Unit 559.1.revised.rev
Continued fraction representations of e,
a widely branched labyrinth
L3.1
e = [2;1,**2**,1,1,**4**,1,1,**6**,1,1,**8**,1,1,**10**,1,1,**12**,…]
= [1;**0**,1,1,**2**,1,1,**4**,1,1,**6**,1,1,**8**,1,1,**10**,1,1,…]
= [0;1,**-1**,-2,**+3**,+2,**-5**,-2,**+7**,+2,**-9**,-2,…] (1)
L3.2
e = 2+1/2*[0;1/2,5,10,14,18,22,26,30,34,38,…]
= 2+1/2*[1;2,3,**2**,3,1,**2**,1,3,**4**,3,1,**4**,1,3,**6**,3,
1,**6**,1,3,**8**,3,1,**8**,1,…] (2)
L3.3
e = 2*[1; 2, **1**, 3, 1, **1**, 1, 3, **3**, 3, 1, **3**, 1, 3, **5**,
3, 1, **5**, 1, 3, **7**, 3, 1, **7**, 1, 3, **9**, 3, 1, **9**, 1, 3, **11**,
3, 1, **11**, 1, 3, **13**, 3, 1, **13**, 1, 3, **15**, 3, 1, **15**,
1, 3, **17**, 3, 1, **17**, 1, 3, **19**, 3, 1, **19**, 1, 3, **21**,
3, 1, **21**, 1, 3, **23**, 3, 1, **23**, 1, 3, **25**, 3, 1, **25**,
1, 3, **27**, 3, 1, **27**, 1, 3, **29**, 3, 1, **29**, 1, 3, **31**,
3, 1, **31**, 1, 3, **33**, 3, 1, **33**, 1, 3, **35**, 3, 1, **35**,
1, 3, **37**, 3, 1, **37**, 1, 3, **39**, 3, 1, **39**, 1, 3, …]
(3)
L.3.4

$$\frac{3-e}{e-2} = \cfrac{\frac{3}{2}}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cfrac{9}{9+\cfrac{10}{10+\cdots}}}}}}}}$$

$$= [0,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2[0;5,10,14,18,22,26,30,34,38,\ldots]$$

$$= 4*[0;\ 10,\ 5,\ 28,\ 9,\ 44,\ 13,\ 60,\ 17,$$
$$76,\ 21,\ 92,\ 25,\ 108,\ 29,\ 124,\ 33,$$
$$140,\ 37,\ 156,\ 41,\ 172,\ 45,\ 188,$$
$$49,\ 204,\ 53,\ 220,\ 57,\ 236,\ 61,$$

252, 65, 268, 69, 284, 73, 300,
77, 316, 81, 332, 85, 348, 89,
364, 93, 380, 97, 396, 101, 412,
105, 428, 109, 444, 113, 460,
117, 476, 121, 492, 125, 508,
129, 524, 133, 540, 137, 556,…] (4.1-4)

$$\frac{1}{e-1} = \frac{1+x}{2+x} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cdots}}}}}}}}} \quad ; x = \frac{3-e}{e-2}$$

$$= [0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 1-\frac{1}{2}[0;1,5,10,14,18,22,26,30,34,38,\ldots]$$

$$= [0;-1,+1,\mathbf{-1},-2,\mathbf{+3},+2,\mathbf{-5},-2,\mathbf{+7},+2,\ldots]$$

$$= 2[0;3,2,3,\mathbf{2},3,1,\mathbf{2},1,3,\mathbf{4},3,1,\mathbf{4},1,3,\mathbf{6},3,1,\mathbf{6},1,\ldots]$$

