Unit 161, part A revised

Thirty-six Views of Mount Phi, part A

161.A.1

$$\Phi = \frac{1}{2}\left(1+\sqrt{5}\right)$$

$$= 1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}} = [1;1,1,1,1,\ldots] \qquad (1.1,2)$$

161.A.2

$$\Phi = \lim_{i\to\infty} \frac{\text{fib}(i+1)}{\text{fib}(i)}$$

$$\text{fib}(i+2) = \text{fib}(i+1) + \text{fib}(i)\ ;\ i=0,1,2,3,\ldots$$

$$\text{fib}(0) = \text{fib}(1) = 1 \qquad (2)$$

161.A.3

$$x^2 - x - 1 = 0 \rightarrow x_1 = \Phi,\ x_2 = -\frac{1}{\Phi} \qquad (3)$$

161.A.4

$$\Phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}} \qquad (4)$$

161.A.5 The Great Seven Wave off Phi

$$3-\Phi=\frac{1}{2}\left(5-\sqrt{5}\right)=$$

$$1+\frac{1}{3}\left(1+\frac{1}{s_1}\left(1+\frac{1}{s_2}\left(1+\frac{1}{s_3}\left(1+\frac{1}{s_4}\left(1+\frac{1}{s_5}(1+\cdots)\right)\right)\right)\right)\right)$$

$s_1=8$, $s_2=7$

$$s_3=7-\frac{1}{8}=\frac{55}{8}\ ,\ s_4=7-\frac{1}{7}=\frac{48}{7}$$

$$s_5=7-\frac{1}{7-\frac{1}{8}}=\frac{377}{55}\ ,\ s_6=7-\frac{1}{7-\frac{1}{7}}=\frac{329}{48}$$

$$s_7=7-\frac{1}{7-\frac{1}{7-\frac{1}{8}}}=\frac{2584}{377}\ ,\ s_8=7-\frac{1}{7-\frac{1}{7-\frac{1}{7}}}=\frac{2255}{329}$$

$$s_9=7-\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{8}}}}=\frac{17711}{2584}\ ,\ s_{10}=7-\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{7}}}}=\frac{15456}{2255}$$

…… (5.1)

$$s_{2k+1}=7-\frac{1}{s_{2k-1}}\ ,\ k=1,2,3,4,\ldots$$

$$s_1=8 \qquad (5.2)$$

$$s_{2k+1}=\frac{f(4k+5)}{f(4k+1)}\ ,\ k=0,1,2,3,\ldots \qquad (5.3)$$

$$\lim_{k\to\infty} s_{2k+1}=2+3\Phi$$

(5.4)

$$\lim_{k\to\infty} \frac{s_{2k+1}-s_{2k+3}}{s_{2k+3}-s_{2k+5}} = (2+3\Phi)^2$$

(5.5)

$$s_{2k} = 7 - \frac{1}{s_{2k-2}}\text{ , k=1,2,3,4,…}$$

$$s_2 = 7 \qquad (5.6)$$

$$\lim_{k\to\infty} s_{2k} = 2+3\Phi$$

(5.7)

$$\lim_{k\to\infty} \frac{s_{2k}-s_{2k+2}}{s_{2k+2}-s_{2k+4}} = (2+3\Phi)^2$$

(5.8)

What about

#

$$1-\frac{1}{3}\left(1-\frac{1}{s_1}\left(1-\frac{1}{s_2}\left(1-\frac{1}{s_3}\left(1-\frac{1}{s_4}\left(1-\frac{1}{s_5}(1+\cdots)\right)\right)\right)\right)\right)$$

$$= 0.703136565612626373742656972373767 91\ldots$$

##

$$1+\frac{1}{3}\left(1-\frac{1}{s_1}\left(1+\frac{1}{s_2}\left(1-\frac{1}{s_3}\left(1+\frac{1}{s_4}\left(1-\frac{1}{s_5}(1+\cdots)\right)\right)\right)\right)\right)$$

$$= 1.286685681407788924214890654500681557241\ldots$$

###

$$1+\frac{1}{3}\left(1+\frac{1}{s_1}\left(1+\frac{1}{s_3}\left(1+\frac{1}{s_5}\left(1+\frac{1}{s_7}\left(1+\frac{1}{s_9}(1+\cdots)\right)\right)\right)\right)\right)$$

