Unit n °193

Infinite branched expressions

193.1
Our initial collection of infinite branched nested expressions. Find more.

$$1 = 4\,\Big|\,\frac{1}{(1+4(3))}\Big(3 + 4{\cdot}5\,\Big|\,\frac{1}{(1+4(3+4{\cdot}5(5)))}\Big(5 + 6{\cdot}7\,\Big|\,\frac{1}{(1+4(3+4{\cdot}5(5+6{\cdot}7)))}\Big(7 + 8{\cdot}9\,\Big|\,\frac{1}{(1+4(3+4{\cdot}5(5+6{\cdot}7(7+8{\cdot}9(9)))))}\Big(9 + 10{\cdot}11\,\Big|\,\frac{1}{(1+4(3+4{\cdot}5(5+6{\cdot}7(7+8{\cdot}9(9+10{\cdot}11(11))))))}\Big(11 + \ldots \tag{1}$$

$$1 = \frac{1}{3}\Bigg(2 + 2{\cdot}3\,\frac{1}{3+2{\cdot}3(6)}\Bigg(6 + 4{\cdot}5\,\frac{3}{(3+2{\cdot}3(6+4{\cdot}5(10)))}\Bigg(10 + 6{\cdot}7\,\frac{(3+2{\cdot}3(6+4{\cdot}5(10+6{\cdot}7(14))))}{(3+2{\cdot}3(6+4{\cdot}5(10+6{\cdot}7(14+8{\cdot}9(18)))))}\Bigg(18 + \ldots \tag{2}$$

$$1 = a_0 \left| \frac{1}{(b_0 + a_0(b_1))} \right. \left(b_1 + a_1 \left| \frac{{}^0q}{(b_0 + a_0(b_1 + a_1(b_2)))} \right. \left(b_2 + a_2 \left| \frac{{}^0q}{(b_0 + a_0(b_1 + a_1(b_2 + a_2(b_3))))} \right. \left(b_3 + a_3 \left| \frac{({}^0q)}{(b_0 + a_0(b_1 + a_1(b_2 + a_2(b_3 + a_3(b_4)))))} \right. \left(b_4 + \ldots \right. \right. \right. \right. \right.$$

$$((({}^1q)^{0}a + {}^0q)(b_1 + a_1(q_2)))$$

$$((({}^1q)^{0}a + {}^0q)(b_1 + a_1(b_2 + a_2(b_3 + a_3(b_4)))))$$

$$((({}^1q)^{0}a + {}^0q)(b_1 + a_1(b_2 + a_2(b_3 + a_3(b_4)))))$$

(3)

$a_i, b_i \in \mathbb{N}$

$a_0 < a_1 < a_2 < a_3 < \ldots$; $b_0 < b_1 < b_2 < b_3 < \ldots$

$$\frac{(n+2)e\ -2(n+1)}{(n+1)e\ -2n} = 1\ \frac{1}{(1\cdot 4+n)}\left(1 + 1\cdot 4\ \frac{1}{((1\cdot 4+n)3\cdot 6+n)}\left(1 + 3\cdot 6\ \frac{1}{(((1\cdot 4+n)3\cdot 6+n)5\cdot 8+n)}\left(1 + 5\cdot 8\ \frac{1}{((((1\cdot 4+n)3\cdot 6+n)5\cdot 8+n)7\cdot 10+n)}\left(1 + 7\cdot 10\ \frac{1}{(((((1\cdot 4+n)3\cdot 6+n)5\cdot 8+n)7\cdot 10+n)9\cdot 12+n)}\left(1 + \ldots\right.\right.\right.\right.\right. \tag{4}$$

$$\frac{1}{e-1} = [0;1,1,2,1,1,4,1,1,6,1,1,8,1,\ldots]$$

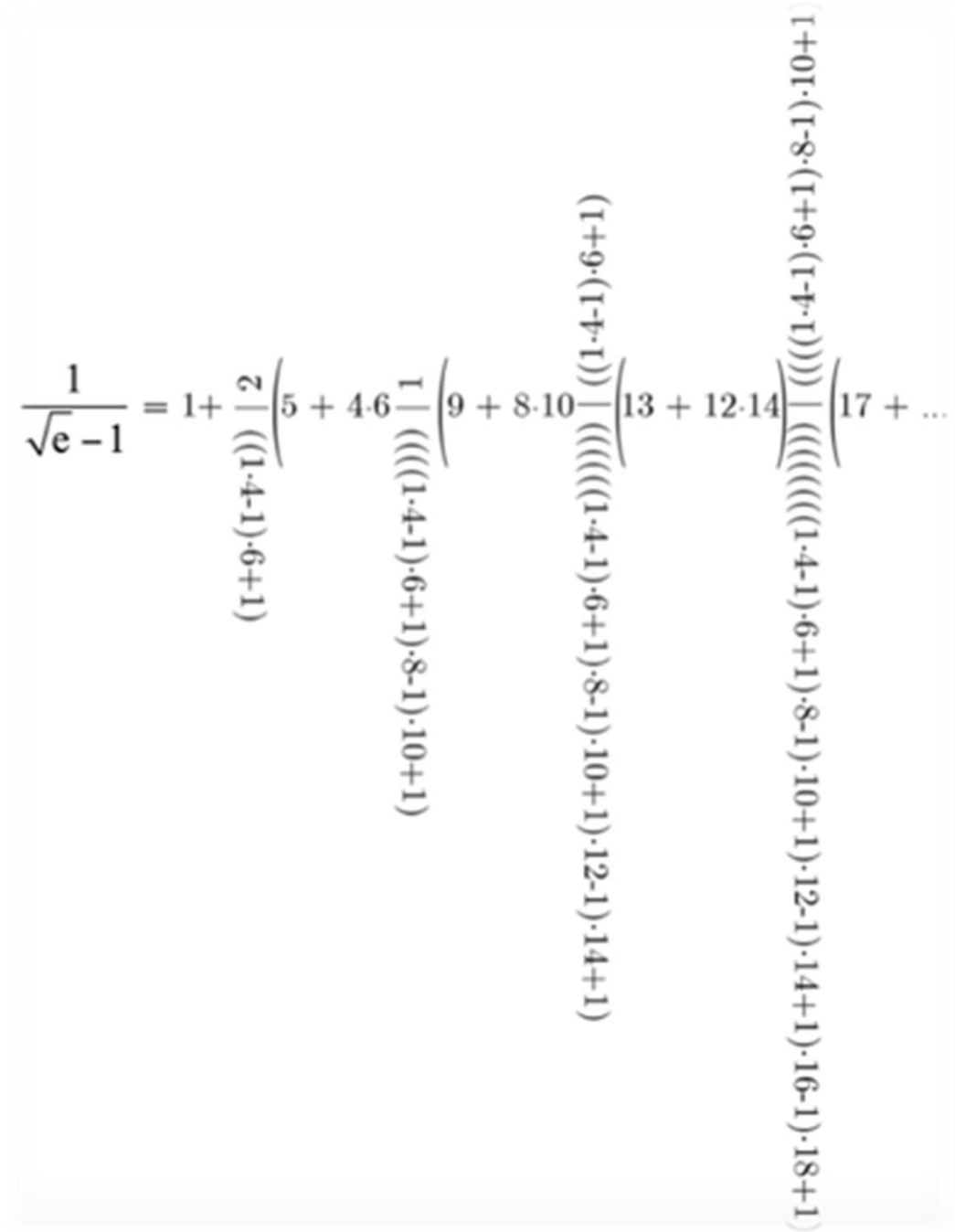
$$= \frac{|1}{1|} + \frac{|2}{2|} + \frac{|3}{3|} + \frac{|4}{4|} + \frac{|5}{5|} + \ldots$$