(5.1-5)

$$\cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cfrac{12}{13+\cdots}}}}}} = \frac{2-\sqrt{e}}{\sqrt{e}-1} = 0.541494082\ldots$$

$$= [0;1,1,\mathbf{5},1,1,\mathbf{9},1,1,\mathbf{13},1,1,\ldots]$$

$$= 2*[0;3,1,\mathbf{2},3,1,\mathbf{4},3,1,\mathbf{6},3,1,\mathbf{8},\ldots]$$

$$= \frac{1}{2} * [1; 12,20,28,\ 36,44,\ 52,\ldots]$$

$$= \frac{1}{4} * [2;\ \ 6,40,14,\ 72,22,104,\ldots]$$

$$= \frac{1}{8} * [4;\ \ 3,80,\ 7,144,11,208,\ldots]$$

$$= \frac{1}{2} * [1; 4 * \mathbf{3},\ \ 4 * \mathbf{5}, 4 * \mathbf{7},\ \ 4 * \mathbf{9}, 4 * \mathbf{11},\ \ 4 * \mathbf{13}, \ldots]$$
$$= \frac{1}{4} * [2; 2 * \mathbf{3},\ \ 8 * \mathbf{5}, 2 * \mathbf{7},\ \ 8 * \mathbf{9}, 2 * \mathbf{11},\ \ 8 * \mathbf{13}, \ldots]$$
$$= \frac{1}{8} * [4; 1 * \mathbf{3}, 16 * \mathbf{5}, 1 * \mathbf{7}, 16 * \mathbf{9}, 1 * \mathbf{11}, 16 * \mathbf{13}, \ldots]$$

(6.1-7)

L.3.5

$$\sqrt[n]{e} = [1;n-1,1,1,3n-1,1,1,5n-1,1,1,7n-1,\ldots]$$

(7)

$$\frac{\sqrt[n]{e}-\frac{1}{\sqrt[n]{e}}}{\sqrt[n]{e}+\frac{1}{\sqrt[n]{e}}} = \tanh\left(\frac{1}{n}\right) = [0;n,3n,5n,7n,9n,11n,\ldots]$$

(8.1)

$$\tanh\left(\frac{1}{1}\right) = [0;1,3,5,7,9,11,13,15,17,19,\ldots]$$
$$= 2[0;2,1.5,10,3.5,18,5.5,26,7.5,34,\ldots]$$
$$= 2[0;2*1,1.5,2*5,3.5,2*9,5.5,2*13,7.5,2*17,\ldots]$$
$$= \frac{1}{2}[1;1,1,10,3,1,1,4,22,6,1,1,7,34,9,1,1,\ldots]$$
$$= \frac{1}{2}[1;1,1,2*5,3,1,1,4,2*11,6,1,1,7,2*17,9,$$
$$1,1,10,2*23,12,1,1,13,2*29,15,1,1,\ldots]$$

(8.2-6)

$$\tanh\left(\frac{1}{2}\right) = [0;2*1,2*3,2*5,2*7,2*9,2*11,\ldots]$$
$$= 2[0;4*1,1*3,4*5,1*7,4*9,1*11,\ldots]$$
$$= \frac{1}{2}[0;1*1,4*3,1*5,4*7,1*9,4*11,\ldots]$$

(8.7-9)

$$\tanh\left(\frac{1}{4}\right) = [0;4*1,4*3,4*5,4*7,4*9,4*11,\ldots]$$

$= 2[0;8*1,2*3,8*5,2*7,8*9,2*11,\ldots]$

$= 4[0;16*1,1*3.16*5,1*7,16*9$

$= \frac{1}{2}[0;2*1,8*3,2*5,8*7,2*9,8*11,\ldots]$

$= \frac{1}{4}[0;1*1,16*3,1*5,16*7,1*9, ,\ldots]$

(8.10-14)

L.3.6

$e^2 = [0;1,-\frac{1}{2},-2,+\frac{3}{2},+2,-\frac{5}{2},-2,+\frac{7}{2},+2,-\frac{9}{2},-2,+\frac{11}{2},+2,\ldots]$

$= [0;1,-1,-2*3,-\frac{5}{2},-2*7,-\frac{9}{2},-2*11,$

$-\frac{13}{2},-2*15,-\frac{17}{2},\ldots]$

$= [7;2,1,1,3,18,5,1,1,6,30,8,1,1,9,42,11,\ldots]$

$= [1;0*3,6*1,2,1,1,1*3,6*3,5,1,1,2*3,6*5,8,$

$1,1,3*3,6*7,11,1,1,4*3,6*9,14,1,1,\ldots]$

nota bene $[\ldots,a,0,b,c,\ldots] = [\ldots,a+b,c,\ldots]$

(9.1-4)

