1, **31**, 7, 1, …]

scf(e/5) =

[0; 1, 1, 5, 4, 2, 2, 2,
2, 2, 1, **1**, 9, 1,
1, 3, 3,
2, 3, 3,
2, 1, 2, 2, 1,
2, 1, 2, 2, 1, **3**, 9, 1,
3, 3, 3,

4, 3, 3,
4, 1, 2, 2, 1,
4, 1, 2, 2, 1, **5**, 9, 1,
5, 3, 3,
6, 3, 3,
6, 1, 2, 2, 1,
6, 1, 2, 2, 1, **7**, 9, 1,
7, 3, 3,
8, 3, 3,
8, 1, 2, 2, 1,
8, 1, 2, 2, 1, **9**, 9, 1,
9, 3, 3,
10, 3, 3,
10, 1, 2, 2, 1,
10, 1, 2, 2, 1, **11**, 9, 1,
11, 3, 3,
12, 3, 3,
12, 1, 2, 2, 1,
12, 1, 2, 2, 1, **13**, 9, 1,
13, 3, 3,
14, 3, 3,
14, 1, 2, 2, 1,
14, 1, 2, 2, 1, **15**, 9, 1,
15, 3, 3,
16, 3, 3,
16, 1, 2, 2, 1,
16, 1, 2, 2, 1, **17**, 9, 1,
17, 3, 3,
18, 3, 3,
18, 1, 2, 2, 1,
18, 1, 2, 2, 1, **19**, 9, 1,
19, 3, 3,
20, 3, 3,
20, 1, 2, 2, 1,
20, 1, 2, 2, 1, **21**, 9, 1,
21, 3, 3,
22, 3, 3,
22, 1, 2, 2, 1,
22, 1, 2, 2, 1, **23**, 9, 1,
23, 3, 3,
24, 3, 3,
24, 1, 2, 2, 1,
24, 1, 2, 2, 1, **25**, 9, 1,

25, 3, 3,
26, 3, 3,
26, 1, 2, …]

scf(e/6) =

[0; 2, 4, 1, 4, 1, 2, 3, 3,
13, 3, 1, **1**, 11, 1, **1**, 3, 1,
21, 1, 3, **2**, 1, 1, 2, 2, **2**, 1, 3,
31, 3, 1, **3**, 11, 1, **3**, 3, 1,
39, 1, 3, **4**, 1, 1, 2, 2, **4**, 1, 3,
49, 3, 1, **5**, 11, 1, **5**, 3, 1,
57, 1, 3, **6**, 1, 1, 2, 2, **6**, 1, 3,
67, 3, 1, **7**, 11, 1, **7**, 3, 1,
75, 1, 3, **8**, 1, 1, 2, 2, **8**, 1, 3,
85, 3, 1, **9**, 11, 1, **9**, 3, 1,
93, 1, 3, **10**, 1, 1, 2, 2, **10**, 1, 3,
103, 3, 1, **11**, 11, 1, **11**, 3, 1,
111, 1, 3, **12**, 1, 1, 2, 2, **12**, 1, 3,
121, 3, 1, **13**, 11, 1, **13**, 3, 1,
129, 1, 3, **14**, 1, 1, 2, 2, 14, 1, 3,
139, 3, 1, **15**, 11, 1, **15**, 3, 1,
147, 1, 3, **16**, 1, 1, 2, 2, **16**, 1, 3,
157, 3, 1, **17**, 11, 1, **17**, 3, 1,
165, 1, 3, **18**, 1, 1, 2, 2, **18**, 1, 3,
175, 3, 1, **19**, 11, 1, **19**, 3, 1,
183, 1, 3, **20**, 1, 1, 2, 2, **20**, 1, 3,
193, 3, 1, **21**, 11, 1, **21**, 3, 1,
201, 1, 3, **22**, 1, 1, 2, 2, **22**, 1, 3,
211, 3, 1, **23**, 11, 1, **23**, 3, 1,
219, 1, 3, **24**, 1, 1, 2, 2, **24**, 1, 3,
229, 3, …]

scf(e/7) =

[0; 2, 1, 1, 2, 1, 4, 1, 3, 5, 1, 1, 2, 1, 1, 4,
1, 2, 1, 3, 1,
1, 13, 1,
1, 4, 1, 1, 1,
1, 1, 3, 1, 2,
2, 2, 1, 3, 1,
2, 4, 1, 1, 1,
2, 1, 1, 1, 4,
3, 2, 1, 3, 1,

