Unit n°72

Part 1 – Nachbereitung zum Alumni-Seminar der Roland Berger Stiftung

Leipzig, 09.-10. Mai 2015
Workshop zum Internetprojekt
„Math for Seals and Nerds only"
www.algorithmicschannel-dresden-guilin.net

Bei der Introduktion bzw. beim Ausblick vergessen:

** Hinweis auf Buddha: wahre Geistesgegenwart.
** Hinweis auf Sokrates' Mäeutik
** Hinweis auf Sherlock Holmes
C. Doyle. Sherlock Holmes. A Study in Scarlet. The Science of Deduction.

> ""From a drop of water," said the writer, "a logician could infer the possibility of an Atlantic or a Niagara without having seen or heard of one or the other. So all life is a great chain, the nature of which is known whenever we are shown a single link of it. Like all other arts, the Science of Deduction and Analysis is one which can only be acquired by long and patient study, nor is life long enough to allow any mortal to attain the highest possible perfection in it. Before turning to those moral and mental aspects of the matter which present the greatest difficulties, let the inquirer begin by mastering more elementary problems."

The "drop of water" is here

$$\text{area}\Delta P_1P_2P_3 = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} .$$

Hausaufgabe zu S. 13 und 14, Gln (2), (9) und (10)

„Versuchen Sie, basierend auf den Gln.(2) und (10), ähnliche klassische Determinantendarstellungen für den Flächeninhalt eines einfachen konvexen 2D Fünfeck, Sechseck, ... zu finden.“

$$\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \Rightarrow \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

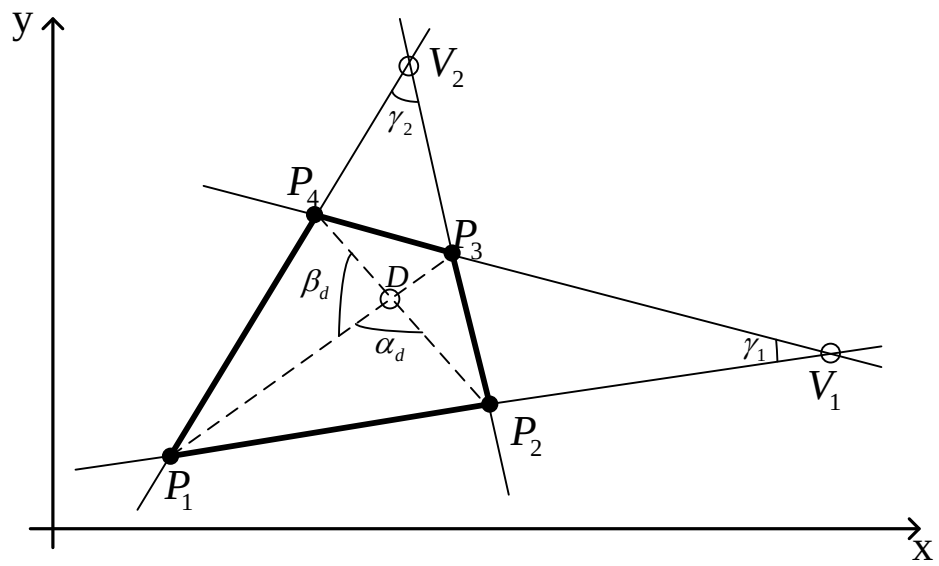
$$\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}$$

$$\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & y_4 & y_5 \end{vmatrix} \Rightarrow \begin{vmatrix} & & ? & & \\ x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & y_4 & y_5 \end{vmatrix}$$

Aus der Diskussion mit den Teilnehmern am Workshop und mit mir selbst ergab sich die Frage nach weiteren Anwendungsbereichen der verkürzten Determinantendarstellung. Hier einige Beispiele.

