Unit n°67

A GUET scf story

incomplete

67.1 Free-running simple continued fraction expansion

The simple continued fraction (scf)

$$a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+...}}}\qquad ...+\frac{1}{a_n}=[a_0;a_1,a_2,a_3,...,a_n]$$

$a_i \in \mathbb{N}$

$n \to \infty$ infinite scf

is a very efficient tool to analyze the "natural inner discrete structure" of real numbers and functions using the following

Expansion $r \in R_+ \to \text{scf}(r)$ algorithm:

Input: $r \in \mathbb{R}_+$ $\text{length}(\text{approx.scf}(r)) \in \mathbb{N}$	
$x:=r$; $i:=0$	
$a_0:=\text{int}(x) \mapsto$	
repeat until	$x:=(x-\text{int}(x))^{-1}$
	$i:=i+1$

	a_i:=int(x) $\mapsto$
x=int(x) or i=length	
n:=i	
Output: approx.scf(r)=$[a_0;a_1,a_2,...a_n]$	

Examples

scf($\frac{1}{2}(1+\sqrt{5})$) = [1;(1)per]

scf($\sqrt{2}$) = [1;(2)per]

scf($\sqrt{3}$) = [1;(1,2)per]

scf($\sqrt{26}$) = [5;(10)per]

scf($\sqrt{35}$) = [5;(1,10)per]

scf($\sqrt{32}$) = [5;(1,1,1,10)per]

$\text{scf}(\frac{1}{4}\sqrt{22*23}) = [5;(1,1,1,1,1,10)\text{per}]$

scf(z) = $[(\underbrace{1;1,1,1,...,1}_{\text{n times}},a)\text{per}]$; $a \in \mathbb{N}$

z is solution of the quadratic equation

$$\begin{vmatrix} z^2 & \text{fib(n)} & 0 \\ z & \text{fib(n-1)} & -1 \\ 1 & \text{fib(n-2)} & a \end{vmatrix} = 0 \ .$$

fib(0) = fib(1) = 1 ; fib(i+2) = fib(i+1)+fib(i)

scf(z) = $[0;(\underbrace{1,1,1,1,...,1}_{\text{n times}},a)\text{per}] = [0;(1^n,a)\text{per}]$; $a \in \mathbb{N}$

z is solution of the quadratic equation

$$\begin{vmatrix} z^2 & \text{fib(n-2)} & a \\ z & \text{fib(n-1)} & -1 \\ 1 & \text{fib(n)} & 0 \end{vmatrix} = 0 \ .$$

$$\mathrm{scf}\left(\frac{(a+T)\mathrm{fib}(n-1)+f(n-2)}{(a+T)\mathrm{fib}(n)+f(n-1)}\right)=[0;1^{n},a,1^{\infty}]\ ;\ T=\frac{1}{2}(\sqrt{5}-1)$$

$$\mathrm{scf}\left(\frac{2393+144*\Phi}{1479+89*\Phi}=1.617984451...\right)=$$

[1; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 17, 1, 1, 1, 1, 1, …]

$$\Phi=\frac{1}{2}(\sqrt{5}+1)=1.618033989...$$

scf(e = 2.7182818284…) = [2;1,2,1,1,4,1,1,6,1,1,8,1,…]

scf($\sqrt{e}$) = [1;1,1,1,1,5,1,1,9,1,1,13,1,…]

How about

[1; 2, 4, 6, 8, …] = 1.446389965… = scf(?)

[1; 1, 3, 5, 7, 9, …] = scf($\frac{2e^2}{e^2+1}$)

[1; 2, 1, 4, 1, 6, 1, 8, 1,…] = scf(?)

[1; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, …] = scf(e-1)

[1; 1, 1, 2, 1, 1, 1, 4, 1, 1, 1, 6, 1, 1, 1, 8, 1,…] = scf(?)

[1; 1, 1, 1, 2, 1, 1, 1, 1, 4, 1, 1, 1, 1, 6, 1, 1, 1, 1, 8, 1,…] = scf(?)

67.2 Simple continued fraction expansion with given fixed (value, position) elements, f.i.

$$[a_0;a_1,2,\ a_2,2,\ a_3,2,\ a_4,2,\ a_5,2,\ a_6,2,\ldots] \qquad (67.2.1)$$

$$[a_0;a_1,1,\ 1,a_2,1,1,\ a_3,1,1,\ a_4,1,1,,\ a_5,1,1,\ldots] \qquad (67.2.2)$$

Modify the given “Expansion $r\in\mathbb{R}_+ \rightarrow$ scf(r) algorithm“ for the special cases equ. (67.2.1) and (67.2.2).
Find convincing examples for the ability of these modifications to analyze the inner discrete structure of real numbers.

67.3 Simple continued fraction expansion with a given pattern of fixed elements, f.i

** a binary message with the elements 1 and 2 (stego)
f.i. “hello”, in International Morse code [1]
(dot=1, dash=2, gap=3) ↦ “11113131211312113222”
↦ (67.3.1.1)

$[a_0;a_1,1,a_2,1,a_3,1,a_4,1,a_5,3,a_6,1,a_7,3,a_8,1,a_9,2,a_{10},1,$
$a_{11},1,a_{12},3,a_{13},1,a_{14},2,a_{15},1,a_{16},1,a_{17},3,a_{18},2,a_{19},2,$
$a_{20},2,a_{21},a_{22},a_{23},a_{24},\ldots]$ (67.3.1.2)

** another scf

$scf(Z)=[A_0;A_1,A_2,A_3,A_4,A_5,A_6,\ldots]” \mapsto$
(67.3.2.1)

$scf(z)\#scf(Z)=[a_0;a_1,A_1,a_2,A_2,a_3,A_3,a_4,A_4,\ldots]$
(67.3.2.2)

(# = folding)

Modify the given “Expansion $r\in\mathbb{R}_+ \rightarrow scf(r)$ algorithm“ for the special cases equ. (67.3.1) and (67.3.2).

Find convincing examples for the ability of these modifications to analyze the inner discrete structure of real numbers.
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References

https://en.wikipedia.org/wiki/Morse_code