Unit n°66

Supplement 2

$$x = \sqrt{a} + \sqrt{b} + \sqrt{c} \qquad \text{(S2.1)}$$

rec.

$$x := \frac{1}{2}\left(x + \frac{(2Ax^2 + 4B - A^2)[2x^4 - (\ )] + 64Cx^2}{x^7}\right)$$

$$A = a+b+c \ ; \ B = ab+ac+bc \ ; \ C = abc \qquad \text{(S2.2)}$$

Example 1

$x(0)=7 \ ; \ a=3 \ ; \ b=5 \ ; \ c=7$

$$\rightarrow x=\sqrt{3} + \sqrt{5} + \sqrt{7} = 6.613870096... \qquad \text{(S2.3)}$$

** **Derive formula (S2.2) from (S2.1).**

** **Find**

$\lim_{t\to\infty} x(t)$ and

$$\lim_{t\to\infty} dx(t) = \frac{x(t) - x(t+1)}{x(t+1) - x(t+2)} \qquad \text{(S2.4)}$$

for the given example (S2.3) using the recurrence formula (S2.2).

Record x(t); t=0,1,2,3,…

7.000000000000000000000000000000000000000000000000,
6.738262604381313422614241150735298581859113610339 7,
6.655012765284334710361051046253049476293547645407 3,
6.627607878743628288627504164372841232683231184475 0,
6.618472170357865566794308298681653809167592725689 6,
6.615413457733803747715400954946184161140578271681 4,
6.614387872453645375016296459860410945774086358614 8,
6.614043824423709381844525447500242296065733815658 7,
6.613928389211108470614181387784129723561452045987 8,
6.613889656162235941081360998419821540506031789331 2,
6.613876659460177333711291404280615338225449501282 4,
6.613872298447377924228325223128591596550710521005 1,
6.613870835116718847454131525101628310793512315026 1,
6.613870344098181247103581648385692331120472614344 3,
6.613870179337564910297624675153697544951446132389 7,
6.613870124052355144394682169657638321870613672675 1,
6.613870105501474671784184951648976094251252262635 8,
6.613870099276750720450457152830336295561512693545 0,
6.613870097188052642550587798217804037520885244858 7,
6.613870096487192700949911430999707213421143486014 6,
6.613870096252020052924001858287123264526368529962 1,
6.613870096173108174668452915517463074655141284746 4,
6.613870096146629395854986341295005155445085031847 9,
6.613870096137744475828233241702008620324320011168 2,
6.613870096134763152390386933085845246387545403865 3,
6.613870096133762773161952546277230116559957818430 1,
6.613870096133427097204094581110756266206445364431 3,
6.613870096133314461570045856947610260613023535154 6,
6.613870096133276666828538254046269281135934065920,
6.613870096133263984851846857885292457797145108816 1,
6.613870096133259729430950044423681174931489003133 3,
6.613870096133258301529965072407983466948842073213 9,
6.613870096133257822399634060774987355914670288967 3,
6.613870096133257661628070407980016718624628541609 8,
6.613870096133257607681379901473248974805498736786 2,
6.613870096133257589579637587827264486789906072931 3,
6.613870096133257583505621337467675342134332109395 3,

6.613870096133257581467493030789699047748179078265 1,
6.613870096133257580783601710242752025945935459217 8,
6.6138700961332575805541228610520544987512300964837
Converges slowly!

Record dx(t); t=0,1,2,3,…

3.143998816787929413390362096587179434474350173648 1,
3.037773536238576341341700288557396246602743812191 3,
2.999754959715523267715847492687265754781865923170 1,
2.986782188655223502093977562998216883701479012694 9,
2.982406907780001724570709418221787337245772179047 1,
2.980936354581576823194548347762141380461181580311 8,
2.980442641236815307754610467291849723970001424989 3,
2.980276945943721728907247075050696715340664667888 4,
2.980221343681389143334292777763875746640218190722 3,
2.980202686029087215686915497886750563876973571835 6,
2.980196425434615860499555935256070726787680727031 3,
2.980194324695348298281672607992694550567680638557 6,
2.980193619794451277096011031303515745355199475220 1,
2.980193383265803978321636603214412167405791123204 9,
2.980193303898914948727492829533660979643519609489 5,
2.980193277267457073058577709932128501670739799934 2,
2.980193268331305695851767939430451467124499117064 0,
2.980193265332791634590788013997964389625844943412 4,
2.980193264326644113555555670562964414353789097076 8,
2.980193263989032612440472545430547947092187718448 2,
2.980193263875747509337867081158148869969648928903 0,
2.980193263837734839326589285191341431676000909321 8,
2.980193263824979737007114853151115017744631364905 4,
2.980193263820699778900239726828989943437344113338 2,
2.980193263819263644861252581521345958572878865957,
2.980193263818781751438600176906135373952954584139 4,
2.980193263818620052848990013458706125169293835760 2,
2.980193263818565795096120997227212964503791652724 5,
2.980193263818547588977233426640676968866908415470 2,
2.980193263818541479937558515387786140309324863402 4,
2.980193263818539430057186970718285717409277474138 6,
2.980193263818538742222476238519727818426092498884 9,