B. 1

Diagonal point D and vanishing points V_1 and V_2 of a convex quadrangle



$$z_d = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ z_1 & z_2 & z_3 & z_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}$$

z = x + jy

$$z_{V_1} = \frac{\begin{vmatrix} x_4 & x_1 & x_3 & x_2 \\ z_4 & z_1 & z_3 & z_2 \\ y_4 & y_1 & y_3 & y_2 \end{vmatrix}}{\begin{vmatrix} x_4 & x_1 & x_3 & x_2 \\ y_4 & y_1 & y_3 & y_2 \end{vmatrix}} \qquad z_{V_2} = \frac{\begin{vmatrix} x_1 & x_2 & x_4 & x_3 \\ z_1 & z_2 & z_4 & z_3 \\ y_1 & y_2 & y_4 & y_3 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_4 & x_3 \\ y_1 & y_2 & y_4 & y_3 \end{vmatrix}}$$

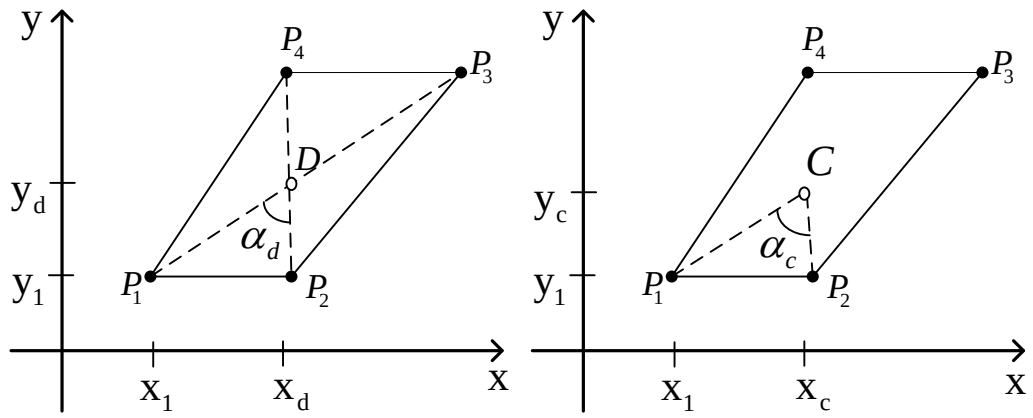
Diagonal angles α_d and β_d and vanishing angles γ_1 and γ_2 of a convex quadrangle

$$\tan\alpha_d = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & x_3 & y_3 \\ y_4 & x_2 & y_2 & x_4 \end{vmatrix}} \qquad \tan\gamma_2 = \frac{\begin{vmatrix} x_1 & x_2 & x_4 & x_3 \\ y_1 & y_2 & y_4 & y_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & x_4 & y_4 \\ y_3 & x_2 & y_2 & x_3 \end{vmatrix}}$$

$$\tan\beta_d = \frac{\begin{vmatrix} x_4 & x_1 & x_2 & x_3 \\ y_4 & y_1 & y_2 & y_3 \end{vmatrix}}{\begin{vmatrix} x_4 & y_4 & x_2 & y_2 \\ y_3 & x_1 & y_1 & x_3 \end{vmatrix}} \qquad \tan\gamma_1 = \frac{\begin{vmatrix} x_4 & x_1 & x_3 & x_2 \\ y_4 & y_1 & y_3 & y_2 \end{vmatrix}}{\begin{vmatrix} x_4 & y_4 & x_3 & y_3 \\ y_2 & x_1 & y_1 & x_2 \end{vmatrix}}$$

B. 2

Diagonal intersection point D and centroid C of a 2D simple convex quadrangle.



$$z_d = x_d + jy_d = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ z_1 & z_2 & z_3 & z_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}} \; ; \quad z_i = x_i + jy_i$$

$$z_c = x_c + jy_c = \frac{1}{3} \cdot \frac{\begin{vmatrix} x_1 & x_1 & x_2 & x_2 & x_3 & x_3 & x_4 & x_4 \\ z_1 & z_1 & z_2 & z_2 & z_3 & z_3 & z_4 & z_4 \\ y_1 & y_1 & y_2 & y_2 & y_3 & y_3 & y_4 & y_4 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}$$