3, 13, 1,
3, 4, 1, 1, 1,
3, 1, 3, 1, 2,
4, 2, 1, 3, 1,
4, 4, 1, 1, 1,
4, 1, 1, 1, 4,
5, 2, 1, 3, 1,
5, 13, 1,
5, 4, 1, 1, 1,
5, 1, 3, 1, 2,
6, 2, 1, 3, 1,
6, 4, 1, 1, 1,
6, 1, 1, 1, 4,
7, 2, 1, 3, 1,
7, 13, 1,
7, 4, 1, 1, 1,
7, 1, 3, 1, 2,
8, 2, 1, 3, 1,
8, 4, 1, 1, 1,
8, 1, 1, 1, 4,
9, 2, 1, 3, 1,
9, 13, 1,
9, 4, 1, 1, 1,
9, 1, 3, 1, 2,
10, 2, 1, 3, 1,
10, 4, 1, 1, 1,
10, 1, 1, 1, 4,
11, 2, 1, 3, 1,
11, 13, 1,
11, 4, 1, 1, 1,
11, 1, 3, 1, 2,
12, 2, 1, 3, 1,
12, 4, 1, 1, 1,
12, 1, 1, 1, 4,
13, 2, 1, 3, 1,
13, 13, 1,
13, 4, 1, 1, 1,
13, 1, 3, 1, 2,
14, 2, 1, 3, 1,
14, 4, 1, 1, 1,
14, 1, 1, 1, 4, …]

scf(e/8) =

[0; 2, 1, 16, 1, 1, 4, 16, 6, 3, 1, 15, 2, 2, 4, 1,

1, 15, 1,
1, 5, 3, **2**, 15, 1,
1, 1, 2, 4, 1,
2, 15, 1,
2, 5, 3, **3**, 15, 1,
2, 1, 2, 4, 1,
3, 15, 1,
3, 5, 3, **4**, 15, 1,
3, 1, 2, 4, 1,
4, 15, 1,
4, 5, 3, **5**, 15, 1,
4, 1, 2, 4, 1,
5, 15, 1,
5, 5, 3, **6**, 15, 1,
5, 1, 2, 4, 1,
6, 15, 1,
6, 5, 3, **7**, 15, 1,
6, 1, 2, 4, 1,
7, 15, 1,
7, 5, 3, **8**, 15, 1,
7, 1, 2, 4, 1,
8, 15, 1,
8, 5, 3, **9**, 15, 1,
8, 1, 2, 4, 1,
9, 15, 1,
9, 5, 3, **10**, 15, 1,
9, 1, 2, 4, 1,
10, 15, 1,
10, 5, 3, **11**, 15, 1,
10, 1, 2, 4, 1,
11, 15, 1,
11, 5, 3, **12**, 15, 1,
11, 1, 2, 4, 1,
12, 15, 1,
12, 5, 3, **13**, 15, 1,

12, 1, 2, 4, 1,
13, 15, 1,
13, 5, 3, **14**, 15, 1,
13, 1, 2, 4, 1,
14, 15, 1,
14, 5, 3, **15**, 15, 1,
14, 1, 2, 4, 1,
15, 15, 1,
15, 5, 3, **16**, 15, 1,
15, 1, 2, 4, 1,
16, 15, 1,
16, 5, 3, **17**, 15, 1,
16, 1, 2, 4, 1,
17, 15, 1,
17, 5, 3, **18**, 15, 1,
17, 1, …]

scf(e^2/n)-Strukturen

scf(e^2/2) =

[3;