2.980193263818538511420427565048045325901041696006 6,
2.980193263818538433975098271959267973127496957194 1,
2.980193263818538407988418067918526177776975150440 9,
2.980193263818538399268621093831059613636571995423 4,
2.980193263818538396342704483002021743210966420610 7,
2.980193263818538395360916943800606174758461050966 3,

Example 2

x(0)=7 ; a=2 ; b=3 ; c=4

$\rightarrow x=\sqrt{2}+\sqrt{3}+\sqrt{4}=5.14626436994197...$

Record x(t); t=0,1,2,3,…

7.000000000000000000000000000000000000000000000000,
5.686922237211657436223730879869053589187206982513 4,
5.310998243438291746059932020403119099320281582255 5,
5.198381578056154726959029608162476103769199698174 8,
5.162993925419426498956193062737554138883537290066 7,
5.151660953277650489195950513540425549766135503215 3,
5.148007988853889560864804406952116432667921382664 8,
5.146828021796469238060334230539276691420042516766 6,
5.146446609940824985326162256359864734713812243564 7,
5.146323295025879555793475612897094189222175772292 1,
5.146283422986969271994576748341562225881637296789 4,
5.146270530656134339146482601251194357368702434337 1,
5.146266361984144632823884041343906316618561860682 2,
5.146265014061190289952332597853255357284627091237 5,
5.146264578215498317287065100863315789149858012917 9,
5.146264437286437658904686576621929222313719796463,
5.146264391717553656313227375604721502108864663179 4,
5.146264376983025269708417352978296522549900516336 0,
5.146264372218670526228210444197967017458261756323 4,
5.146264370678134203001224683609745954222662571310 0,
5.146264370180007532527152676749652244311141261920 6,
5.146264370018940132707359755597715723319250207514 3,
5.146264369966859590096639668016241870920174656425 1,

5.14626436995001954101318379166394376220013599953 48,
5.14626436994457437399071823041104250126877074788 85,
5.14626436994281369955588121908701018350771812579 43,
5.14626436994224439204837924175695795521976294477 31,
5.14626436994206030859053757339048209879211689528 39,
5.14626436994200078589146122007311678374789187304 76,
5.14626436994198153944885918270533184846615590134 44,
5.14626436994197531618347334467149540825222052458 74,
5.14626436994197330391383664462905439479777686918 50,
5.14626436994197265325395959687391019564691015556 85,
5.14626436994197244286551667722973383964396620096 90,
5.14626436994197237483719884858378661351258091388 07,
5.14626436994197235284049616675999750844339555164 16,
5.14626436994197234572794444835181277936745323854 60,
5.14626436994197234342812739491529106329998469042 44,
5.14626436994197234268449013381975021599881799423 23,
5.14626436994197234244403780076858080883126616410 01
Converges slowly!

Record dx(t); t=0,1,2,3,…

3.49293416897449032866225482351460795472103723161 92,
3.33808493172622244539296975431883743521570922449 63,
3.18237173112901375578142630542358875560076512197 99,
3.12253945337789981162831667860684953937130551801 11,
3.10240419207479987981979495050734035916594614543 37,
3.09581898985141329348123652064018599569123991227 19,
3.09368217049050456880156824364658024546192715279 91,
3.09299046115418130104466270445581882023465589696 35,
3.09276671862456829894413005918488742191758692335 43,
3.09269436386531258446554309569190964090430679253 16,
3.09267096734111135171012718208734056293025837492 79,
3.09266340207003871513538243208082101210518361377 64,
3.09266095585822289461651620060398677857120233284 35,
3.09266016488375294757271585691245923784332213122 00,
3.09265990912500460944188038237808482760049482423 50,
3.09265982642635682261737286528605176608339120063 55,
3.09265979968605654692534944674688721270838520249 47,
3.09265979103968012954835008471128493606621792159 82,