Duplication effect!

$$z_d + 3z_c = z_1 + z_2 + z_3 + z_4$$

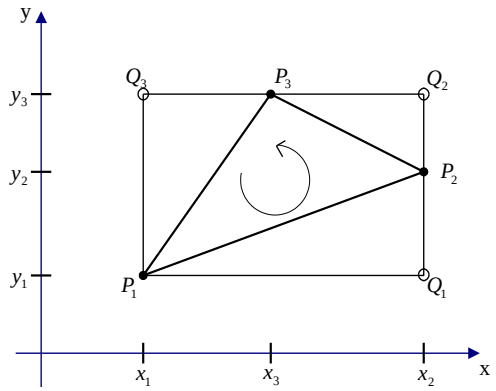
Central angles of the D and C based sectors

$$\tan\alpha_d = \frac{\begin{vmatrix} x_1 & x_2 & x_d \\ y_1 & y_2 & y_d \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & x_d & y_d \\ y_d & x_2 & y_2 & x_d \end{vmatrix}} = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & x_3 & y_3 \\ y_4 & x_2 & y_2 & x_4 \end{vmatrix}}$$

$$\tan\alpha_c = \frac{\begin{vmatrix} x_1 & x_2 & x_c \\ y_1 & y_2 & y_c \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & x_c & y_c \\ y_c & x_2 & y_2 & x_c \end{vmatrix}} = ?$$

Auch das Ungeheuer von Loch Ness existiert höchst-wahrscheinlich gar nicht, aber es lockt trotzdem alljährlich Tausende von Touristen nach Schottland, realisiert so Einnahmen von Hotels, Restaurants, Imbissständen, Tankstellen, Fremdenführern, Postkartenverkäufern, - real pounds, hardpower. Aber viel wichtiger ist: es verleiht der Phantasie ihrer Besucher Flügel, inspiriert und konspiriert Bankangestellte, VerkäuferInnen, Manager, Schichtarbeiter, die „mal die Seele baumeln lassen“ wollen, vielleicht aber auch ernste und nicht so ernste Mathematiker, Physiker, Biologen zu allerlei schweifenden Gedanken, - der Softpower Aspekt von Nessy.
Ähnlich ist es wohl mit einem „schönen“ (beautiful) statischen oder dynamischen Formelausdruck, sei er nun korrekt und effizient, nur in Spezialfällen richtig oder gänzlich falsch.
If you find however, wherever such a formula expression, regard it as a present of the gods, try immediately to extend, to mutate it in all possible directions!
Die Spur ist ausgelegt. Die Inszenierung kann beginnen.
The trail is marked. The production of the play can start.

Variations on the theme



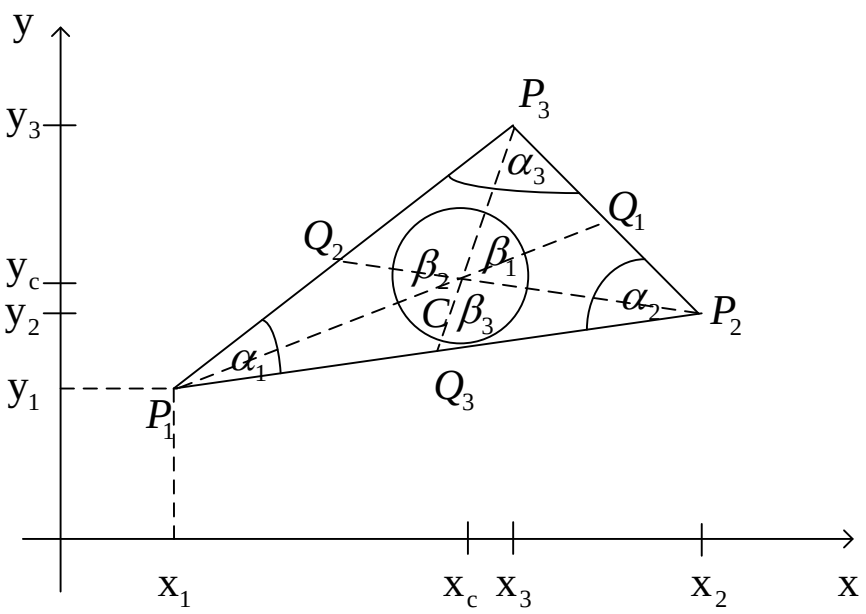
Given a planar triangle $\triangle P_1P_2P_3$ by the coordinates of its vertices $P_i = (x_i, y_i)$; $i = 1(1)3$, the simplest interaction between the points P_1, P_2 and P_3.