1, 2, 3, 1, 1, 1,
8, 1, 1, 2, 3, 1, 2, 1, 1,
14, 1, 1, 3, 1, 3, 4, 1, 1,
20, 1, 1, 5, 3, 1, 5, 1, 1,
26, 1, 1, 6, 1, 3, 7, 1, 1,
32, 1, 1, 8, 3, 1, 8, 1, 1,
38, 1, 1, 9, 1, 3, 10, 1, 1,
44, 1, 1, 11, 3, 1, 11, 1, 1,
50, 1, 1, 12, 1, 3, 13, 1, 1,
56, 1, 1, 14, 3, 1, 14, 1, 1,
62, 1, 1, 15, 1, 3, 16, 1, 1,
68, 1, 1, 17, 3, 1, 17, 1, 1,
74, 1, 1, 18, 1, 3, 19, 1, 1,
80, 1, 1, 20, 3, 1, 20, 1, 1,
86, 1, 1, 21, 1, 3, 22, 1, 1,
92, 1, 1, 23, 3, 1, 23, 1, 1,
98, 1, 1, 24, 1, 3, 25, 1, 1,

104, 1, 1, 26, 3, 1, 26, 1, 1,
110, 1, 1, 27, 1, 3, 28, 1, 1,
116, 1, 1, 29, 3, 1, 29, 1, 1,
122, 1, 1, 30, 1, 3, 31, 1, 1,
128, 1, 1, 32, 3, 1, 32, 1, 1,
134, 1, 1, 33, 1, 3, 34, 1, 1,
140, 1, 1, 35, 3, 1, 35, 1, 1,
146, 1, 1, 36, 1, 3, …]

scf($e^2/3$) =

[2; 2, 6, 3, 1,
5, 2, 1, **1**, 5, 1, **1**, 2, 1,
9, 2, 1, **2**, 5, 1, **2**, 2, 1,
13, 2, 1, **3**, 5, 1, **3**, 2, 1,
17, 2, 1, **4**, 5, 1, **4**, 2, 1,
21, 2, 1, **5**, 5, 1, **5**, 2, 1,
25, 2, 1, **6**, 5, 1, **6**, 2, 1,
29, 2, 1, **7**, 5, 1, **7**, 2, 1,
33, 2, 1, **8**, 5, 1, **8**, 2, 1,
37, 2, 1, **9**, 5, 1, **9**, 2, 1,
41, 2, 1, **10**, 5, 1, **10**, 2, 1,
45, 2, 1, **11**, 5, 1, **11**, 2, 1,
49, 2, 1, **12**, 5, 1, 12, 2, 1,
53, 2, 1, **13**, 5, 1, **13**, 2, 1,
57, 2, 1, **14**, 5, 1, **14**, 2, 1,
61, 2, 1, **15**, 5, 1, **15**, 2, 1,
65, 2, 1, **16**, 5, 1, **16**, 2, 1,
69, 2, 1, **17**, 5, 1, **17**, 2, 1,
73, 2, 1, **18**, 5, 1, **18**, 2, 1,
77, 2, 1, **19**, 5, 1, **19**, 2, 1,
81, 2, 1, **20**, 5, 1, **20**, 2, 1,
85, 2, 1, **21**, 5, 1, 21, 2, 1,
89, 2, 1, **22**, 5, 1, **22**, 2, 1,
93, 2, 1, **23**, 5, 1, **23**, 2, 1,
97, 2, 1, **24**, 5, 1, 24, …]