$$\frac{1}{e-1} = 1{\cdot}2\,\frac{1}{|((1{\cdot}2{\cdot}1){\cdot}3{+}1)}\,1 + 3{\cdot}4\,\frac{1}{|((((1{\cdot}2{\cdot}1){\cdot}3{+}1){\cdot}4{\cdot}1){\cdot}5{+}1)}\,1 + 5{\cdot}6\,\frac{1}{|(((((1{\cdot}2{\cdot}1){\cdot}3{+}1){\cdot}4{\cdot}1){\cdot}5{+}1){\cdot}6{\cdot}1){\cdot}7{+}1)}\,1 + 7{\cdot}8\,\frac{1}{|((((((((1{\cdot}2{\cdot}1){\cdot}3{+}1){\cdot}4{\cdot}1){\cdot}5{+}1){\cdot}6{\cdot}1){\cdot}7{+}1){\cdot}8{\cdot}1){\cdot}9{+}1)}\,1 + \ldots$$

(5)

$$\frac{1}{\sqrt{e}-1} = [1;1,1,5,1,1,9,1,1,13,1,1,17,1,\ldots]$$

$$= 1 + \frac{|2}{3|} + \frac{|4}{5|} + \frac{|6}{7|} + \frac{|8}{9|} + \frac{|10}{11|} + \ldots$$



$$\frac{1}{\sqrt{e}-1} = 1 + \frac{2}{|((1\cdot4-1)\cdot6+1)} \left(5 + 4\cdot6 \frac{1}{|((((1\cdot4-1)\cdot6+1)\cdot8-1)\cdot10+1)} \left(9 + 8\cdot10 \frac{1}{|(((((1\cdot4-1)\cdot6+1)\cdot8-1)\cdot10+1)\cdot12-1)\cdot14+1)} \left(13 + 12\cdot14 \frac{1}{|((((((((1\cdot4-1)\cdot6+1)\cdot8-1)\cdot10+1)\cdot12-1)\cdot14+1)\cdot16-1)\cdot18+1)} \left(17 + \ldots \right. \right. \right. \right.$$

(6)

193.2
Collection of infinite branched continued fractions[1)]
see Unit 12.C. Find more.

193.3
Hybrid forms

$$\sqrt[k]{n} = 1 + \left(\frac{0\cdot k+1}{1\cdot k}\right)\left(\frac{n-1}{n}\right)(1+\left(\frac{1\cdot k+1}{2\cdot k}\right)\left(\frac{n-1}{n}\right)(1+\left(\frac{2\cdot k+1}{3\cdot k}\right)\left(\frac{n-1}{n}\right)(1+\left(\frac{3\cdot k+1}{4\cdot k}\right)\left(\frac{n-1}{n}\right)(1+\cdots$$

$$= 1+\frac{1}{k}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1}{k+1})}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1}{k+1}(1+\frac{1}{2k+1}))}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1}{k+1}(1+\frac{1}{2k+1}(1+\frac{k+1}{3k+1})))}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1}{k+1}(1+\frac{1}{2k+1}(1+\frac{1}{3k+1}(1+\frac{2k+1}{4k+1}))))}\cdot\frac{n-1}{n}(1+\cdots$$

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$$\sqrt[k]{n} = 1+\frac{1}{k}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1|}{|k+1})}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1|}{|k+1}-\frac{1|}{|2})}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1|}{|k+1}-\frac{1|}{|2}-\frac{1|}{|2})}\cdot\frac{n-1}{n}(1+\frac{1}{k(1-\frac{k-1|}{|k+1}-\frac{1|}{|2}-\frac{1|}{|2}-\frac{1|}{|2})}\cdot\frac{n-1}{n}(1+\cdots$$

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More you can find in Unit 12.C.
Your turn is to continue this collection.