3.092659788243906571474921884537961175882717427266 9,
3.092659787339903624243583453125986244159752873676 5,
3.092659787047597644942431927789512994638485400346 8,
3.092659786953081597434357995491789917931813752134 4,
3.092659786922520186213879107562111241122418782443 9,
3.092659786912638267953635693196994793106053554469 5,
3.092659786909442986564416219741988054256045522396 1,
3.092659786908409804252300510789184372577961691784 8,
3.092659786908075728607611103999082091435986332250 2,
3.092659786907967706494702435692110732751745973439 4,
3.092659786907932777947642082704130165274188895371 0,
3.092659786907921483932306734205555221960168526329 0,
3.092659786907917832054598374800982006377833997137 4,
3.092659786907916651233570527244160818172627631929 9,
3.092659786907916269419498107672042526400583303405 2,
3.092659786907916145961342756767709660600948496348 7,
3.092659786907916106041608981479252396574235668628 6,
3.092659786907916093133712049433985772032166758779 1,
3.092659786907916088959991749940338555511669016917 5,
3.092659786907916087610434867815069176771778403811 2,

Another approach

Example

$$x = \sqrt{3} + \sqrt{5} + \sqrt{7} = 6.61387009...$$

$$z = x^2 = 43.7432776... \qquad \text{(S2.5.1)}$$

rec. z

$$z := \frac{3z^4 - 120z^3 + 782z^2 - 3481}{4z^3 - 180z^2 + 1564z - 3180} \qquad \text{(S2.5.2)}$$

$$z(0) = 50$$

Record z(t); t=0,1,2,3,…

t	z(t)
0	50.0
1	45.60485522…
2	43.97189332…
3	43.7473052…
4	43.74327893…
5	43.74327763…

$\rightarrow \quad (\sqrt{3}+\sqrt{5}+\sqrt{7})^2$

General case

$x = \sqrt{a}+\sqrt{b}+\sqrt{c}$

$z = x^2$

rec. z

$$z := z - \frac{[(z-A)^2-4B]^2-64Cz}{4(z-A)[(z-A)^2-4B]-64C}$$

A=a+b+c ; B=ab+ac+bc ; C=abc (S2.6)

Find a determinant representation of this recurrence formula.

Test of the recurrence formula by sampling:

Example 1

a=3 ; b=5 ; c=17 ; z(0)=60

Record z(t); t=0,1,2,3,…

60.000000000000000000000000000000000000000000000000,
68.404970263381478334749362786745964316057774001699,
65.869862660251014228646573878150550397279055775660,
65.476858009700747161970767153144056662494431352892,
65.467917028073176878105322775239382927776016563230,
65.467912464087497208119727228142552380454165945119,
65.467912464086308386361879066485546935196260377291,
65.467912464086308386361878985824916093691795827834,
65.467912464086308386361878985824916093691795827834,…

→ $(sqr(3)+sqr(5)+sqr(17))^2$

Example 2

a=3 ; b=5 ; c=17 ; z(0)=50

Record z(t); t=0,1,2,3,…

50.000000000000000000000000000000000000000000000000,
-7.352953586497890295358649789029535864978902953586 5,
-3.060257465565547533578499692840729612376748813002 5,
-0.795619089320818066741205391688019011261548468630 77,
-0.056055456176727439238779012658180695205892418758191,
0.023146523606901714886226963091027275011903951149846,
0.024020814847111175067956660646210161490685524521262,
0.024020920743357600897144470047747427302537848926164,
0.024020920743359154355182613304348048818781141826556,
0.024020920743359154355182613304682349639890993332786,
0.024020920743359154355182613304682349639890993332786,…

→ $(sqr(3)-sqr(5)-sqr(17))^2$!!!

Example 3

a=3 ; b=5 ; c=17 ; z(0)=70

Record z(t); t=0,1,2,3,…

70.000000000000000000000000000000000000000000000000,
66.338256911383947214566107074250396725966758540048,

65.508443823331238992619673650038860546658582527362,
65.468005932434930930323384277060542017848039700110,
65.467912464584910546031288741924274295117165896829,
65.467912464086308386376067484204737503613842124063,
65.467912464086308386361878985824916093691807317330,
65.467912464086308386361878985824916093691795827834,
65.467912464086308386361878985824916093691795827834,…

