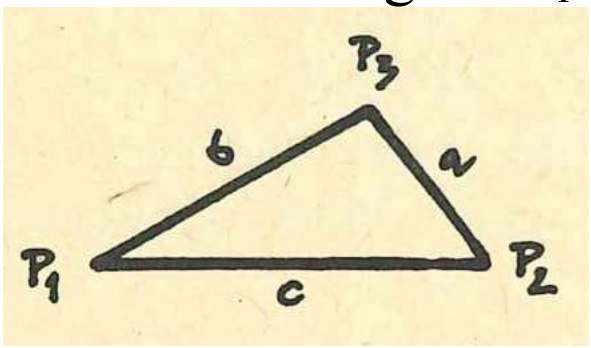
Unit 555.1.corr.corr

A synopsis:

Length of a line segment $\underline{P_1P_2} = a_{12} = a_{21}$

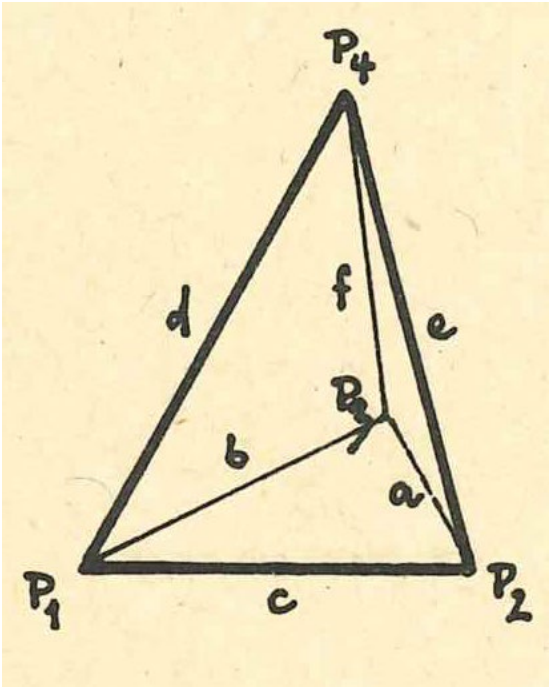
$L^2(\text{line segment } P_1P_2) = \frac{D_1}{-(-2)^1 * (1!)^2}$

Area of a triangle $\Delta P_1P_2P_3$



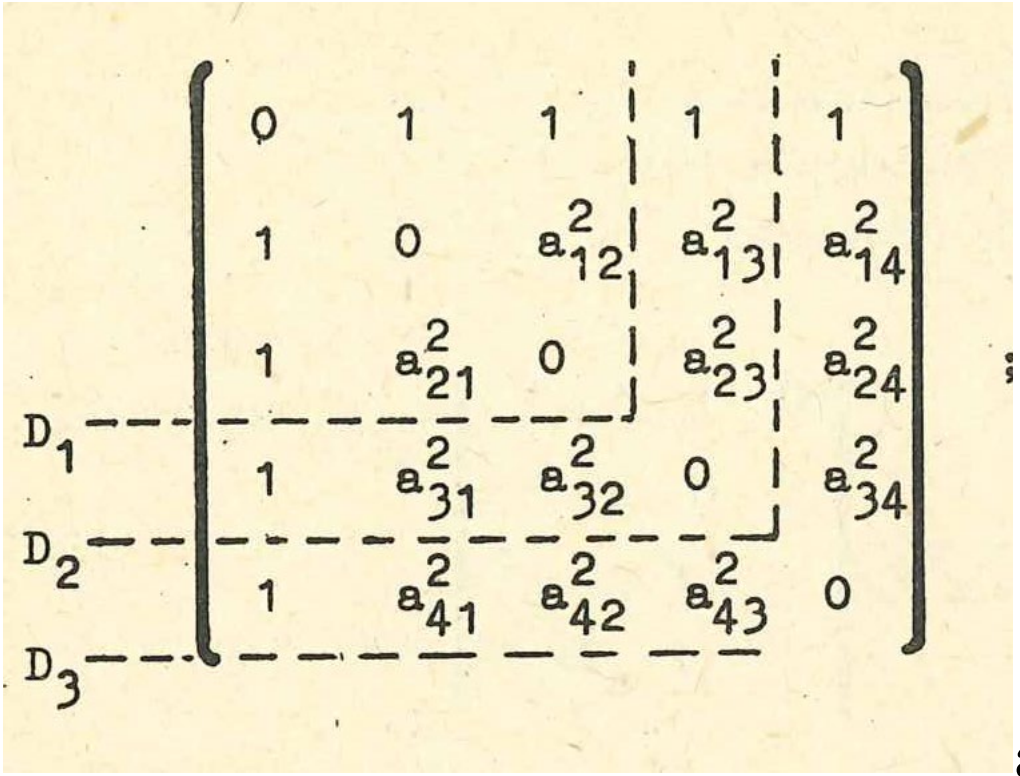
$A^2(\text{triangle } P_1P_2P_3) = \frac{D_2}{-(-2)^2 * (2!)^2}$