193.4
A collection of infinite continued fractions[1)] waiting for their conversion into branched infinite expressions
193.4.1

$$f(3,4)=\frac{|1}{1|}+\frac{|3}{3|}+\frac{|5}{5|}+\frac{|7}{7|}+\frac{|9}{9|}+\frac{|11}{11|}+\cdots=[0;1,1,3,1,1,7,1,1,11,1,1,15,\ldots]=\frac{1}{2}[1;8,16,24,32,40,\ldots]$$

$$f(5,8)=\frac{|1}{1|}+\frac{|5}{5|}+\frac{|9}{9|}+\frac{|13}{13|}+\frac{|17}{17|}+\frac{|21}{21|}+\cdots=[0;1,1,5,1,1,13,1,1,21,1,1,29,\ldots]=\frac{1}{2}[1;12,28,44,60,76,\ldots]$$

$$f(7,12)=\frac{|1}{1|}+\frac{|7}{7|}+\frac{|13}{13|}+\frac{|19}{19|}+\frac{|25}{25|}+\frac{|31}{31|}+\cdots=[0;1,1,7,1,1,19,1,1,31,1,1,43,\ldots]=\frac{1}{2}[1;16,40,64,88,112,\ldots]$$

$$f(2,2)=\frac{|1}{1|}+\frac{|2}{2|}+\frac{|3}{3|}+\frac{|4}{4|}+\frac{|5}{5|}+\frac{|6}{6|}+\cdots=[0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10\ldots]=\frac{1}{e-1}=\frac{1}{2}[1;6,10,14,18,\ldots]$$

$$f(4,6)=\frac{|1}{1|}+\frac{|4}{4|}+\frac{|7}{7|}+\frac{|10}{10|}+\frac{|13}{13|}+\frac{|16}{16|}+\cdots=[0;1,1,4,1,1,10,1,1,16,1,1,22,1,1,28\ldots]=\frac{1}{2}[1;10,22,34,46,58\ldots]$$

$$f(6,10)=\frac{|1}{1|}+\frac{|6}{6|}+\frac{|11}{11|}+\frac{|16}{16|}+\frac{|21}{21|}+\frac{|26}{26|}+\cdots=[0;1,1,6,1,1,16,1,1,26,1,1,36,1,1,46,\ldots]=\frac{1}{2}[1;14,34,54,74,94,\ldots]$$

(7)

$$\frac{|1}{1|}+\frac{|n+1}{n+1|}+\frac{|n+1}{2n+1|}+\frac{|n+1}{3n+1|}+\frac{|4n+1}{4n+1|}+\cdots=$$

$$[0;1,1,n+1,1,1,3n+1,1,1,5n+1,1,1,7n+1,\ldots]=$$

$$\frac{1}{2}[1;2(n+2),2(3n+2),2(5n+2),2(7n+2),\ldots] \qquad n=1,2,3,\ldots$$

(8)

Find a proof in 2by2 matrix representation.

Gaps

$$\frac{1|}{|1}+\frac{2|}{|2}+\frac{3|}{|3}+\frac{4|}{|4}+\frac{5|}{|5}+\frac{6|}{|6}+\ldots$$

$$=[0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,\ldots]$$

$$=\frac{1}{2}[1;6,10,14,18,22,\ldots]$$

$$=\frac{1}{e-1}$$

$$\frac{1|}{|1}+\frac{3|}{|3}+\frac{5|}{|5}+\frac{7|}{|7}+\frac{9|}{|9}+\frac{11|}{|11}+\ldots$$

$$=[0;1,1,3,1,1,7,1,1,9,1,1,11,1,1,\ldots]$$

$$=\frac{1}{2}[1;8,16,24,32,40,\ldots]$$

$$\frac{2|}{|2}+\frac{4|}{|4}+\frac{6|}{|6}+\frac{8|}{|8}+\frac{10|}{|10}+\frac{12|}{|12}+\ldots$$

$$=[0;1,1,3,1,1,5,1,1,7,1,1,9,1,1,\ldots]$$

$$\frac{1|}{|1}+\frac{4|}{|4}+\frac{7|}{|7}+\frac{10|}{|10}+\frac{13|}{|13}+\frac{16|}{|16}+\ldots$$

$$=[0;1,1,4,1,1,10,1,1,16,1,1,22,1,1,\ldots]$$

$$\frac{1|}{|1}+\frac{5|}{|5}+\frac{9|}{|9}+\frac{13|}{|13}+\frac{17|}{|17}+\frac{21|}{|21}+\ldots$$

$$=[0;1,1,5,1,1,13,1,1,21,1,1,29,1,1,\ldots]$$

$$\frac{1|}{|1}+\frac{7|}{|7}+\frac{13|}{|13}+\frac{19|}{|19}+\frac{25|}{|25}+\frac{31|}{31}+\ldots$$

$$=[0;1,1,7,1,1,19,1,1,31,1,1,43,1,1,\ldots]$$

Generalization

$$f(a,b) =$$

$$[0;1,1,a,1,1,a+b,1,1,a+2b,1,1,a+3b,1,1,a+4b,1,1,...] =$$

$$\frac{1}{2}[1;2(a+1),2(a+b+1),2(a+2b+1),2(a+3b+1),2(a+4b+1),...]$$

$$y(a,b) =$$

$$[2;1,a,1,1,a+b,1,1,a+2b,1,1,a+3b,1,1,a+4b,1,1,...]$$

$$\frac{y(a,b)-1}{y(a,b)+1} =$$

$$[0;2,2(a+1),2(a+b+1),2(a+2b+1),2(a+3b+1),2(a+4b+1),...]$$

to

$$f(2,2) =$$

$$[0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,...] =$$

$$\frac{1}{2}[1;6,10,14,18,22,...] =$$

$$\frac{1}{e-1} =$$

$$\frac{1|}{|1} + \frac{2|}{|2} + \frac{3|}{|3} + \frac{4|}{|4} + \frac{5|}{|5} + \frac{6|}{|6} + \frac{7|}{|7} + \frac{8|}{|8} + ... =$$

$$\frac{1|}{|1} + \frac{1|}{|1} + \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + ...$$

$$f(3,2) =$$

$$[0;1,1,3,1,1,5,1,1,7,1,1,9,1,1,11,1,1,...] =$$

$$\frac{1}{2}[1;8,16,24,32,40,...] =$$

$$? =$$

$$\frac{2}{|2} + \frac{4|}{|4} + \frac{6|}{|6} + \frac{8|}{|8} + \frac{10|}{|10} + \frac{12|}{|12} + \frac{14|}{|4} + \frac{16|}{|16} + ...$$

$$f(4,2) =$$

$$[0;1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,...] =$$

$$\frac{1}{2}[1;10,14,18,22,26,...] =$$

$$\frac{3e-8}{3-e} =$$

$$\frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \frac{7|}{|8} + \frac{8|}{|9} + \frac{9|}{|10} + ...$$