Find a formula expression for the area of $\triangle P_1P_2P_3$ in function of the coordinates x_i and y_i taking a simplest and shortest way.

The aforementioned semantics of the 2D 3point arrangement as the geometric object triangle (with area, sum of side squares, angles, special points, etc.) provokes immediately the question for other interpretations, f.e.

* three sphere capacitor
* three input automaton
* three phase magnetic rotating field, three phase induction motor, three phase reluctance motor
* 3-body problem (mass center, potential, moments, Newtonian 3-body problem)
* triple synchronization and resonance effects:
  - 3 mechanical coupled (f.e. on a suitable common metal baseplate) mechanical but also electronic clockworks
  - 3 electrical or magnetic coupled systems (oscillation circuits, motors, etc.)
  - 3 acoustic coupled biological systems
* local triple operators for transitivity and metrics
  - $(P_1 \text{rel} P_2)\&(P_2 \text{rel} P_3) \rightarrow (P_1 \text{rel} P_3)$
  - $\text{dist}(P_1,P_2)+\text{dist}(P_2,P_3) \geq \text{dist}(P_1,P_3)$.

B. 3



Find shortened determinant expressions for the angles $\alpha_1,\ \alpha_2,\ \alpha_3$ and its centroid "patterns" $\beta_1,\ \beta_2,\ \beta_3$ of the 2D triangle $\triangle P_1P_2P_3$ given by $P_i = (x_i, y_i)\ ;\ i = 1(1)3$.

Solution:

$$tan\ \alpha_1 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & x_2 & y_2 \\ y_3 & x_1 & y_1 & x_3 \end{vmatrix}}$$

$$tan\ \beta_1 = \frac{3\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}}{\begin{vmatrix} x_3 & y_3 & x_2 & y_2 & x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_2 & y_2 \\ y_3 & x_2 & y_2 & x_3 & y_3 & x_1 & y_1 & x_3 & y_3 & x_2 & y_2 & x_3 \end{vmatrix}}$$

left wing core right wing

Addition to Unit 70.12

The area^2 of a triangle $\Delta P_1P_2P_3$ is given by

$$(area\Delta P_1P_2P_3)^2 = -\frac{1}{16}[(d_{12}^4 + d_{12}^4 + d_{12}^4) - 2(d_{12}^2 d_{13}^2 + d_{12}^2 d_{23}^2 + d_{13}^2 d_2^2)];$$

$$d_{ij} = \overline{P_iP_j}$$

$$= -\frac{1}{16}\begin{vmatrix} 0 & d_{12}^2 & d_{13}^2 & d_{23}^2 \\ d_{12}^2 & 0 & 1 & 1 \\ d_{13}^2 & 1 & 0 & 1 \\ d_{23}^2 & 1 & 1 & 0 \end{vmatrix}$$