$= 1.382095813980376720612946359554228972 74\ldots$

####

$$1+\frac{1}{3}\left(1+\frac{1}{s_2}\left(1+\frac{1}{s_4}\left(1+\frac{1}{s_6}\left(1+\frac{1}{s_8}\left(1+\frac{1}{s_{10}}(1+\cdots)\right)\right)\right)\right)\right)$$

$= 1.389083066895122823191013817988912088 1\ldots$

(6.1,2,3,4)

161.A.6 2-5-7

$\Phi = \frac{1}{2}\left(1+\sqrt{5}\right) =$

$$1+\frac{1}{\sigma_0}\left(1+\frac{1}{\tau_0}\left(1+\frac{1}{\sigma_1}\left(1+\frac{1}{\tau_1}\left(1+\frac{1}{\sigma_2}\left(1+\frac{1}{\tau_2}(1+\cdots)\right)\right)\right)\right)\right)$$

$\sigma_0 = 2$, $\tau_0 = 5$

$\sigma_1 = 7-\frac{1}{2} = \frac{13}{2}$, $\tau_1 = 7-\frac{1}{5} = \frac{34}{5}$

$\sigma_2 = 7-\frac{1}{7-\frac{1}{2}} = \frac{89}{13}$, $\tau_2 = 7-\frac{1}{7-\frac{1}{5}} = \frac{233}{34}$

$\sigma_3 = 7-\frac{1}{7-\frac{1}{7-\frac{1}{2}}} = \frac{610}{89}$, $\tau_3 = 7-\frac{1}{7-\frac{1}{7-\frac{1}{5}}} = \frac{1597}{233}$

$$\sigma_4 = 7 - \frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{2}}}} = \frac{4181}{610},\ \tau_4 = 7 - \frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{5}}}} = \frac{10946}{1597}$$

…….. (7.1)

$$\sigma_{k+1} = 7 - \frac{1}{\sigma_k},\ k=0,1,2,3,\ldots$$

$$\sigma_0 = 2 \quad (7.2)$$

$$\sigma_k = \frac{\text{fib}(4k+2)}{\text{fib}(4k-2)},\ k=1,2,3,4,\ldots \quad (7.3)$$

$$\lim_{k\to\infty} \sigma_k = 2 + 3\Phi \quad (7.4)$$

$$\tau_{k+1} = 7 - \frac{1}{\tau_k},\ k=0,1,2,3,\ldots$$

$$\tau_0 = 5 \quad (7.5)$$

$$\tau_k = \frac{\text{fib}(4k+4)}{\text{fib}(4k)},\ k=1,2,3,4,\ldots \quad (7.6)$$

$$\lim_{k\to\infty} \tau_k = 2 + 3\Phi$$

161.A.7 ln(Φ) meets π

$\sqrt{\frac{5}{2}} * \ln(\Phi) = 0.760862702\ldots$ // $\frac{\pi}{2} = 1.570796327$

$$= 1 \mp \frac{1}{3}\left(1 \mp \frac{2}{5}\left(1 \mp \frac{3}{7}\left(1 \mp \frac{4}{9}\left(1 \mp \frac{5}{11}\left(1 \mp \frac{6}{13}(1 \mp \cdots)\right)\right)\right)\right)\right)$$

$$= 1 \mp \frac{1}{c_0}\left(1 \mp \frac{1}{c_1}\left(1 \mp \frac{1}{c_2}\left(1 \mp \frac{1}{c_3}\left(1 \mp \frac{1}{c_4}\left(1 \mp \frac{1}{c_5}(1+\cdots)\right)\right)\right)\right)\right)$$

$$c_0 = 3\,,\ c_1 = 3 - \frac{1}{2}\,, c_2 = 3 - \frac{1}{2-\frac{1}{2}}\,, c_3 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2}}}\,,$$

$$c_4 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}\,, c_5 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}}\,, \ldots$$

(8)