$$f(3,4) =$$

$$[0;1,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,...] =$$

$$\frac{1}{2}[1;8,16,24,32,40,...] =$$

$$? =$$

$$\frac{1|}{|1} + \frac{1|}{|1} + \frac{1|}{|3} + \frac{3|}{|5} + \frac{5|}{|7} + \frac{7|}{|9} + \frac{9|}{|11} + \frac{11|}{|13} + ... =$$

$$\frac{1|}{|1} + \frac{3|}{|3} + \frac{5|}{|5} + \frac{7|}{|7} + \frac{9|}{|9} + \frac{11|}{|11} + \frac{13|}{|13} + \frac{15|}{|15} + ...$$

$$f(5,4) =$$

$$[0;1,1,5,1,1,9,1,1,13,1,1,117,1,1,21,1,1,...] =$$

$$\frac{1}{2}[1;12,20,28,36,40,...] =$$

$$\frac{2-\sqrt{e}}{\sqrt{e}-1} =$$

$$\frac{2|}{|3} + \frac{4|}{|5} + \frac{6|}{|7} + \frac{8|}{|9} + \frac{10|}{|11} + \frac{12|}{|13} + \frac{14|}{|15} + \frac{16|}{|17} + ...$$

$$f(n-1,2n) =$$

$$[0;1,1,n\text{-}1,1,1,3n\text{-}1,1,1,5n\text{-}1,1,1,7n\text{-}1,1,1,9n\text{-}1,1,1,...] =$$

$$\frac{1}{2}[1;2n,6n,10n,14n,18n,...] =$$

$$\frac{\sqrt[n]{e}}{\sqrt[n]{e}+1}$$

193.4.2

Head extension

$$\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+\frac{8|}{|9}+...=$$

$$[0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,...]=$$

$$2[0;5,10,14,18,22,26,30,...]=$$

$$\frac{3-e}{e-2}$$

$$\frac{1|}{|2}+\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+\frac{8|}{|9}+...=$$

$$[0;2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,...]=$$

$$\frac{1}{2}[0;1,5,10,14,18,22,26,30,...]=$$

$$\frac{e-2}{e-1}$$

$$\frac{1|}{|2}+\frac{1|}{|2}+\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+...=$$

$$[0;2,2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,...]=$$

$$2[0;4,1,5,10,14,18,22,26,30,...]=$$

$$\frac{e-1}{3e-4}$$

$$\frac{1|}{|2}+\frac{1|}{|2}+\frac{1|}{|2}+\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+\frac{8|}{|9}+\ldots=$$

$$[0;2,2,2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,\ldots]=$$

$$\frac{1}{2}[0;1,4,1,5,10,14,18,22,26,30,\ldots]=$$

$$\frac{3e-4}{7e-9}$$

$$\frac{1|}{|2}+\frac{1|}{|2}+\frac{1|}{|2}+\frac{1|}{|2}+\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+\ldots=$$

$$[0;2,2,2,2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,\ldots]=$$

$$2[0;4,1,4,1,5,10,14,18,22,26,30,\ldots]=$$

$$\frac{7e-9}{17e-22}$$

$$\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+\frac{8|}{|9}+\frac{9|}{|10}+...=$$

$$[0;1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,...]=$$

$$\frac{1}{2}[0;10,14,18,22,26,30,...]=$$

$$\frac{3e-8}{3-e}$$

Transcendent and natural numbers are near neighbors via infinite continued fraction mirroring:

$$\frac{2}{|1|}+\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\frac{2}{|1|}+\frac{2}{|1|}+\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\frac{2}{|1|}+\frac{2}{|1|}+\frac{2}{|1|}+\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\vdots$$

$$\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\frac{2}{|1|}+\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\frac{2}{|1|}+\frac{2}{|1|}+\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\frac{2}{|1|}+\frac{2}{|1|}+\frac{2}{|1|}+\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

$$\vdots$$

$$\frac{3}{|2|}+\frac{4}{|3|}+\frac{5}{|4|}+\frac{6}{|5|}+\frac{7}{|6|}+\frac{8}{|7|}+\frac{9}{|8|}+\ldots=1$$

193.4.3
Transcendent and natural numbers are near neighbors via infinite continued fraction mirroring

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$$\frac{3-e}{e-2} = \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2[0;5,10,14,18,22,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

$$\frac{2-\sqrt{e}}{\sqrt{e}-1} = \frac{2|}{|3} + \frac{4|}{|5} + \frac{6|}{|7} + \frac{8|}{|9} + \frac{10|}{|11} + \frac{12|}{|13} + \ldots$$

$$= [0;1,1,5,1,1,9,1,1,13,1,1,17,1,1,\ldots]$$

$$= 2[0;3,1,2,3,1,4,3,1,6,3,1,8,3,\ldots]$$

$$1 = \frac{3|}{|2} + \frac{5|}{|4} + \frac{7|}{|6} + \frac{9|}{|8} + \frac{11|}{|10} + \frac{13|}{|12} + \ldots$$

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$$\frac{3-e}{e-2} = \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2[0;5,10,14,18,22,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

$$\frac{3e-8}{3-e} = \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \frac{7|}{|8} + \ldots$$

$$= [0;1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= \frac{1}{2}[1;10,14,18,22,26,30,\ldots]$$

$$1 = \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7}{|6} + \frac{8|}{|7} + \ldots$$