→ $(sqr(3)+sqr(5)+sqr(17))^2$

Example 4

a=3 ; b=15 ; c=17 ; z(0)=100

Record z(t); t=0,1,2,3,…

100.000000000000000000000000000000000000000000000000,
95.526601899196493791088385682980277574872169466764,
94.666846449470290371353114631758327324426810674829,
94.636739755177346385155288188688688617005716352108,
94.636703567479304344162691362580492358530184499358,
94.636703567427062172594440867386376544268250355285,
94.636703567427062172594331989083415653310751886698,
94.636703567427062172594331989083415653310751886225,
94.636703567427062172594331989083415653310751886225,…

→ $(sqr(3)+sqr(15)+sqr(17))^2$

Example 5

a=3 ; b=15 ; c=17 ; z(0)=50

Record z(t); t=0,1,2,3,…

50.000000000000000000000000000000000000000000000000,
41.161578947368421052631578947368421052631578947368,
39.355654506322518978972652766378697603719997753192,
39.238683243558944236992799385834562776886700137787,
39.238174132917383823353537800740977751130710552613,

39.238174123258185827163624233039363702355700412920,
39.238174123258185823686648718839039713409175940436,
39.238174123258185823686648718839039712958646931276,
39.238174123258185823686648718839039712958646931276,…

→ $(\mathrm{sqr}(3)-\mathrm{sqr}(15)-\mathrm{sqr}(17))^2$!!!

Example 6

a=3 ; b=15 ; c=17 ; z(0)=200

Record z(t); t=0,1,2,3,…

200.000000000000000000000000000000000000000000000,
161.340792860547640485143795349822572956940839997 88,
133.421691883939584048306138070795962811070159266 77,
114.095648972464148143814681330098758640230648721 73,
102.012912886132233181762687144318677398110126092 57,
96.189298670263609827906026736461593240203718021645,
94.725426620695426498847786511631341600191650707373,
94.637016099486221429132479660282998567330058230880,
94.636703571323616292655891396559979671172107855730,
94.636703567427062173200035827748056495591993152435,
94.636703567427062172594331989083415653325387790250,
94.636703567427062172594331989083415653310751886225,
94.636703567427062172594331989083415653310751886225,…

→ $(\mathrm{sqr}(3)+\mathrm{sqr}(15)+\mathrm{sqr}(17))^2$

Example 7

a=3 ; b=15 ; c=17 ; z(0)=78

Record z(t); t=0,1,2,3,…

78.000000000000000000000000000000000000000000000000,
209.285532994923857868020304568527918781725888324 87,
168.135663328401529515522204098647911291991023398 52,
138.267009990081124150661816550357330499055695155 96,
117.351293656921940538193853811730168039589462197 39,

103.897243007393559074108952659125792040104332231 66,
96.918228924674352525563946605566240515261595589060,
94.821579271285641925308622842886875877221540378148,
94.638053590897869419388764932664195865039190160928,
94.636703640129647591348309200144298093809109679783,
94.636703567427062383456472011694947691972236797888,
94.636703567427062172594331989083417427075912738700,
94.636703567427062172594331989083415653310751886225,
94.636703567427062172594331989083415653310751886225,…

→ $(sqr(3)+sqr(15)+sqr(17))^2$

Example 8

a=3 ; b=15 ; c=17 ; z(0)=1

Record z(t); t=0,1,2,3,…

1.000000000000000000000000000000000000000000000000,
1.823529411764705882352941176470588235294117647058,
2.136379386815989355214281754171979819260409158951 0,
2.194060056185422046534762978724271651966599415277 8,
2.196109579352927469277824504158628002170939665613 4,
2.196112162566311080524118276541125783470828133165 2,
2.196112162570414184315741683868269997485469465422 2,
2.196112162570414184315752035691899171976668429046 9,
2.196112162570414184315752035691899171976668429112 8,
2.196112162570414184315752035691899171976668429112 8,…

→ $(sqr(a)+sqr(b)-sqr(c))^2$

Example 9

a=3 ; b=15 ; c=17 ; z(0)=5

Record z(t); t=0,1,2,3,…

5.000000000000000000000000000000000000000000000000,
4.200000000000000000000000000000000000000000000000,
3.959013337157607916248386634160332711888713609637 2,

3.92947792519625034556096810805874025723621209231 40,
3.92901026434321049088301548890075259454088873749 96,
3.92901014674434525583048002028263390351842607511 19,
3.92901014674433781940326725641538174853440828086 09,
3.92901014674433781940326725638564546175393275338 55,
3.92901014674433781940326725638564546175393275338 55,…

