Unit 2, addendum 1

Example 3
A story about the first take-off of the young dragon Carl Friedrich Gauss and more

http://www.answers.com/Carl+Friedrich+Gauss?cat=technology

Problem

Add the natural numbers from 1 to 10 (100) as **efficiently** as possible.

Classical solution

(((((((1 + 2) + 3) + 4) + 5) + 6) + 7) + 8) + 9) + 10 (1)

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

3 6 10 15 21 28 36 45 **55**

Efficient solution (for this special case!) by suitable "tuning up" (tune up = make small changes to an engine so that it works as well as possible), here: creative exhausting of the freedom given by the commutative and the associative laws of addition, creative bracketing:

(1+10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6) = 5*11 = **55** (2)

↓ ↓ ↓ ↓ ↓

11 11 11 11 11

Colorencoding, opposite pairs → invariant!!!

Efficient solution for 1 to 100:

(1 + 100) + (2 + 99) + (3 + 98) + .. + (50 + 51) = 50*101 = **5050**

↓ ↓ ↓ ↓

101 101 101 101 (3)

A similar approach to find the efficient solution by creative exhausting of the freedom given by the commutative and the associative laws of addition, creative bracketing:

$s_{10} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

$s_{10} = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$

$2s_{10} = 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 = 10*11 = 110$

$s_{10} = \frac{1}{2} * 10 * 11 = 55$

$\rightarrow$ general solution $s_n = \sum_{i=1}^{n} i = \frac{1}{2} * n*(n+1)$. (4,5)

Another approach: palindromic summation

$1+2+3+4+5+6+5+4+3+2+1 = 36 = 6^2$

$= s_5 + 6 + s_5$

Generalization $6 \rightarrow n$

$1+2+3+....+n+...+3+2+1 = n^2$

$= 2s_{n-1} + n \rightarrow s_n = \frac{1}{2} n(n+1)$ (6,7)

Another remarkable palindromic sum

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 5^3 + 4^3 + 3^3 + 2^3 + 1^3 = 666$

$= \frac{1}{2} * 36 * 37 = \frac{1}{2} * 6^2 * (6^2 + 1)$

$= 1+2+3+4+...+35+6^2$, (8)

unique or generalizable?

The latter is the case, generalization:

$1^3 + 2^3 + 3^3 + ...^3 + n^3 + ... + 3^3 + 2^3 + 1^3$

$= \frac{1}{2} * n^2 * (n^2 + 1)$

$= 1+2+3+4+...+n^2$. (9)

Prime laterals:

$2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 = 666$. (10)

$2+3+5+7+11+13+17+19 = 77$ (11)

More!

Even more remarkable sums

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i^1 \right)^2 \qquad (12)$$

$$\sum_{i=1}^{n} (-1)^{n-i} i^2 = \sum_{i=1}^{n} i^1 \qquad (14)$$

Almost palindromic continued fractions
163 scf curiosities

$$\frac{3^5}{163} = 1 + \cfrac{1}{2 + \cfrac{1}{2*13 + \cfrac{1}{1 + \cfrac{1}{2}}}}$$

$$\frac{3^7}{163} = 13 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{13}}}}}$$

$$\frac{13}{163} = \cfrac{1}{12 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6}}}}$$

$$\frac{13+12}{163} = \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{12}}}}$$

(15.1,2,3,4)