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$$\frac{3-e}{e-2} = \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2[0;5,10,14,18,22,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

$$\frac{e-2}{e-1} = \frac{1|}{|2} + \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= \frac{1}{2}[0;1,5,10,14,18,22,26,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

$$\frac{e-1}{3e-4} = \frac{1|}{|2} + \frac{1|}{|2} + \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2[0;4,1,5,10,14,18,22,26,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{2|}{|1} + \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

$$\frac{3e-4}{7e-9} = \frac{1|}{|2} + \frac{1|}{|2} + \frac{1|}{|2} + \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,2,2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots] \; \frac{1|}{|2} +$$

$$= \frac{1}{2}[0;1,4,1,5,10,14,18,22,26,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{2|}{|1} + \frac{2|}{|1} + \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

$$\frac{7e-9}{17e-22} = \frac{1|}{|2} + \frac{1|}{|2} + \frac{1|}{|2} + \frac{1|}{|2} + \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \ldots$$

$$= [0;2,2,2,2,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2[0;4,1,4,1,5,10,14,18,22,26,\ldots]$$

$$1 = \frac{2|}{|1} + \frac{2|}{|1} + \frac{2|}{|1} + \frac{2|}{|1} + \frac{2|}{|1} + \frac{3|}{|2} + \frac{4|}{|3} + \frac{5|}{|4} + \frac{6|}{|5} + \frac{7|}{|6} + \ldots$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$\left(1+\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\ldots\right)^{-1}+$$

$$\left(1+\frac{2|}{|1}+\frac{3|}{|2}+\frac{4|}{|3}+\frac{5|}{|4}+\frac{6|}{|5}+\frac{7|}{|6}+\ldots\right)=\mathrm{e}$$

$$\left(\frac{-1|}{|1}+\frac{1|}{|3}+\frac{3|}{|5}+\frac{5|}{|7}+\frac{7|}{|9}+\frac{9|}{|11}+\frac{11|}{|13}+\ldots\right)+$$

$$\left(\frac{1|}{|-1}+\frac{3|}{|1}+\frac{5|}{|3}+\frac{7|}{|5}+\frac{9|}{|7}+\frac{11|}{|9}+\frac{13|}{|11}+\ldots\right)=2$$

$$\left(\frac{1|}{|2}+\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\ldots\right)+$$

$$\left(\frac{1|}{|1}+\frac{2|}{|2}+\frac{3|}{|3}+\frac{4|}{|4}+\frac{5|}{|5}+\frac{6|}{|6}+\frac{7|}{|7}+\ldots\right)=1$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$a = \frac{1|}{|3} + \frac{3|}{|5} + \frac{5|}{|7} + \frac{7|}{|9} + \frac{9|}{|11} + \frac{11|}{|13} + \ldots$$

$$= [0;3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,\ldots]$$

$$= 0.28319234169228049286689887344036139 57\ldots$$

$$b = \frac{3|}{|1} + \frac{5|}{|3} + \frac{7|}{|5} + \frac{9|}{|7} + \frac{11|}{|9} + \frac{13|}{|11} + \ldots$$

$$= [1;2,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,\ldots]$$

$$= 1.35980199995545872556536992420668842 38\ldots$$

$$c = \frac{1|}{|1} + \frac{3|}{|3} + \frac{5|}{|5} + \frac{7|}{|7} + \frac{9|}{|9} + \frac{11|}{|11} + \ldots$$

$$= [0;1,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,\ldots]$$

$$= 0.56201675095896235730828218149342828 13\ldots$$

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \frac{2c \mp 1}{1 \mp c}$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Lateral

Transcendent-natural-number neighborhood

$$\frac{2*2}{1*3}*\frac{4*4}{3*5}*\frac{6*6}{5*7}*\frac{8*8}{7*9}*\frac{10*10}{9*11}*\frac{12*12}{11*13}*\ldots=\frac{\pi}{2}$$

$$\frac{3*3}{1*5}*\frac{5*5}{3*7}*\frac{7*7}{5*9}*\frac{9*9}{7*11}*\frac{11*11}{9*13}*\frac{13*13}{11*15}*\ldots=3$$

$$\prod_{i=2}^{\infty}\frac{i^2+1}{i^2-1}=\frac{\sinh(\pi)}{\pi}$$

$$\prod_{i=2}^{\infty}\frac{i^2}{i^2-1}=2$$

$$\prod_{i=2}^{\infty}\frac{i^2+1}{i^2}=?$$

$$2\prod_{i=2}^{\infty}\frac{p_i^2}{p_i^2-1}=\left(\frac{\pi}{2}\right)^2\ ;\ p_2=3,\ p_3=5,\ \text{prime sequence}$$

$$2\prod_{i=2}^{\infty}\frac{p_i^2+1}{p_i^2-1}=2$$

$$2\prod_{i=2}^{\infty}\frac{p_i^2+1}{p_i^2}=?$$

64.7

$$x(n) = \left(\frac{2^{n+1}+1}{2^{n+1}-1}\right)^{2^n}$$

$$\lim_{n\to\infty} x(n) = e$$

$$\lim_{n\to\infty} \frac{x(n)-x(n+1)}{x(n+1)-x(n+2)} = \mathbf{4}$$

64.8

Input: $x := 0\,;\ y := 1$
$i = 0(1)n$
$z := y - x$
$x := \frac{1}{2}(x+y)$
$y := \sqrt{x\,y}$
$d := \frac{z}{y-x}$
Output: $y(n)\,;\ d(n)$

$$\lim_{n\to\infty} y(n) = \frac{2}{\pi}$$

$$\lim_{n\to\infty} d(n) = \mathbf{4}$$

193.4.4
Triple concerto 1
Infinite sum, continued fraction, and determinant

$$\frac{\sum_{i=0}^{\infty} \frac{1}{i!(i+1)}}{\sum_{i=0}^{\infty} \frac{1}{i!i!}} = [0,1,2,3,4,5,6,7,8,9,...]$$

$$= 0.697774658\ldots$$

$$= -\lim_{\dim \to \infty} \begin{vmatrix} 0 & \frac{1}{0!1!} & \frac{1}{1!2!} & \frac{1}{2!3!} & \frac{1}{3!4!} & \frac{1}{4!5!} & \cdots \\ 1 & 1 & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \cdots \\ 1 & 0 & 1 & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \cdots \\ 1 & 0 & 0 & 1 & \frac{1}{1!1!} & \frac{1}{2!2!} & \cdots \\ 1 & 0 & 0 & 0 & 1 & \frac{1}{1!1!} & \cdots \\ 1 & 0 & 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{vmatrix}$$