161.A.8 $(\ln(\Phi))^2$ meets π^2

$(\ln(\Phi^2))^2 = 0.926259282\ldots$ // $\left(\frac{\pi}{3}\right)^2 = 1.096622711\ldots$

$$= 1 \mp \frac{1}{3} * \frac{1}{4} * \left(1 \mp \frac{2}{5} * \frac{2}{6} * \left(1 \mp \frac{3}{7} * \frac{3}{8} * \left(1 \mp \frac{4}{9} * \frac{4}{10}\left(1 \mp \frac{5}{11} * \frac{5}{12} * \left(1 \mp \frac{6}{13} * \frac{6}{14} * (1 \mp \cdots)\right)\right)\right)\right)\right)$$

$$= 1 \mp \frac{1}{c_0} * \frac{1}{d_0} * \left(1 \mp \frac{1}{c_1} * \frac{1}{d_1} * \left(1 \mp \frac{1}{c_2} * \frac{1}{d_2} * \left(1 \mp \frac{1}{c_3}\ \frac{1}{d_3}(1 \mp \cdots)\right)\right)\right)$$

$$c_0 = 3\,,\ c_1 = 3 - \frac{1}{2}\,, c_2 = 3 - \frac{1}{2-\frac{1}{2}}\,, c_3 c = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2}}}\,,$$

$$c_4 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}\,, c_5 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}}\,, \ldots$$

$$d_0 = 4\,,\ d_1 = 3 = c_0\ \,, d_2 = 3 - \frac{1}{3}\,, d_3 = 3 - \frac{1}{2} = c_1\,,$$

$$d_4 = 3 - \frac{1}{2-\frac{1}{3}}\,,\ d_5 = 3 - \frac{1}{2-\frac{1}{2}} = c_2\,,\ d_6 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{3}}}\,,$$

$$d_7 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2}}} = c_3\,,\ d_8 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{3}}}}\,,$$

$$d_9 = 3 - \frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}} = c_4\,,\ \ldots$$

(9)

161.A.9

$\mathrm{fib}(n+2) = \mathrm{fib}(n+1) + \mathrm{fib}(n)$; $n=0,1,2,3,\ldots$

$\mathrm{fib}(0) = \mathrm{fib}(1) = 1$

$$\Phi = \lim_{n\to\infty} \frac{\mathrm{fib}(n+1)}{\mathrm{fib}(n)}$$

$\mathrm{scf}(\Phi^5) = [11;11_{\mathrm{per}}]$

FibonacciBomb$(m,n) = \mathrm{scf}\left(\left(\frac{\mathrm{fib}(n+1)}{\mathrm{fib}(n)}\right)^m\right)$; $m,n \in \mathbb{N}$