Extension

The sum of the area^2 of the faces of a tetrahedron T$P_1P_2P_3P_4$ is given by

$$(area_{\triangle}P_1P_2P_3)^2 + (area_{\triangle}P_1P_2P_4)^2 + (area_{\triangle}P_2P_3P_4)^2 + (area_{\triangle}P_3P_1P_4)^2$$

$$= -\frac{1}{8}(d_{12}^4 + d_{13}^4 + d_{14}^4 + d_{23}^4 + d_{24}^4 + d_{34}^4 - d_{12}^2 d_{13}^2 - d_{12}^2 d_{14}^2 - d_{12}^2 d_{23}^2 - d_{12}^2 d_{24}^2 - d_{13}^2 d_{14}^2 - d_{13}^2 d_{23}^2 - d_{13}^2 d_{34}^2 - d_{14}^2 d_{24}^2 - d_{14}^2 d_{34}^2 - d_{23}^2 d_{24}^2 - d_{23}^2 d_{34}^2 - d_{24}^2 d_{34}^2);\ d_{ij} = \overline{P_iP_j}$$

Find determinant representations for this string expression.

Possible solutions:

$$(area_{\triangle}P_1P_2P_3)^2+(area_{\triangle}P_1P_2P_4)^2+(area_{\triangle}P_2P_3P_4)^2+(area_{\triangle}P_3P_1P_4)^2=$$

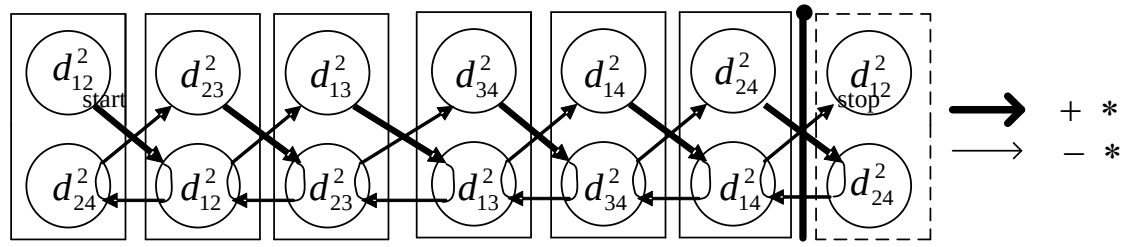
$\frac{1}{?}$

	d_{12}^2	d_{13}^2	d_{14}^2	d_{23}^2	d_{34}^2	d_{24}^2
d_{12}^2	0					
d_{13}^2		0				
d_{14}^2			0			
d_{23}^2				0		
d_{34}^2					0	
d_{24}^2						0

$=$

Find the contents 0 or 1 of all empty cells.

$-\frac{1}{8}$



$$=-\frac{1}{8}(+\ d_{12}^2*d_{12}^2-d_{12}^2*d_{24}^2-d_{24}^2*d_{23}^2+d_{23}^2*d_{23}^2-d_{23}^2*d_{12}^2-d_{12}^2*d_{13}^2$$
$$+d_{13}^2*d_{13}^2-d_{13}^2*d_{23}^2-d_{23}^2*d_{34}^2+d_{34}^2*d_{34}^2-d_{34}^2*d_{13}^2-d_{13}^2*d_{14}^2$$
$$+\ d_{14}^2*d_{14}^2-d_{14}^2*d_{34}^2-d_{34}^2*d_{24}^2+d_{24}^2*d_{24}^2-d_{24}^2*d_{14}^2-d_{14}^2*d_{12}^2)$$

More!

Creativity means

** to make a potentially existing vertex of the "knowledge graph" "visible", real existing
  ** using extrapolation, extension, evolution
  ** by a breakthrough in research or development to new results, theories, methods, …
  ** by chance, serendipity, intuition, stepping stones, …
** to connect
  ** adjacent, neighboring
  ** far distant (far-fetched)

  already existing vertices of the knowledge graph by not yet thought, not yet existing edges.

A simple example

"Grow or go" connect with

2x5 (2xn) determinant $\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & y_4 & y_5 \end{vmatrix} \Rightarrow$

"Growing determinant" $= \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{vmatrix}$

with $x_6 = \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}$; $y_6 = \begin{vmatrix} x_1 & y_2 \\ y_1 & x_2 \end{vmatrix}$

or other recurrence formulas.

Explore application possibilities of this dynamic extension of the shortened determinant.
To be continued.