Lateral

[0;1,2,3,4,5,6,7,8,9,…]
= 2*[0;2,1,6,2,10,3,14,4,18,5,22,6,…]
= 2*([0;1,2,3,4,5,6,7,8,9,…]entangled
[0;2,6,10,14,18,22,26,30,…])

$$[0;2,6,10,14,18,22,26,30,\ldots] = \frac{e-1}{e+1}$$

$$= \frac{1|}{|1} + \frac{4|}{|1} + \frac{2|}{|2} + \frac{3|}{|3} + \frac{4|}{|4} + \frac{5|}{|5} + \frac{6|}{|6} + \frac{7|}{|7} + \ldots$$

$$= \frac{1|}{|2} + \frac{1|}{|5} + \frac{4|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \frac{7|}{|8} + \ldots$$

$$= 0.462118715\ldots$$

2*[0;2,1,6,1,10,1,14,1,18,1,22,1,…]
= [0;1,2,3,2,5,2,7,2,9,2,11,2,…]
= 0.695972387…

$$[0;1,1,3,1,5,1,7,1,9,1,11,1,\ldots] = \tan(1) - 1$$

$$[0;1,3,5,7,9,11,\ldots] = \tanh(1) = \frac{e^2-1}{e^2+1}$$

2*[0;2,1,4,1,6,1,8,1,10,1,12,1,14,1,…]
= [0;1,2,2,2,3,2,4,2,5,2,6,2,7,2,…]

$= 0.706778468\ldots$

$[0;1,1,2,1,3,1,4,1,5,1,6,1,7,1,\ldots]$

$$= \frac{1}{|1} + \frac{2}{|2} + \frac{2}{|2} + \frac{3}{|3} + \frac{3}{|3} + \frac{4}{|4} + \frac{4}{|4} + \frac{5}{|5} + \frac{5}{|5} + \frac{6}{|6} + \frac{6}{|6} + \ldots$$

$$= \frac{1}{|1} + \frac{1}{|1} + \frac{1}{|2} + \frac{2}{|2} + \frac{2}{|3} + \frac{3}{|3} + \frac{3}{|4} + \frac{4}{|4} + \frac{4}{|5} + \frac{5}{|5} + \frac{5}{|6} + \ldots$$

$= 0.575920323\ldots$

$$\frac{1}{|2} + \frac{2}{|3} + \frac{3}{|4} + \frac{4}{|5} + \frac{5}{|6} + \frac{6}{|7} + \frac{7}{|8} + \ldots = \frac{3-e}{e-1}$$

$$= [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,\ldots]$$

$$= 2*[0;5,10,14,18,22,26,30,\ldots]$$

$= 0.392211191\ldots$

$$\frac{1}{|3} + \frac{2}{|4} + \frac{3}{|5} + \frac{4}{|6} + \frac{5}{|7} + \frac{6}{|8} + \frac{7}{|9} + \frac{8}{|10} + \ldots$$

$= [0;3,2,3,1,2,1,3,4,3,1,4,1,3,6,3,1,6,1,\ldots]$

$= 0.290616693\ldots$

193.4.5
Triple concerto 2

$$\frac{\sum_{i=0}^{\infty}\frac{1}{i!(i+1)}}{\sum_{i=0}^{\infty}\frac{1}{i!i!}}=[0,1,2,3,4,5,6,7,8,9,...]$$

$$=0.697774658\ldots$$

$$\frac{\sum_{i=0}^{\infty}\frac{(-1)^i}{i!(i+1)}}{\sum_{i=0}^{\infty}\frac{(-1)^i}{i!i!}}=[0;1,-2,3,-4,5,-6,7,-8,9,...]$$

$$=2.575920322\ldots$$

$$=\lim_{\dim\to\infty}\begin{vmatrix} 0 & \frac{1}{0!1!} & \frac{1}{1!2!} & \frac{1}{2!3!} & \frac{1}{3!4!} & \frac{1}{4!5!} & \cdots \\ -1 & 1 & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \cdots \\ 1 & 0 & 1 & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \cdots \\ -1 & 0 & 0 & 1 & \frac{1}{1!1!} & \frac{1}{2!2!} & \cdots \\ 1 & 0 & 0 & 0 & 1 & \frac{1}{1!1!} & \cdots \\ -1 & 0 & 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{vmatrix}$$

Lateral

$$[0;1,-2,3,-4,5,-6,7,-8,9,\ldots]$$

$$=[2;1,1,2,1,3,1,4,1,5,1,6,1,7,1,8,1,\ldots]$$

$$=2+\frac{1}{|1}+\frac{2}{|2}+\frac{2}{|2}+\frac{3}{|3}+\frac{3}{|3}+\frac{4}{|4}+\frac{4}{|4}+\frac{5}{|5}+\frac{5}{|5}+\frac{6}{|6}+\frac{6}{|6}+\ldots$$

$$=2+\frac{1}{|1}+\frac{1}{|1}+\frac{1}{|2}+\frac{2}{|2}+\frac{2}{|3}+\frac{3}{|3}+\frac{3}{|4}+\frac{4}{|4}+\frac{4}{|5}+\frac{5}{|5}+\frac{5}{|6}+\ldots$$

193.4.6

Double concerto 1

$$\frac{1|}{|1}+\frac{2|}{|2}+\frac{3|}{|3}+\frac{4|}{|4}+\frac{5|}{|5}+\frac{6|}{|6}+\frac{7|}{|7}+\ldots$$

$$=[0;1,1,2,1,1,4,1,1,6,1,1.,8,1,1,\ldots]$$

$$=\frac{1}{2}[1;6,10,14,18,22,26,30,\ldots]$$

$$=\frac{1}{e-1}=0.581976706\ldots$$

$$\frac{1|}{|1}+\frac{1|}{|1}+\frac{2|}{|2}+\frac{2|}{|2}+\frac{3|}{|3}+\frac{3|}{|3}+\frac{4|}{|4}+\frac{4}{4}+\frac{5|}{|5}+\frac{5|}{|5}+\frac{6|}{|6}+\frac{6|}{|6}+\ldots$$

$$=[0;1,1,1,2,1,3,1,4,1,5,1,6,1,7,\ldots]$$

$$=[0;-1,1,-2,3,-4,5,-6,7,\ldots]$$

$$=0.634549847\ldots$$

$$\frac{1|}{|2}+\frac{2|}{|3}+\frac{3|}{|4}+\frac{4|}{|5}+\frac{5|}{|6}+\frac{6|}{|7}+\frac{7|}{|8}+\frac{8|}{|9}+\frac{9|}{|10}+\ldots$$