$e^{-2} = 1-2[0;+1,+1,-3,-1,+5,+1,-7,-1,+9,+1,\ldots]$

(10)

$\sinh(1) =$

$[0;1,-7,+\frac{21}{6},-\frac{6*43}{20},+\frac{20*73}{6*42},-\frac{6*42*111}{20*72},$

$+\frac{20*72*157}{6*42*110},-\frac{6*42*110*211}{20*72*156},+\frac{20*72*156*273}{6*42*110*210},\ldots]$

$$= \cfrac{1}{1+\cfrac{1}{-7+\cfrac{6}{21+\cfrac{20}{-43+\cfrac{42}{73+\cfrac{72}{-111+\cfrac{110}{157+\cfrac{156}{-211+\cdots}}}}}}}} \qquad (11.1,2)$$

Lateral

1)

cosh(1) = ???

2)

sin(1) =

$$[0;1,\ 5,\frac{19}{6},\frac{6*41}{20},\frac{20*71}{6*42},\frac{6*42*109}{20*72},\frac{20*72*155}{6*42*110},\frac{6*42*110*209}{20*72*156},\frac{20*72*156*271}{6*42*110*210},\ldots]$$

$$= \cfrac{1}{1+\cfrac{1}{5+\cfrac{6}{19+\cfrac{20}{41+\cfrac{42}{71+\cfrac{72}{109+\cfrac{110}{155+\cfrac{156}{209+\cdots}}}}}}}} \qquad (12.1,2)$$

3) e rekursiv, korrigiert

$$e = \cfrac{1}{e+\cfrac{2}{-1+\cfrac{1}{+7+\cfrac{6}{-21+\cfrac{20}{+43+\cfrac{42}{-73+\cfrac{72}{+111+\cfrac{110}{-157+\cfrac{156}{+211+\cdots}}}}}}}}}} \quad (14)$$

4)

$$\frac{5}{7} * \frac{19}{21} * \frac{41}{43} * \frac{71}{73} * \frac{109}{111} * \frac{155}{157} * \frac{209}{211} * \frac{271}{273} * \frac{341}{343} * \ldots = 0.53980\ldots = ??? \quad (15)$$

5)

$$\left(\frac{6*6}{5*7}\right) * \left(\frac{20*20}{19*21}\right) * \left(\frac{42*42}{41*43}\right) * \left(\frac{72*72}{71*73}\right) * \left(\frac{110*110}{109*111}\right) * \ldots = ???$$

Differenzenquotient = ???

Konvergenzverbesserung ???

vgl. Wallissches Produkt [a,b] (16)

[a] https://de.wikipedia.org/wiki/Wallissches_Produkt

[b] https://mathworld.wolfram.com/WallisFormula.html

6) Open

$$\cfrac{1}{1+\cfrac{1}{7+\cfrac{6}{21+\cfrac{20}{43+\cfrac{42}{73+\cfrac{72}{111+\cfrac{110}{157+\cfrac{156}{211+\cdots}}}}}}}} = ??? \quad (17.1)$$

$$\cfrac{1}{1+\cfrac{1}{-5+\cfrac{6}{19+\cfrac{20}{-41+\cfrac{42}{71+\cfrac{72}{-109+\cfrac{110}{155+\cfrac{156}{-209+\cdots}}}}}}}} = ??? \qquad (17.2)$$

$$\cfrac{1}{1+\cfrac{1}{5+\cfrac{6}{21+\cfrac{20}{41+\cfrac{42}{73+\cfrac{72}{109+\cfrac{110}{157+\cfrac{156}{209+\cdots}}}}}}}} = ??? \qquad (17.3)$$

$$\cfrac{1}{1+\cfrac{1}{7+\cfrac{6}{19+\cfrac{20}{43+\cfrac{42}{71+\cfrac{72}{111+\cfrac{110}{155+\cfrac{156}{211+\cdots}}}}}}}} = ??? \qquad (17.4)$$

.....

L.3.8
Find more!

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