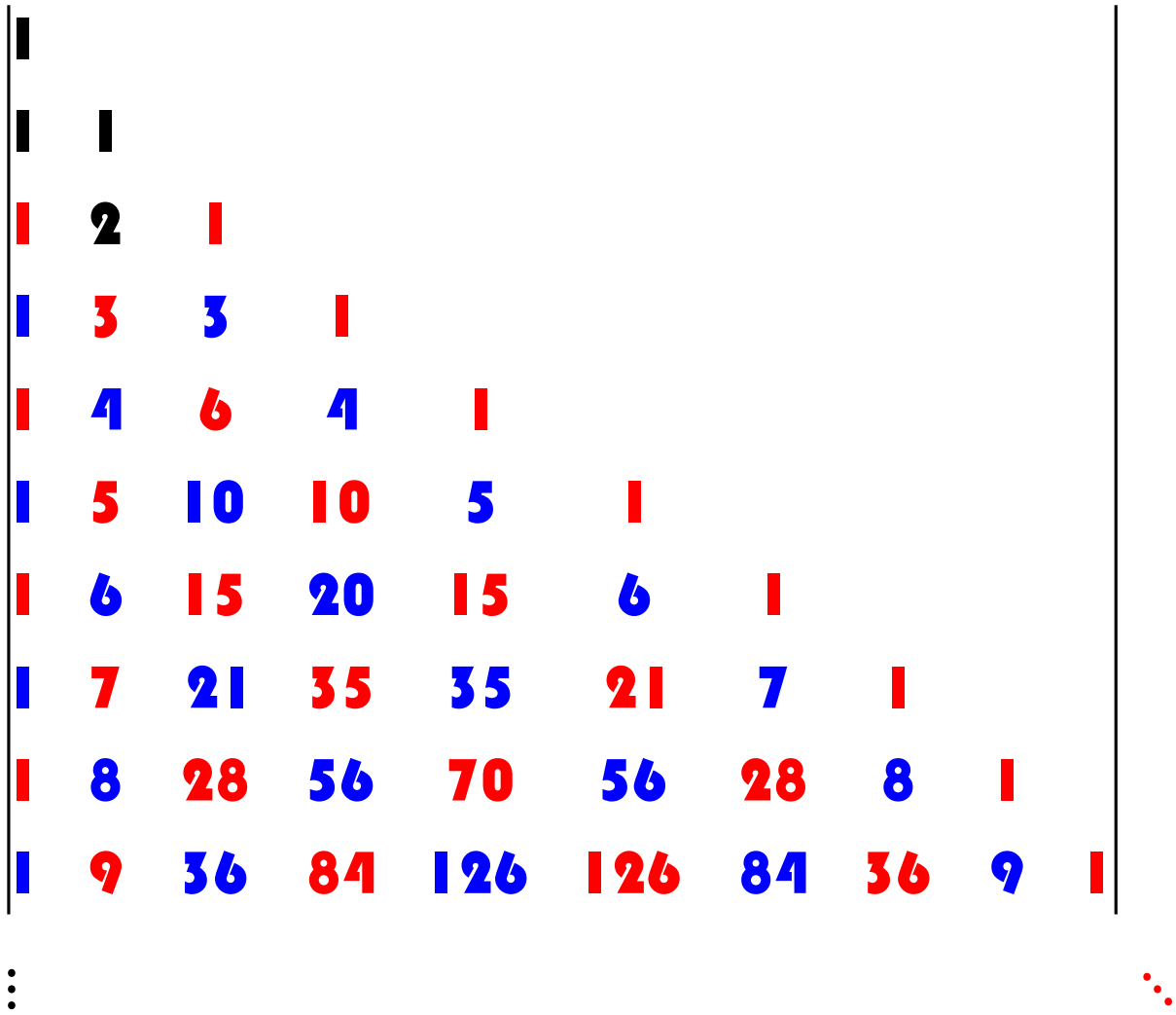
FibonacciBomb(5,50) =
[11, 11, 11, 11, 11, 11, 11, 11, 11, 13, 437845991669110338 04,
1, 5, 1, 10, 11, 11, 11, 11, 11, 11, 11, 11, 4, 1, 10, 11, 11, 11, 11,
11, 11, 11, 11, ...]
FibonacciBomb(5,55) =
[11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 12, 1,
538514970164685621 5201, 6, 1, 10, 11, 11, 11, 11, 11, 11, 11,
11, 11, 4, 1, 10, 11, 11, 11, 11, 11, 11, 11, 11, 11, ...]
FibonacciBomb(5,99) =
[11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11,
11, 11, 8, 1324677387061709725635162843114104137 9815, 1,
1, 5, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11,
11, 11, 11, 1, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11,
11, 11, 11, 11, 11, 11, ...]

(10)

See also Unit 30.1

161.A.10

+ Pascal's triangle, see Units 30, 42, 72, [3,4]



(11.1)

Red part kxk determinant

$$dr(k=7)=\begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 6 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 10 & 1 & 0 & 0 \\ 0 & 0 & 1 & 15 & 15 & 1 & 0 \\ 0 & 0 & 0 & 7 & 35 & 21 & 0 \\ 0 & 0 & 0 & 1 & 28 & 70 & 28 \end{vmatrix}$$

(11.2)

** The quotient of two immediately consecutive full row ⋮ column sums of dr(k) converges to $\Phi + 1$ ⋮ 2 for k to ∞.

** k	1	2	3	4	5	6	7	8
dr(k)	1	5	17	155	2073	38227	929569	28820619

Genocci numbers [5]

** $\lim_{k\to\infty}(2k+4)(2k+3)\frac{dr(k)}{dr(k+1)} = \pi^2$

(11.3,4,5)

And the blue part determinant, how about that?