→ $(sqr(a)-sqr(b)+sqr(c))^2$

Example 10

a=3 ; b=15 ; c=17 ; z(0)=25

Record z(t); t=0,1,2,3,…

25.00000000000000000000000000000000000000000000000,
-123.88000000000000000000000000000000000000000000000,
-86.09099693826856316449271669004583802589086323692 4,
-58.27264920961238152849702225228763524190808209571 7,
-38.03354454119325822639007045985186171022990188727 1,
-23.57575876090648739097423875748056495314657538052 8,
-13.52941558518955359990399056060872169051790790958 1,
-6.82097183836820039798330890395407514523720711509 91,
-2.57453497270712289912323984369425559206081695511 54,
-0.05932113019576332364602766872882888009761020006 4716,
1.31033130303039759298611904386471445074592558758 19,
1.95636774642245077060945613803514241015702058828 75,
2.16842734782734971343115820885910628747639970769 19,
2.19565551811688642591005966797924596611707520167 63,
2.19611203442039537420304356344250037533578271848 66,
2.19611216257040408646047237171495012755363239351 94,
2.19611216257041418431575203562920168376448577679 43,
2.19611216257041418431575203569189917197666842911 28,
2.19611216257041418431575203569189917197666842911 28,

→ $(sqr(a)+sqr(b)-sqr(c))^2$

Example 11

a=3 ; b=15 ; c=17 ; z(0)=26

Record z(t); t=0,1,2,3,…

26.000000000000000000000000000000000000000000000000,
384.385135135135135135135135135135135135135135135514,
298.130029514435981477887730900559477731919766523 53,
233.841820974639297398738163273917793556241210866 66,
186.197149591730128485799925090816272738022503995 16,
151.290139803923533450820215862768694635288520948 95,
126.333089319388575251304936101844631841467586857 35,
109.457259332803191836654096652760594149349851115 84,
99.508413450652211189879610178850117425338269236539,
95.386357287068133197768461948890513456046567806512,
94.658250036654752583540595497690488038716171009251,
94.636722066319836911944029317679665501593362707850,
94.636703567440713973213253558866393840415567955567,
94.636703567427062172594339424045783071980083408514,
94.636703567427062172594331989083415653310751886228,
94.636703567427062172594331989083415653310751886225,
94.636703567427062172594331989083415653310751886225,…

→ $(sqr(a)+sqr(b)+sqr(c))^2$

Example 12

a=3 ; b=15 ; c=17 ; z(0)=30

Record z(t); t=0,1,2,3,…

30.000000000000000000000000000000000000000000000000,
50.245135640785781103835360149672591206735266604303,
41.213965557155773020734897139554068777356311103021,
39.361624677035086949549811572850450508122822170994,
39.238736013381044743050835776924158390480512197584,
39.238174135023432563447529527847820060146815771014,
39.238174123258185828845126618482134683640400553920,
39.238174123258185823686648718839039713950306584834,
39.238174123258185823686648718839039712958646931276,

39.238174123258185823686648718839039712958646931276,…

→ $(sqr(a)-sqr(b)-sqr(c))^2$

Example 13

a=3 ; b=15 ; c=17 ; z(0)=69

Record z(t); t=0,1,2,3,…

69.000000000000000000000000000000000000000000000000,
28.888544891640866873065015479876160990712074303406,
58.040752601192491585203918215875897251906270967729,
41.975428724433633408655288581476766580586605939178,
39.461366584354903678824715863501470072049676611862,
39.239995021381219831202803047958319994803050698555,
39.238174246802261443546837222451527392497183514583,
39.238174123258186392491562292372211831563582076119,
39.238174123258185823686648718839051770150577228357,
39.238174123258185823686648718839039712958646931276,
39.238174123258185823686648718839039712958646931276,…

→ $(sqr(a)-sqr(b)-sqr(c))^2$

Example 14

a=3 ; b=15 ; c=17 ; z(0)=70

Record z(t); t=0,1,2,3,…

70.000000000000000000000000000000000000000000000000,
24.131869764928397730343150499864901378005944339368,
-43.873794834231634456426989791825022036069499660899,
-27.712713710821752474189985285629318664757289018123,
-16.368211071522845649195148733169880780429412385064,
-8.683395909036372656015659213669034472289858610173 9,
-3.726742391493141310374753688739063791716561024603 3,
-0.722832094089832177111080298209574647172427923538 85,
0.963311672005063799640358789770784818809696593732 22,
1.807085187544959571099615839733892896546021479302 3,

