Unit n°72

Part 2 – Nachbereitung zum Alumni-Seminar der Roland Berger Stiftung

Leipzig, 09.-10. Mai 2015
Workshop zum Internetprojekt
„Math for Seals and Nerds only”
www.algorithmicschannel-dresden-guilin.net

B. 6

Ganzzahlige Testwerte für die Tetraederformeln:
http://mathworld.wolfram.com/HeronianTetrahedron.html

B. 7

Determinant and matrix product representation of
$area\Delta P_1P_2P_3$ and $\left(area\Delta P_1P_2P_3\right)^2$

$$2area\Delta P_1P_2P_3 = x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3$$

$$2area\Delta P_1P_2P_3 = \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

$$\left(2area\Delta P_1P_2P_3\right)=\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}$$

coupling matrix $\underline{c}$

$$\det\underline{c}=0$$

Extension to n-gon

$$2area\ \text{n-gon}\ P_1P_2P_3P_4...P_n = \begin{vmatrix}x_1 & x_2 & x_3 & x_4 & x_n\\ y_1 & y_2 & y_3 & y_4 & y_n\end{vmatrix}$$

$$\left(2area\ \text{n-gon}\ P_1P_2P_3P_4...P_n\right) =$$

$$\begin{pmatrix}x_1 & x_2 & x_3 & x_4 & x_n\end{pmatrix}\begin{pmatrix}0 & 1 & 0 & 0 & -1\\ -1 & 0 & 1 & 0 & 0\\ 0 & -1 & 0 & 1 & 0\\ 0 & 0 & -1 & 0 & 1\\ 1 & 0 & 0 & -1 & 0\end{pmatrix}\begin{pmatrix}y_1\\ y_2\\ y_3\\ y_4\\ y_n\end{pmatrix}$$

$$-16(area\Delta P_1P_2P_3)^2 = d_{12}^4+d_{13}^4+d_{23}^4 - 2(d_{12}^2d_{13}^2+d_{12}^2d_{23}^2+d_{13}^2d_{23}^2);$$

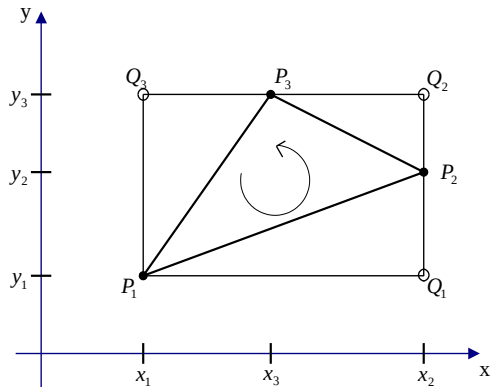
$$d_{ij}=\overline{P_iP_j}$$

$$-16(area\Delta P_1P_2P_3)^2 = \begin{vmatrix}0 & d_{12}^2 & d_{13}^2 & d_{23}^2\\ d_{12}^2 & 0 & 1 & 1\\ d_{13}^2 & 1 & 0 & 1\\ d_{23}^2 & 1 & 1 & 0\end{vmatrix}$$

$$\left(16(area\Delta P_1P_2P_3)^2\right) = \begin{pmatrix} d_{12}^2 & d_{13}^2 & d_{23}^2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} d_{12}^2 \\ d_{13}^2 \\ d_{23}^2 \end{pmatrix}$$

$$\left(8(area\Delta P_1P_2P_3)^2\right) = \begin{pmatrix} d_{12}^2 & d_{13}^2 & d_{23}^2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} d_{12}^2 \\ d_{13}^2 \\ d_{23}^2 \end{pmatrix}$$

B. 8



Given a planar triangle $\Delta P_1P_2P_3$ by the coordinates of its vertices $P_i = (x_i, y_i)$; $i = 1(1)3$.

Find shortened determinant formula expressions for the coordinates of its

* 1 orthocenter OC = intersection point of the altitudes
* 2 circumcenter CC = intersection point of the Perpendicular bisectors of the sides

* 3 incenter IC = intersection point of the angle bisectors in function of the coordinates x_i and y_i .

Solutions

***** 1

$$x_{oc} = -\frac{\begin{vmatrix} x_2x_3 & x_3x_1 & x_1x_2 & y_2y_3 & y_3y_1 & y_1y_2 \\ y_1 & y_2 & y_3 & y_1 & y_2 & y_3 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}}$$

$$y_{oc} = -\frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_1 & x_2 & x_3 \\ x_2x_3 & x_3x_1 & x_1x_2 & y_2y_3 & y_3y_1 & y_1y_2 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}}$$

***** 2

$$x_{cc} = \frac{1}{2}\frac{\begin{vmatrix} x_1^2 & x_2^2 & x_3^2 & y_1^2 & y_2^2 & y_3^2 \\ y_1 & y_2 & y_3 & y_1 & y_2 & y_3 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}}$$

$$y_{cc} = \frac{1}{2}\frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^2 & y_1^2 & y_2^2 & y_3^2 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}}$$

* 3

$$x_{ic} = \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \ ? \ ; \ y_{ic} = \frac{ay_1 + by_2 + cy_3}{a+b+c} = ?$$

$$a = \overline{P_2P_3} \ ; \ b = \overline{P_1P_3} \ ; \ c = \overline{P_1P_2}$$

IC is also centroid of the point masses $m_1 = a$ in P_1, $m_2 = b$ in P_2, and $m_3 = c$ in P_3 .

B. 9

Solid of revolution generated by revolving the 2D polygon $P_1P_2P_3\ldots P_n$ around the x-axis

** Volume

volume rotx polygon $P_1P_2P_3\ldots P_n = \pi I(0,2,n)$

** Centroid

centroid rotx polygon $P_1P_2P_3\ldots P_n = (x_{cx}, y_{cx}, z_{cx})$

$$x_{cx} = \frac{I(1,2,n)}{I(0,2,n)}; \quad y_{cx} = z_{cx} = 0$$

$$I(0,2,n) = \frac{1}{3}\begin{vmatrix} x_1 & x_1 & x_2 & x_2 & x_3 & x_3 & \ldots & x_n & x_n \\ y_1 & y_1 & y_2 & y_2 & y_3 & y_3 & \ldots & y_n & y_n \\ y_1 & y_1 & y_2 & y_2 & y_3 & y_3 & \ldots & y_n & y_n \end{vmatrix}$$

- +

$$I(1,2,n) = \frac{1}{12}\begin{vmatrix} 2x_1 & x_1 & 2x_1 & 2x_2 & x_2 & 2x_2 & \ldots & 2x_n & x_n & 2x_n \\ x_1 & x_1 & x_1 & x_2 & x_2 & x_2 & \ldots & x_n & x_n & x_n \\ y_1 & y_1 & y_1 & y_2 & y_2 & y_2 & \ldots & y_n & y_n & y_n \\ y_1 & y_2 & y_1 & y_2 & y_2 & y_2 & \ldots & y_n & y_n & y_n \end{vmatrix}$$