$$=[0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1\ldots]$$

$$=2[0;5,10,14,18,22,26,30,\ldots]$$

$$=\frac{3-e}{e-2}=0.392211191\ldots$$

$$\frac{1}{|2|}+\frac{1}{|2|}+\frac{2}{|3|}+\frac{2}{|3|}+\frac{3}{|4|}+\frac{3}{|4|}+\frac{4}{|5|}+\frac{4}{5}+\frac{5}{|6|}+\frac{5}{|6|}+\frac{6}{|7|}+\frac{6}{|7|}+\ldots$$

$$=0.41839748\ldots$$

$$=?$$

$$\frac{1}{|1|}+\frac{1}{|2|}+\frac{2}{|2|}+\frac{2}{|3|}+\frac{3}{|3|}+\frac{3}{|4|}+\frac{4}{|4|}+\frac{4}{|5|}+\frac{5}{|5|}+\ldots$$

$$=[0;1,2,1,3,1,4,1,5,1,6,1,7,1,8,1,9,1,\ldots]$$

$$=[-1;-2,1,-2,3,-4,5,-6,7,-8,9,\ldots]$$

$$\frac{1}{|1|}+\frac{2}{|1|}+\frac{2}{|2|}+\frac{3}{|2|}+\frac{3}{|3|}+\frac{4}{|3|}+\frac{4}{|4|}+\frac{5}{|4|}+\frac{5}{|5|}+\ldots$$

$$=0.461487412\ldots=?$$

193.4.7
Double concerto 2

[0;1,2,3,4,5,6,7,8,9,…]
= 2*[0;2,1,6,2,10,3,14,4,18,5,22,6,…]

2*[0;2,1,6,1,10,1,14,1,18,1,22,1,…]
= [0;1,2,3,2,5,2,7,2,9,2,11,2,…]
= 0.695972387…

[0;1,1,3,1,5,1,7,1,9,1,11,1,…] = tan(1) – 1

2*[0;2,1,4,1,6,1,8,1,10,1,12,1,14,1,…]
= [0;1,2,2,2,3,2,4,2,5,2,6,2,7,2,…]
= 0.706778468…

$$\frac{1|}{|3}+\frac{2|}{|4}+\frac{3|}{|5}+\frac{4|}{|6}+\frac{5|}{|7}+\frac{6|}{|8}+\frac{7|}{|9}+\frac{8|}{|10}+...$$

= [0;3,2,3,1,2,1,3,4,3,1,4,1,3,6,3,1,6,1,...]

= 0.290616693...

[0;1,1,2,1,3,1,4,1,5,1,6,1,7,1,…]
= 0.575920323…

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
How about that?
[0;2,1^2,6,2^2,10,3^2,14,4^2,18,5^2,22,6^2,26,7^2,…]
[0;2,1,6,3,10,5,14,7,18,9,22,11,26,13,…]
[0;2,2,6,6,10,10,14,14,18,18,22,22,26,26,…]
[0;1,1,3,3,5,5,7,7,9,9,11,11,13,13,…]
[0;1,1,2,2,3,3,4,4,5,5,6,6,7,7,8,8,9,9,10,10,…]

193.4.8
Find infinite branched nested expressions for

$$\sqrt{2} = 1 + \frac{2}{5}(1 + \frac{1}{29}(1 + \frac{5}{169}(1 + \frac{29}{985}(1 + \frac{169}{5741}(1 + \ldots$$

$$\sqrt{3} = 1 + \frac{2}{3}(1 + \frac{1}{11}(1 + \frac{3}{41}(1 + \frac{11}{153}(1 + \frac{41}{571}(1 + \ldots$$

$$\Phi = \frac{1}{2}(\sqrt{5} + 1) =$$

$$1 + \frac{1}{\text{fib}(2)}(1 + \frac{\text{fib}(0)}{\text{fib}(4)}(1 + \frac{\text{fib}(2)}{\text{fib}(6)}(1 + \frac{\text{fib}(4)}{\text{fib}(8)}(1 + \frac{\text{fib}(6)}{\text{fib}(10)}(1 + \ldots$$

$$1 + \frac{1}{2}(1 + \frac{1}{5}(1 + \frac{2}{13}(1 + \frac{5}{34}(1 + \frac{13}{89}(1 + \ldots$$

$$\Phi^{-1} = \frac{1}{2}(\sqrt{5} - 1) =$$

$$\frac{1}{\text{fib}(4)}(3 + \frac{\text{fib}(0)}{\text{fib}(8)}(3 + \frac{\text{fib}(4)}{\text{fib}(12)}(3 + \frac{\text{fib}(8)}{\text{fib}(16)}(3 + \frac{\text{fib}(12)}{\text{fib}(20)}(3 + \ldots$$

$$\frac{1}{5}(3 + \frac{1}{34}(3 + \frac{5}{233}(3 + \frac{34}{1597}(3 + \frac{233}{10946}(3 + \ldots$$

$$\frac{\text{fib}(n+2)e - 2\text{fib}(n+1)}{\text{fib}(n+1)e - 2\text{fib}(n)}$$

$$\sqrt[3]{2} = 1 + \frac{1}{6} * \frac{1}{1}(1 + \frac{1}{6} * \frac{4}{2}(1 + \frac{1}{6} * \frac{7}{3}(1 + \frac{1}{6} * \frac{10}{4}(1 + \frac{1}{6} * \frac{13}{5}(1 + \ldots$$

$2^{\sqrt{2}}$, π, π^2, e^{π}, $T(\pi)$, $T(\pi^2)$, ...