Volume of a tetrahedron



$V^2(\text{tetrahedron } P_1P_2P_3P_4) = \frac{D_3}{-(-2)^3 * (3!)^2}$

D_1, D_2, and D_3 are minors of the matrix Jade1



$a_{ij} = a_{ji}$

Problems

** Extension to the dimensions 4 and 5, discussion.

** Coefficients → determinant representation

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2 = -(-2)^1 * (1!)^2$$

$$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 4 \\ 1 & 1 & 0 & 5 \\ 1 & 4 & 5 & 0 \end{vmatrix} = -16 = -(-2)^2 * (2!)^2$$

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 4 & 9 \\ 1 & 1 & 0 & 5 & 10 \\ 1 & 4 & 5 & 0 & 13 \\ 1 & 9 & 10 & 13 & 0 \end{vmatrix} = 288$$

$$= -(-2)^3 * (3!)^2$$

** Extension to the dimensions 4 and 5.
** Find the construction formula of the coefficient determinants → **matrix Jade2**.
** Link to Bessel functions of the first kind [1]

$$-J_0\left(\sqrt{2x}\right) = -\sum_{i=0}^{\infty} \frac{\left(-\frac{x}{2}\right)^i}{(i!)^2}$$

$$= \sum_{i=0}^{\infty} \frac{x^i}{-(-2)^i * (i!)^2}$$

[1] https://en.wikipedia.org/wiki/Bessel_function#Bessel_functions_of_the_first_kind:_J%CE%B1

** Discuss $\sum_{i=0}^{k} \frac{D_i}{-(-2)^i * (i!)^2}$, k=0(1)∞ .
** Discuss applications.

Matrix Jade2

Δ_0= Δ_3= Δ_5=

	-1	$\Delta_1=$	-16	$\Delta_4=$	-9216
	↓	2	↓	288	↓
	↓	↓	↓	↓	↓
0	**1**	**1**	**1**	**1**	**1**
1	**0**	$\mathbf{1^2}$	$\mathbf{2^2}$	$\mathbf{3^2}$	$\mathbf{4^2}$
			+1=	+1=	+1=
1	**1**	**0**	5	10	17
				+3=	+3=
1	**4**	5	**0**	13	20
					+5=
1	**9**	10	13	**0**	25
1	**16**	17	20	25	**0**

=

$\Delta_0=$		$\Delta_2=$		$\Delta_4=$
-1	$\Delta_1=$	-16	$\Delta_3=$	-9216

$$
\begin{array}{cccccc}
 & \downarrow & 2 & \downarrow & 288 & \downarrow \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\left| \begin{array}{cccccc}
0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1^2 & 2^2 & 3^2 & 4^2 \\
1 & 1^2 & 0 & 1^2+2^2 & 1^2+3^2 & 1^2+4^2 \\
1 & 2^2 & 2^2+1^2 & 0 & 2^2+3^2 & 2^2+4^2 \\
1 & 3^2 & 3^2+1^2 & 3^2+2^2 & 0 & 3^2+4^2 \\
1 & 4^2 & 4^2+1^2 & 4^2+2^2 & 4^2+3^2 & 0
\end{array} \right|
\end{array}
$$