scf($e^2/4$) =

[1; 1, 5, 1, 1, 4, 1, 3, 1, 3, 1, 7, 2, 3,

7, 3, 1, **1**, 2, 1, 1, 1, **1**, 1, 3,
10, 3, 1, **2**, 7, 1, **2**, 3, 1,
12, 1, 3, **3**, 2, 1, 1, 1, **3**, 3, 1,
15, 1, 3, **4**, 7, 1, **3**, 1, 3,
19, 3, 1, **4**, 2, 1, 1, 1, **4**, 1, 3,
22, 3, 1, **5**, 7, 1, **5**, 3, 1,
24, 1, 3, **6**, 2, 1, 1, 1, **6**, 3, 1,
27, 1, 3, **7**, 7, 1, **6**, 1, 3,
31, 3, 1, **7**, 2, 1, 1, 1, **7**, 1, 3,
34, 3, 1, **8**, 7, 1, **8**, 3, 1,
36, 1, 3, **9**, 2, 1, 1, 1, **9**, 3, 1,
39, 1, 3, **10**, 7, 1, **9**, 1, 3,
43, 3, 1, **10**, 2, 1, 1, 1, **10**, 1, 3,
46, 3, 1, **11**, 7, 1, **11**, 3, 1,
48, 1, 3, **12**, 2, 1, 1, 1, **12**, 3, 1,
51, 1, 3, **13**, 7, 1, **12**, 1, 3,
55, 3, 1, **13**, 2, 1, 1, 1, **13**, 1, 3,
58, 3, 1, **14**, 7, 1, **14**, 3, 1,
60, 1, 3, **15**, 2, 1, 1, 1, **15**, 3, 1,
63, 1, 3, **16**, 7, 1, **15**, 1, 3,
67, 3, 1, **16**, 2, 1, …]

scf($\sqrt[11]{e}$) =

[1;
10, 1, 1,
32, 1, 1,
54, 1, 1,
76, 1, 1,
98, 1, 1,
120, 1, 1,
142, 1, 1,
164, 1, 1,
186, 1, 1,
208, 1, 1,
230, 1, 1,

252, 1, 1,
274, 1, 1,
296, 1, 1,
318, 1, 1,
340, 1, 1, …]

scf(3*$\sqrt[11]{e}$) =

[3; 3, 1, 1,

98, 1, 1, **17**, 1, 5, **25**, 1, 1,
296, 1, 1, **39**, 1, 5, **47**, 1, 1,
494, 1, 1, **61**, 1, 5, **69**, 1, 1,
692, 1, 1, **83**, 1, 5, **91**, 1, 1,
890, 1, 1, **105**, 1, 5, **113**, 1, 1,
1088, 1, 1, **127**, 1, 5, **135**, 1, 1,
1286, 1, 1, **149**, 1, 5, **157**, 1, 1,
1484, 1, 1, **171**, 1, 5, **179**, 1, 1,
1682, 1, 1, **193**, 1, 5, **201**, 1, 1,
1880, 1, 1, **215**, 1, 5, **223**, 1, 1,
2078, 1, 1, **237**, 1, 5, **245**, 1, 1,
2276, 1, 1, **259**, 1, 5, **267**, 1, 1,
2474, 1, 1, **281**, 1, 5, **289**, 1, 1,
2672, 1, 1, **303**, 1, 5, **311**, 1, 1,
2870, 1, 1, **325**, 1, 5, **333**, 1, 1,
3068, 1, 1, **347**, 1, 5, **355**, 1, 1,
3266, 1, 1, **369**, 1, 5, **377**, 1, 1,
3464, 1, 1, **391**, 1, 5, **399**, 1, 1,
3662, 1, 1, **413**, 1, 5, **421**, 1, 1,
3860, 1, 1, **435**, 1, 5, **443**, 1, 1,
4058, 1, 1, **457**, 1, 5, **465**, 1, 1,
4256, 1, 1, **479**, 1, 5, **487**, 1, 1,
4454, 1, 1, **501**, 1, 5, **509**, 1, 1,
4652, 1, 1, **523**, 1, 5, **531**, 1, 1,
4850, 1, 1, **545**, 1, 5, **553**, 1, 1,

5048, 1, 1, **567**, 1, 5, **575**, 1, 1,
5246, 1, 1, **589**, 1, 5, **597**, 1, 1,
5444, 1, 1, **611**, 1, 5, **619**, 1, 1, …]

(64-77)

Eine jeweils geeignete 2D-Darstellung der scf der vorliegenden e-Derivate offenbart in jedem Falle eine charakteristische Struktur (vgl. auch [10]):

** „backbone“ = gebündelte synchrone infinite Folgen von natürlichen Zahlen
** mit systematisch angelagerten kurzen „snips“ aus natürlichen Zahlen als „Verbindungselementen“,
** eine gewisse Ähnlichkeit mit acyklischen Molekül-Strukturen, im Gegensatz zu cyklischen ↔
z.B. $\sqrt{19}$ = [4;(2,1,3,1,2,8)per] ,
** eine gewisse Ähnlichkeit mit der DNA-Struktur.
** Vgl. auch Unit 67 und Unit 79.3 .