$$db(k=7) = \begin{vmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 10 & 5 & 0 & 0 & 0 \\ 0 & 0 & 6 & 20 & 6 & 0 & 0 \\ 0 & 0 & 1 & 21 & 35 & 7 & 0 \\ 0 & 0 & 0 & 8 & 56 & 56 & 8 \\ 0 & 0 & 0 & 1 & 36 & 126 & 84 \end{vmatrix} \quad (12.1)$$

** The quotient of two immediately consecutive full row ⋮ column sums of db(k) converges to $\Phi + 1$ ⋮ 2 for k to ∞.

(12.2)

** $\lim_{k\to\infty}(2k+6)(2k+5)(2k+4)\frac{db(k)}{db(k+1)} = 8\pi^2$

(12.3)

** The factorization structures of dr(k) and db(k) are significantly different, f.e.

dr(44)=
1072771528756695855121403860094670387715552459410849551811936537936175943921814277386876515839 5=
3^2*5*73*151*331*587*631*23311*18837001*1758317910439*228470113952790571815807754364820900071198161742204795399,

db(44)=
2169645193342054344666010266722899331483838791507731370292234248559040341135427164176446772794311764228163830494003200000000000=
2^{41}*3^{20}*5^{11}*7^6*11^3*13^4*17^2*19*23^2*29*#37* 41* 43*
587*1758317910439*228470113952790571815807754364820900071198161742204795399,

db(60)=
3202038795875305372573380603795651510721360294615212272814646698595497982965902865052299090312128883622730786855919429131346317329723978216834826787731268125642146819102374163906560000000000000000=
2^{56}*3^{28}*5^{14}*7^9*11^5*13^4*17^3*19^3*23^2*29^2*31^2*37*41^2*43*47*53*59*61^2*
1545314586433142560447*526537874022904967154842565343405479455117412302583734 7340340007402607918072110058 93,

db(91)=
16226746127521460151409283603640746343786746329896355613071597837529649822921624294812470022983719756381195576138620550770611985038379126744234924171993542084933500440071674020411965859593737068684705839173067798247518540652039482102497078426475786949508284107878471351858712399957819731050601930651507650409267200000000000000000000 = **$2^{87}*3^{44}*5^{21}*7^{15}*11^{8}*13^{8}*17^{5}*19^{4}*23^{4}*29^{3}*31^{2}*37^{2}*41^{2}*43^{2}*47*53*59*61^{2}*67*71*73*79*83*89*$**731071444752614233110 89 841618 633327*3627027615648746666477*21221741142274196480934616 01*26850209236810652921894528581705791897631179146790541629102109514840673820757594605345640828167368 0 375113,

db(100)=

… $2^{96}*3^{48}*5^{23}*7^{16}*11^{8}*13^{7}*17^{5}*19^{5}*23^{4}*29^{3}*31^{3}*37^{2}*41*43^{2}*47^{2}*53*59*61*67^{2}*71*73*79*83*89*97*$… .

(14.1,2,3,4,5)

The factorization of db(k) brings to light **the complete (rarer complete – 1, #) sequence of primes from 2 to about k,** the power of the prime factor 2 is about k , of 3 about int(k/2), of 5 about int(k/4) and so on to 1 („small noise”)

and

a small number of increasingly large primes (“factor bombs”), comparable with “monster waves” in Fibonacci simple continued fractions (see Unit 30).

Problems

** For discussion!

Primes and nonprimes

db(7)=43751232=2^6*3^3*7*3617

db(11)=…$2^9*3^4*5^3*11$*…

db(13)=…$2^{10}*3^5*5^2*7^3*11*13$*…

db(17)=…$2^{15}*3^7*5^2*7*11*$#17*…

db(19)=…$2^{17}*3^8*5^3*7^3*$#13*17*19*…

db(43)=…$2^{40}*3^{20}*5^9*7^6*11^5*13^3*17^3*19^2*$#29*31*37* 41*43*…

db(47)=…$2^{45}*3^{21}*5^9*7^7*11^4*13^2*17*19^2*23^2*29*31*37*41*43*47$*…

For comparison

db(43)=…$2^{40}*3^{20}*5^9*7^6*11^5*13^3*17^3*19^2*$#29*31*37* 41*43*…

db(44)=…$2^{41}*3^{20}*5^{11}*7^6*11^3*13^4*17^2*19*23^2*29*$#37*41*43*…

db(45)=…$2^{41}*3^{21}*5^9*7^6*11^4*13^3*17^2*19^2*23^3*29*31^2*37*41*43$*…

db(47)=…$2^{45}*3^{21}*5^9*7^7*11^4*13^2*17*19^2*23^2*29*31*37*41*43*47$*…

db(48)=…$2^{46}*3^{23}*5^{12}*7^{10}*11^5*13^3*17^2*19^2*23^2*29*31*37*41*43*47$*…