Matrix Jade2 = matrix Jade1

with $a_{ij} = (i-1)^2 + (j-1)^2$

$\Delta_0 = -(-2)^0 * (0!)^2 = -1$

$\Delta_1 = -(-2)^1 * (1!)^2 = 2$

$\Delta_2 = -(-2)^2 * (2!)^2 = -16$

$\Delta_3 = -(-2)^3 * (3!)^2 = 288$
$\Delta_4 = -(-2)^4 * (4!)^2 = -9216$
$\Delta_5 = -(-2)^5 * (5!)^2 = 460800$
$\Delta_6 = -(-2)^6 * (6!)^2 = -33177600$
$\Delta_7 = -(-2)^7 * (7!)^2 = 3251404800$
$\Delta_8 = -(-2)^8 * (8!)^2 = -416179814400$
$\Delta_i = -(-2)^i * (i!)^2$

** Lateral
Extend the matrix construction principle of Jade2 (f.e. 3rd power, factorial, …, **and more**) and check possible applications.

Matrix Jade3

$\Delta_0=$		$\Delta_2=$		$\Delta_4=$
-1	$\Delta_1=$	-32	$\Delta_3=$	-221184
↓	2	↓	1728	↓
↓	↓	↓	↓	↓

$$\begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1^3} & \mathbf{2^3} & \mathbf{3^3} & \mathbf{4^3} \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1^3 & 0 & 1^3+2^3 & 1^3+3^3 & 1^3+4^3 \\ 1 & 2^3 & 2^3+1^3 & 0 & 2^3+3^3 & 2^3+4^3 \\ 1 & 3^3 & 3^3+1^3 & 3^3+2^3 & 0 & 3^3+4^3 \\ 1 & 4^3 & 4^3+1^3 & 4^3+2^3 & 4^3+3^3 & 0 \end{vmatrix}$$

Matrix Jade3 = matrix Jade1

with $a_{ij} = (i-1)^3 + (j-1)^3$

$\Delta_0 = -(-2)^0 * (0!)^3 = -1$

$\Delta_1 = -(-2)^1 * (1!)^3 = 2$

$\Delta_2 = -(-2)^2 * (2!)^3 = -32$

$\Delta_3 = -(-2)^3 * (3!)^3 = 1728$

$\Delta_4 = -(-2)^4 * (4!)^3 = -221184$

$\Delta_5 = -(-2)^5 * (5!)^3 = 55296000$

$\Delta_6 = -(-2)^6 * (6!)^3 = 23887872000$

$\Delta_7 = -(-2)^7 * (7!)^3 = 16387080192000$

$\Delta_8 = -(-2)^8 * (8!)^3 = 16780370116608000$

$\Delta_i = -(-2)^i * (i!)^3$

** $2 \rightarrow 3 \rightarrow k \rightarrow \Delta_i = -(-2)^i * (i!)^k$

Proof by mathematical induction!

Next

Matrix Jade!

	$\Delta_0=$ -1 ↓ ↓	$\Delta_1=$ 2 ↓	$\Delta_2=$ -8 ↓ ↓	$\Delta_3=$ 96 ↓	$\Delta_4=$ -4608 ↓ ↓
0	**1**	**1**	**1**	**1**	**1**
1	**0**	**1!**	**2!**	**3!**	**4!**
1	**1!**	**0**	**1!+2!**	**1!+3!**	**1!+4!**
1	**2!**	**2!+1!**	**0**	**2!+3!**	**2!+4!**
1	**3!**	**3!+1!**	**3!+2!**	**0**	**3!+4!**
1	**4!**	**4!+1!**	**4!+2!**	**4!+3!**	**0**