Diskutieren Sie ein entsprechendes 3D-Modell, z.B. „Datenzylinder“ in Unit 19.

Diskutieren Sie
** Zahl → scf(Zahl) = eindimensionale Zeichenkette → 2D-Darstellung = Zahlen-Muster
** Wie steht es mit der Umkehrung Muster → Zahl?, Anwendungen?

[10] E.P. Stoschek, D. Schönfeld. Computergraphik im Spannungsfeld zwischen Algorithmus und Phantasie. Thun, Frankfurt am Main 2000, ISBN 3-8171-1640-3

Probleme

** Struturanalyse für

scf(e*(3/5)) =

[1; 1, 1, 1, 2, 2, 4, 8, 3, 1,
29, 1, 2, 3, 4, 3, 3, 5, 3, 1, 2, 1, 3, 4, 1,
29, 3, 3, 3, 2, 3, 3, 2, 1, 4, 3, 1, 1, 1, 4, 3, 1, 1, 1, 2, 1, 2, 1,
1, 1, 1, 2, 3, 3, 1, 1, 2, 3, 3, 1, 1, 1, 1, 3, 4, 2, 4, 3, 1, 1, 2,
29, 1, 1, 1, 3, 3, 2, 2, 2, 3, 3, 1, 2, 4, 3, 1, 1, 2, 1, 1, 3, 4, 3,