** Sacra conversazione in db(k): Pascal's triangle, Φ, π^2, prime sequence. „But where is e?“ [6]

** Discuss #.

** Find similar determinant structures.

** Pascal's triangle and Riemann zeta function [7,8]?

References

[1] https://en.wikipedia.org/wiki/Thirty-six_Views_of_Mount_Fuji

[2] https://en.wikipedia.org/wiki/The_Great_Wave_off_Kanagawa
[3] Stover, Christopher and Weisstein, Eric W. "Pascal's Triangle." From *MathWorld*--A Wolfram Web Resource. https://mathworld.wolfram.com/PascalsTriangle.html
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[8] https://en.wikipedia.org/wiki/Riemann_zeta_function
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161.A.11

Aus der „TUD Schülerinformatik“ vor 34 Jahren Diskussion aus heutiger Sicht

2 Fibonacci-Zahlendarstellung (Fibonacci-Kode)

In [1,2,3] wird gezeigt:

- Jede positive Zahl z kann in einem Positionssystem mit den irrationalen Basen

$$\sigma = \frac{1}{2}(\sqrt{5}+1) = \tau + 1 = \frac{1}{\tau} \tag{1.1}$$

$$\tau = \frac{1}{2}(\sqrt{5}-1) = \sigma - 1 = \frac{1}{\sigma} \tag{1.2}$$

(Teilungsverhältnis des Goldenen Schnittes bei $\lessdot$ Außenteilung / Innenteilung, vgl. Gl. $(1{,}5. \lessdot \frac{6}{5})$ und den Ziffern 0 und 1 dargestellt werden:

$$z = \sum_{i=-\infty}^{+\infty} a_i \sigma^i \quad ; \quad a_i \in \{0,1\} \tag{2.1}$$

$$= \sum_{i=0}^{\infty} a_i \sigma^i + \sum_{i=1}^{\infty} a_{-i} \tau^i \tag{2.2}$$

$$= \ldots a_2 a_1 a_0 \cdot a_{-1} a_{-2} \ldots \tag{2.3}$$

Fibonacci-Zahlendarstellung (Fibonacci-Kode).

- Ist z eine natürliche Zahl, so ist die Summe (2.1) bzw. Zeichenkette (2.3) stets endlich

$$z_{nat} = \sum_{i=-n}^{m} a_i \sigma^i = \sum_{i=0}^{m} a_i \sigma^i + \sum_{i=1}^{n} a_{-i} \tau^i \tag{3.1}$$

$$= a_m \ldots a_1 a_0 \cdot a_{-1} a_{-2} \ldots a_{-n} \quad , \tag{3.2}$$

z.B.

z_{nat}	Binärdarstellung	Fibonacci-Darstellung
0	0	0
1	1	1
2	1 0	1 . 1 1
3	1 1	1 1 . 0 1
4	1 0 0	1 0 1 . 0 1
5	1 0 1	1 1 0 . 1 0 0 1
6	1 1 0	1 1 1 . 1 0 0 1
7	1 1 1	1 0 1 1 . 0 0 0 1
8	1 0 0 0	1 1 0 1 . 0 0 0 1

(3.3)