2.131837037629509198129552705406915090808968066248 2,
2.193747696219732289702210793515460484886425266496 7,
2.196108734333336508258987135043358147419263687115 7,
2.196112162563187620709709754128438340577880111147 2,
2.196112162570414184315719924567191234814284955786 5,
2.196112162570414184315752035691899171976668428478 8,
2.196112162570414184315752035691899171976668429112 8,
2.196112162570414184315752035691899171976668429112 8,…

$\rightarrow (sqr(a)+sqr(b)-sqr(c))^2$

Example 15

a=3 ; b=15 ; c=17 ; z(0)=71

Record z(t); t=0,1,2,3,…

71.000000000000000000000000000000000000000000000000,
17.296875000000000000000000000000000000000000000000,
5.178891524980542971764330919581804488109887741047 3,
4.265478248248617477717646397786708146692526338833 2,
3.972706533678069628965516481495070246608258994308 6,
3.929987329752871859544056531256075867300619914446 9,
3.929010659627456387702775712288909810773272838966 1,
3.929010146744479266691149994893750916680720435139 5,
3.929010146744337819403267267144036433542515121609 4,
3.929010146744337819403267256385645461753932753385 5,
3.929010146744337819403267256385645461753932753385 5,…

$\rightarrow (sqr(a)-sqr(b)+sqr(c))^2$

Conclusion

For given a, b, c, and depending on the initial value z(0) we obtain for z(∞) recursively four different sqr combinations!

1° $(sqr(a)+sqr(b)+sqr(c))^2=94.63670357\ldots$
2° $(sqr(a)-sqr(b)+sqr(c))^2=3.92901047\ldots$
3° $(sqr(a)+sqr(b)-sqr(c))^2=2.196112163\ldots$
4° $(sqr(a)-sqr(b)-sqr(c))^2=39.23817412\ldots$

The following tables show the details.

a=3, b=15, c=17

z(0)	z(∞)=(+sqr(a)	○sqr(b)	Δsqr(c))^2
1	+	+	-
2	+	+	-
3	+	+	-
4	+	-	+
↕			
19	+	-	+
20	+	+	-
↕			
25	+	+	-
26	+	+	+
27	+	+	+

28	+	+	-
29	+	-	-
↕			
69	+	-	-
70	+	+	-
71	+	-	+
72	+	-	+
73	+	+	-
↕			
76	+	+	-
77	+	+	+
↕			
inf	+	+	+

a=3, b=5, c=17

1° $(sqr(a)+sqr(b)+sqr(c))^2$= 65.46791246…

2° $(sqr(a)-sqr(b)+sqr(c))^2$ = 13.09780125…

3° $(sqr(a)+sqr(b)-sqr(c))^2$ = 0.02402092…

4° $(sqr(a)-sqr(b)-sqr(c))^2$ = 21.41026537…

z(0)	z(∞)=(	+sqr(a)○sqr(b)Δsqr(c))^2	
1	+	+	-
↕			
4	+	+	-
5			
6	+	-	-

7	+	-	+
↕			
16	+	-	+
17	+	+	-
18	+	-	-
↕			
47	+	-	-
48	+	-	+
49	+	-	+
50	+	+	-
↕			
52	+	+	-
53	+	+	+
↕			
inf	+	+	+

a=17, b=25, c=3

1° $(sqr(a)+sqr(b)+sqr(c))^2$= 117.8344212…

2° $(sqr(a)-sqr(b)+sqr(c))^2$ = 0.731292525…

3° $(sqr(a)+sqr(b)-sqr(c))^2$ = 54.62769132…

4° $(sqr(a)-sqr(b)-sqr(c))^2$ = 6.806594962…

z(0)	z(∞)=(+sqr(a)○sqr(b)Δsqr(c))^2		
1	+	-	+
↕			
3	+	-	+

4	+	-	-
↕			
26	+	-	-
27	+	-	+
↕			
35	+	-	+
36	+	+	+
↕			
38	+	+	+
39	+	-	-
40	+	+	-
↕			
87	+	+	-
88	+	+	+
89	+	-	+
90	+	-	-
91	+	-	-
92	+	-	+
↕			
96	+	-	+
97	+	+	+
↕			
inf	+	+	+

A very difficult problem:
Find the relationship between a, b, c, z(0) and z(inf)=(+sqr(a)○sqr(b)Δsqr(c))^2 , basing on the above tables.

Now

$$x = \sqrt{a} + \sqrt{b} + \sqrt{c} + \sqrt{d} + \ldots$$

$$z = x^2$$

rec. z:=

See [1].

Find a general solution, see [1].

Check the application Krypto, see [2,3].

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