193.4.9
Appendix 1
Doublings

$$\mathrm{scf}\left(\frac{(n+2)e-2(n+1)}{(n+1)e-2n}\right)=$$

$$\mathrm{scf}\left(1+\frac{1}{n+3+\frac{1}{\frac{e-2}{6-2e}}}\right)=[1;\ n+3,$$

1, 3, 1, 1, 1, 3,
3, 3, 1, 3, 1, 3,
5, 3, 1, 5, 1, 3,
7, 3, 1, 7, 1, 3,
9, 3, 1, 9, 1, 3,
11, 3, 1, 11, 1, 3,
13, 3, 1, 13, 1, 3,
15, 3, 1, 15, 1, 3,

17, 3, 1, 17, 1, 3,
19, 3, 1, 19, 1, 3,
21, 3, 1, 21, 1, 3,
23, 3, 1, 23, 1, 3,
25, 3, 1, 25, 1, 3,
27, 3, 1, 27, 1, 3, …]

n=0,1,2,3,4,…

Appendix 2

Lateral

Doublings

$$\text{scf}\left(\frac{\text{fib}(n+2)e-2\text{fib}(n+1)}{\text{fib}(n+1)e-2\text{fib}(n)}\right)=[1;??]$$

fib(0)=fib(1)=1; fib(n+2)=fib(n+1)+fib(n)

n=0,1,2,3,4,…

n=2

$$\text{scf}\left(\frac{\text{fib}(4)e-2\text{fib}(3)}{\text{fib}(3)e-2\text{fib}(red2)}\right)=\text{scf}\left(\frac{5e-6}{3e-4}\right)=$$

[1; 1, 4,
1, 3, 1, 1, 1, 3,
3, 3, 1, 3, 1, 3,
5, 3, 1, 5, 1, 3,
7, 3, 1, 7, 1, 3,

9, 3, 1, 9, 1, 3,
11, 3, 1, 11, 1, 3,
13, 3, 1, 13, 1, 3,
15, 3, 1, 15, 1, 3,
17, 3, 1, 17, 1, 3,
19, 3, 1, 19, 1, 3,
21, 3, 1, 21, 1, 3,
23, 3, 1, 23, 1, 3,
25, 3, 1, 25, 1, 3,
27, 3, 1, 27, 1, 3, …]

n=5

$$\mathrm{scf}\left(\frac{\mathrm{fib}(7)e-2\mathrm{fib}(6)}{\mathrm{fib}(6)e-2\mathrm{fib}(\mathrm{red5})}\right)=\mathrm{scf}\left(\frac{21e-26}{13e-16}\right)=$$

[1; 1, 1, 1, 1, 4,
1, 3, 1, 1, 1, 3,
3, 3, 1, 3, 1, 3,
5, 3, 1, 5, 1, 3,
7, 3, 1, 7, 1, 3,
9, 3, 1, 9, 1, 3,
11, 3, 1, 11, 1, 3,
13, 3, 1, 13, 1, 3,
15, 3, 1, 15, 1, 3,
17, 3, 1, 17, 1, 3,
19, 3, 1, 19, 1, 3,

21, 3, 1, 21, 1, 3,
23, 3, 1, 23, 1, 3,
25, 3, 1, 25, 1, 3,
27, 3, 1, 27, 1, 3, …]

Conjecture

$$\text{scf}\left(\frac{\text{fib}(n+2)e-2\text{fib}(n+1)}{\text{fib}(n+1)e-2\text{fib}(n)}\right)=$$

[1; $1^{\overline{n-1}}$, 4,
1, 3, 1, 1, 1, 3,
3, 3, 1, 3, 1, 3,
5, 3, 1, 5, 1, 3,
7, 3, 1, 7, 1, 3,
9, 3, 1, 9, 1, 3,
11, 3, 1, 11, 1, 3,
13, 3, 1, 13, 1, 3,
15, 3, 1, 15, 1, 3,
17, 3, 1, 17, 1, 3,
19, 3, 1, 19, 1, 3,
21, 3, 1, 21, 1, 3,
23, 3, 1, 23, 1, 3,
25, 3, 1, 25, 1, 3,
27, 3, 1, 27, 1, 3, …]
n=0,1,2,3,4,…
Find a proof.

Appendix 3

Transformation of the infinite scf
$[a_0;a_1,a_2,a_3,a_4,\ldots]$
into an infinite nested expression:

$$[a_0;a_1,a_2,a_3,a_4,...] =$$

$$a_0+\frac{1}{d_2}(a_2+\frac{d_0}{d_4}(a_4+\frac{d_2}{d_6}(a_6+\frac{d_4}{d_8}(a_8+\frac{d_6}{d_{10}}(a_{10}+...$$

$$d_0=1, \qquad \underset{\downarrow}{d_2} \qquad \underset{\downarrow}{d_4} \qquad \underset{\downarrow}{d_6} \qquad \underset{\downarrow}{d_{2n}}$$

$$d_{2n}=\begin{vmatrix} a_1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -1 & a_2 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & a_3 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -1 & a_4 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & -1 & a_5 & 1 & \ddots & 0 \\ 0 & 0 & 0 & 0 & -1 & a_6 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & a_{2n} \end{vmatrix}$$

Difference quotient

$$\lim_{n\to\infty} \frac{a_{2n}}{a_{2(n-1)}} * \frac{d_{2(n-2)}}{d_{2n}}$$

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193.5

193.5.1

Discuss applications of Unit 193 in sc&eng.

193.5.2

Discuss internal aspects of Unit 193

** **Aesthetic aspect** (collection of "beautiful formulas"),

** **Taxonomic aspect,**

** **Heuristic aspect.**

193.5.3

Discuss the difference between

** **a collection of "beautiful formulas"** and

** **a collection of art objects.**

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1) In this unit we use for the infinite {generalized / simple} continued fraction {(gcf) / (scf)} the notations

$$a_0 + \cfrac{b_1}{a_1 + \cfrac{b_2}{a_2 + \cfrac{b_3}{a_3 + \ddots}}} = a_0 + \frac{b_1|}{|a_1} + \frac{b_2|}{|a_2} + \frac{b_3|}{|a_3} + \cdots = a_0 + \left.\frac{b_i|}{|a_i}\right|_{i=1}^{\infty}$$

..... (1)

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ddots}}} = [a_0; a_1, a_2, a_3, \dots] = [a_0; a_i]_{i=1}^{\infty}$$

..... (2)

$\left.\frac{b_i|}{|a_i}\right|_{i=1}^{\infty}$ is the mirror image of $\left.\frac{a_i|}{|b_i}\right|_{i=1}^{\infty}$.

..... (3)

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References

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Links to

** Unit 12.C
** Deb 20