Aufgaben

1. Geben Sie möglichst einfache Algorithmen für die
   - Dezimal-Fibonacci-Konvertierung
   - Binär-Fibonacci-Konvertierung

   an und schreiben Sie die zugehörigen BASIC-Programme.
   Lösungshinweis:
   Aus dem durch die Gl. (1,6) beschriebenen Zusammenhang zwischen dem Goldenen Schnitt und der Fibonacci-Folge ergeben sich die Beziehungen

$$\sigma^i = f_{i-1} + f_i\sigma \quad ; i = 1,2,.. \tag{4.1}$$

$$\tau^i = (-1)^i(f_{i-1} - f_i\tau) \quad ; i = 1,2,.. \tag{4.2}$$

Das Einsetzen von Gl. (4) in (2.2) liefert

$$z = \sum_{i=0}^{\infty} a_i\sigma^i + \sum_{i=1}^{\infty} a_{-1}\tau^i \tag{2.2}$$

$$= a_0 + \sum_{i=1}^{\infty} (a_i f_{i+1} + (-1)^i a_{-i} f_{i-1}) \tag{5.1}$$

$$= a_0 + \sum_{i=1}^{\infty} (a_i f_{i-1} + (-1)^i a_{-i} f_{i+1}) \qquad (5.2)$$

$$= a_0 + \sum_{i=1}^{\infty} \frac{1}{2} (a_i + (-1)^i a_{-i})(f_{i+1} + f_{i-1}) \qquad (5.3)$$

Dabei besteht (als Nebenbedingung) zwischen den a_i und a_{-i} stets die Beziehung

$$\sum_{i=1}^{\infty} (a_i - (-1)^i a_{-i}) f_i = 0 \,. \qquad (5.4)$$

2. Aus Gl. (3.3) ersieht man bereits qualitativ, daß die Fibonacci-Darstellung einer Zahl z gegenüber der zugehörigen Binärdarstellung eine beträchtliche Redundanz besitzen kann.
   - Bestimmen Sie diese Redundanz quantitativ.
   - Diskutieren Sie auf Grundlage der Veröffentlichungen [1,2,3] von А.П. Стахов , wie diese Redundanz für eine Erhöhung der Zuverlässigkeit moderner Rechner eingesetzt werden könnte ("... фибоначчиевая арифметика ... имеет огромное практическое значение в качестве системы счисления для ЭВМ будущего."

3. Untersuchen Sie die in Tabelle 1,1,3 beschriebenen Näherungsbeziehungen im Lichte der Fibonacci-Zahlendarstellung.

Literatur

[1] Стахов, А.П.: Алгоритмическая теория измерения. Москва 1979

[2] Стахов, А.П.: Золотая пропорция в цифровой технике. Автоматика и вычислительная техника

[3] Стахов, А.П.: Коды золотой пропорции или системы счисления для ЭВМ будущего. Техника – молодежи 57 (1985) 7, S. 40-44

Probleme

** Generieren Sie die Fibonacci-Kode Darstellungen mathematischer Konstanten [4,5], insbesondere e und π.

** Untersuchen Sie, ob sich aus deren 2D/Zeichenketten- und 3D-Darstellungen [6] analog zu [6] Informationen über ihre innere Struktur gewinnen lassen.

References

[4] Weisstein, Eric W. "Constant." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/Constant.html

[5] https://en.wikipedia.org/wiki/Mathematical_constant
https://de.wikipedia.org/wiki/Mathematische_Konstante

[6] E.P. Stoschek, D. Schönfeld. Computergraphik im Spannungsfeld zwischen Algorithmus und Phantasie. Signa in silico. Kap. 1
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Unit 161, Part B
Nachtrag zu Unit 161.A.10
Durch gezieltes Einfügen von „Fremdzahlen“
** in das statische Pascalsche Dreieck
** während des rekursiven (dynamischen) Aufbaus
des Pascalschen Dreiecks
** Ideeneinzugsgebiete [1,2,3,4,5]
kann die Länge der von der zugehörigen Determinante db(k) erzeugten Primzahlsequenz signifikant vergrößert werden.
Beispiel 1
Originales Pascalsches Dreieck

1								
1	1							
1	2	1						
1	3	3	1					
1	4	6	4	1				
1	5	10	10	5	1			
1	6	15	20	15	6	1		
1	7	21	35	35	21	7	1	
1	8	28	56	70	56	28	8	1
1	9	36	84	126	126	84	36	9	1

(1.1)

Zugehörige Determinante db(k)

$$\mathbf{db(k=7)} = \begin{vmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 10 & 5 & 0 & 0 & 0 \\ 0 & 0 & 6 & 20 & 6 & 0 & 0 \\ 0 & 0 & 1 & 21 & 35 & 7 & 0 \\ 0 & 0 & 0 & 8 & 56 & 56 & 8 \\ 0 & 0 & 0 & 1 & 36 & 126 & 84 \end{vmatrix} \qquad (1.2)$$

** The quotient of two immediately consecutive full $\begin{matrix}\text{row}\\ \vdots \\ \text{column}\end{matrix}$ sums of db(k) converges to $\begin{matrix}\Phi + 1\\ \vdots \\ 2\end{matrix}$ for k to ∞.