Matrix Jade! = matrix Jade1

with $a_{ij} = (i-1)! + (j-1)!$

$\Delta_0 = -(-2)^0 * (0!) = -1$

$\Delta_1 = -(-2)^1 * (1!) = 2$

$\Delta_2 = -(-2)^2 * (1!*2!) = -8$

$\Delta_3 = -(-2)^3 * (1!*2!*3!) = 96$

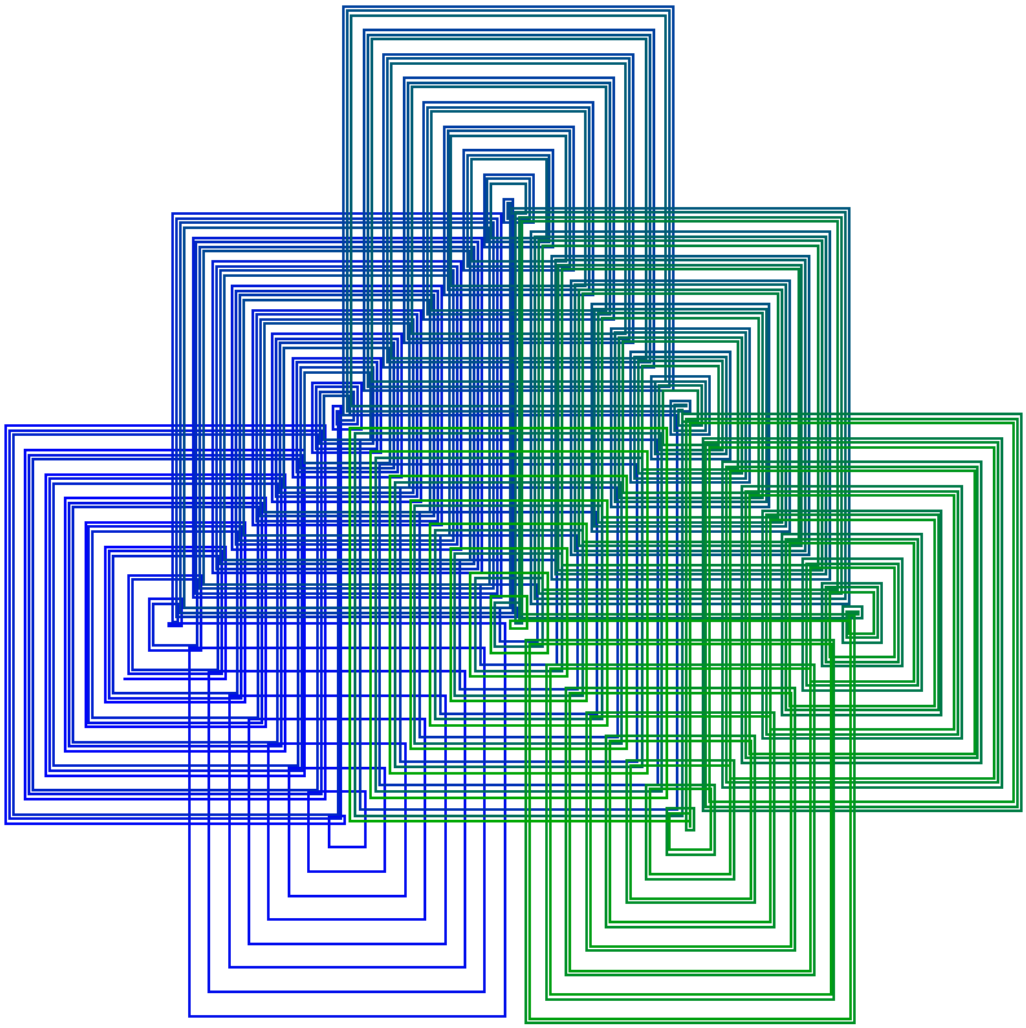
$\Delta_4 = -(-2)^4 * (1!*2!*3!*4!) = -4608$

$\Delta_5 = -(-2)^5 * (1!*2!*3!*4!*5!) = 1105920$

$\Delta_i = -(-2)^i * (1!*2!*3!*4!*\ldots*i!)$

$= -(-2)^i *$„((i!)!)“

Proof by mathematical induction!





25.10.2019 EPSalg DScalc