29, 1, 2, 2, 3, 3, 1, 2, 1, 3, 3, 2, 3, 4, 3, 1, 1, 3, 4, 3, 1, 1, 3, 2,
1, 2, 1, 1, 1, 3, 2, 3, 3, 1, 3, 2, 3, 3, 1, 3, 1, 1, 3, 4, 4, 4, 3, 1, 1, **4**,
29, 1, 3, 1, 3, 3, 2, 4, 2, 3, 3, 1, 4, 4, 3, 1, 1, 4, 1, 1, 3, 4, **5**,
29, 1, 4, 2, 3, 3, 1, 4, 1, 3, 3, 2, 5, 4, 3, 1, 1, 5, 4, 3, 1, 1, 5, 2, 1,
2, 1, 1, 1, 5, 2,3, 3, 1, 5, 2, 3, 3, 1, 5, 1, 1, 3, 4, 6, 4, 3, 1, 1, **6**,
29, 1, 5, 1, 3, 3, 2, 6, 2, 3, 3, 1, 6, 4, 3, 1, 1, 6, 1, 1, 3, 4, **7**,
29, 1, 6, 2, 3, 3, 1, 6, 1, 3, 3, 2, 7, 4, 3, 1, 1, 7, 4, 3, 1, 1, 7, 2, 1,
2, 1, 1, 1, 7, 2, 3, 3, 1, 7, 2, 3, 3, 1, 7, 1, 1, 3, 4, 8, 4, 3, 1, 1, **8**,
29, 1, 7, 1, 3, 3, 2, 8, 2, 3, 3, 1, 8, 4, 3, 1, 1, 8, 1, 1, 3, 4, **9**,
29, 1, 8, 2, 3, 3, 1, 8, 1, 3, 3, 2, 9, 4, 3, 1, 1, 9, 4, 3, 1, 1, 9, 2, 1,
2, 1, 1, 1, 9, 2, 3, 3, 1, 9, 2, 3, 3, 1, 9, 1, 1, 3, 4, 10, 4, 3, 1, 1, **10**,
29, 1, 9, 1, 3, 3, 2, 10, 2, 3, 3, 1, 10, 4, 3, 1, 1, 10, 1, 1, 3, 4, **11**,
29, 1, 10, 2, 3, 3, 1, 10, 1, 3, 3, 2, 11, 4, 3, 1, 1, 11, 4, 3, 1, 1, 11, 2,
1, 2, 1, 1, 1, 11, 2, 3, 3, 1, 11, 2, 3, 3, 1, 11, 1, 1, 3, 4, 12, 4, 3, 1, 1, **12**,
29, 1, 11, 1, 3, 3, 2, 12, 2, 3, 3, 1, 12, 4, 3, 1, 1, 12, 1, 1, 3, 4, **13**,
29, 1, 12, 2, 3, 3, 1, 12, 1, 3, 3, 2, 13, 4, 3, 1, 1, 13, 4, 3, 1, 1, 13, 2, 1,
2, 1, 1, 1, 13, 2, 3, 3, 1, 13, 2, 3, 3, 1, 13, 1, 1, 3, 4, 14, 4, 3, 1, 1, **14**,
29, 1, 13, 1, 3, 3, 2, 14, 2, 3, 3, 1, 14, 4, 3, 1, 1, 14, 1, 1, 3, 4, **15**,
29, 1, 14, 2, 3, 3, 1, 14, 1, 3, 3, 2, 15, 4, 3, 1, 1, 15, 4, 3, 1, 1, 15, 2, 1,
2, 1, 1, 1, 15, 2, 3, 3, 1, 15, 2, 3, 3, 1, 15, 1, 1, 3, 4, 16, 4, 3, 1, 1, **16**,
29, 1, 15, 1, 3, 3, 2, 16, 2, 3, 3, 1, 16, 4, 3, 1, 1, 16, 1, 1, 3, 4, **17**,
29, 1, 16, 2, 3, 3, 1, 16, 1, 3, 3, 2, 17, 4, 3, 1, 1, 17, 4, 3, 1, 1, 17, 2, 1,
2, 1, 1, 1, 17, 2, 3, 3, 1, 17, 2, 3, 3, 1, 17, 1, 1, 3, 4, 18, 4, 3, 1, 1, **18**,
29, 1, 17, 1, 3, 3, 2, 18, 2, 3, 3, 1, 18, 4, 3, 1, 1, 18, 1, 1, 3, 4, 19,
29, 1, 18, 2, 3, 3, 1, 18, 1, 3, 3, 2, 19, 4, 3, 1, 1, 19, 4, 3, 1, 1, 19, 2,1,
2, 1, 1, 1, 19, 2, 3, 3, 1, 19, 2, 3, 3, 1, 19, 1, 1, 3, 4, 20, 4, 3, 1, 1, **20**,
29, 1, 19, 1, 3, 3, 2, 20, 2, 3, 3, 1, 20, 4, 3, 1, 1, 20, 1, 1, 3, 4, **21**,
29, 1, 20, 2, 3, 3, 1, 20, 1, 3, 3, 2, 21, 4, 3, 1, 1, 21, 4, 3, 1, 1, 21, 2, 1,
2, 1, 1,1, 21, 2, 3, 3, 1, 21, 2, 3, 3, 1, 21, 1, 1, 3, 4, 22, 4, 3, 1, 1, **22**, …]

(78)

,

hier sortiert nach dem „Leuchtturm“ 29.

** Vergleichen Sie

scf(e) =

[2; 1,

2, 1, 1,
4, 1, 1,
6, 1, 1,
8, 1, 1,
10, 1, 1,
12, 1, 1,
14, 1, 1,
16, 1, 1, …]

und

scf(?) =

[2; 1,
1, 1, **2**, 1, 1,
1, 1, **4**, 1, 1,
1, 1, **6**, 1, 1,
1, 1, **8**, 1, 1,
1, 1, **10**, 1, 1,
1, 1, **12**, 1, 1,
1, 1, **14**, 1, 1,
1, 1, **16**, 1, 1, …] . (79.1,2)