(1.3)

$$** \lim_{k\to\infty} (2k+6)(2k+5)(2k+4)\frac{db(k)}{db(k+1)} = 8\pi^2$$

(1.4)

** Factorization of db(k)

db(16)=96269372301568677170319360000=

$2^{15} * 3^{7} * 5^{4} * 7^{3} * 11 * 13 * 17^{2}$ *151628697551

db(17)=61714359945187404279241981132800=

$2^{15} * 3^{7} * 5^{2} * 7 * 11$#$* 17$ *26315271553053477373

(1.5)

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

1								
1	2							
1	3	2						
1	4	5	2					
1	5	9	7	2				
1	6	14	16	9	2			
1	7	20	30	25	11	2		
1	8	27	50	55	36	13	2	
1	9	35	77	105	91	49	15	2
1	10	44	112	182	196	140	64	17	2

Modified Pascal's triangle

(2.1)

Zugehörige Determinante db(k)

$$db(k=7)=\begin{vmatrix} 1 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 7 & 0 & 0 & 0 & 0 \\ 0 & 1 & 14 & 9 & 0 & 0 & 0 \\ 0 & 0 & 7 & 30 & 11 & 0 & 0 \\ 0 & 0 & 1 & 27 & 55 & 13 & 0 \\ 0 & 0 & 0 & 9 & 77 & 91 & 15 \\ 0 & 0 & 0 & 1 & 44 & 182 & 140 \end{vmatrix} \qquad (2.2)$$

** The quotient of two immediately consecutive full

$$\begin{matrix} \text{row} \\ \vdots \\ \text{column} \end{matrix} \quad \text{sums of db(k) converges to} \quad \begin{matrix} \Phi + 1 \\ \vdots \\ 2 \end{matrix} \quad \text{for k to } \infty. \tag{2.3}$$

$$** \lim_{k\to\infty} (2k+5)(2k+4)(2k+3)\frac{db(k)}{db(k+1)} = 4\pi^2 \tag{2.4}$$

** Factorization of db(k)

db(16)=1904422387003889133045020700093 75 =

$3^8*5^5*7^3*11^2*13*17^2*19*23*29*31*$ 151628697551

db(17)=22489293407860857419060956461864562 5=

$3^8*5^4*7^2*11^2*$#$17*23*29*31$*263152715530534773 73 (2.5)

Loten Sie das „Doping/dotieren/rändern/lokal verfälschen…“ des pascalschen Dreiecks experimentell umfassend aus und finden Sie dazu ein zugehöriges systematisches (zahlentheoretisch begründetes) Modellkonzept.

Diskutieren Sie Anwendungen.

Lateral 1

Systematische „Einfach/mehrfach/fehler“–Suche und –Korrektur im Pascalschen Dreieck mittels lokaler Operatoren mit geringsten Aufwand.

Lateral 2

PDKrypt ?

Lateral 3

Compare and merge

xx Pascal’s triangle,

xx Langton’s ant [6]

xx Conway's game of life [7]

Lateral 4

A recursively generated 2D “crystal structure”
(esp. Pascal’s triangle)
doped with “foreign atoms”
(⇨ foreign numbers)
generates a prime sequence.
(⇨ via factorization of the determinant db(k)).

References

[1] https://en.wikipedia.org/wiki/Pearl#Cultured_pearls
[2] https://de.wikipedia.org/wiki/Perle#Zuchtperlen
[3] https://de.wikipedia.org/wiki/Dotierung
[4] https://en.wikipedia.org/wiki/Doping_(semiconductor)
[5] E.S. Rajendran. Nanodynamik.
Winterwork 2019, ISBN-13: 978-3960145516
[6] https://en.wikipedia.org/wiki/Langton%27s_ant
[7] https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life

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In a magic scenery Photo Cornelia Koch-Stoschek