Unit 15, part 47

Aphorisms, advices, anecdotes, brain teasers, curious stories, epigrams, epigraphs, epitaphs, ideas, crazy-creative ideas, koans, memorabilia, mottos, poems, proverbs, quotations, riddles, sayings, slogans, scurrilities, besun, etc.
collected for encouragement and the advantage of my students and myself.

Wort-Fundstücke, Gehörtes, Gelesenes, Zugelaufenes, …

Nur wer seinen eigenen Weg geht, kann von niemandem überholt werden.
Marlon Brando

Wie im Großen – so im Kleinen.
Wie oben – so unten, wie unten – so oben. Wie innen – so außen, wie außen – so innen.

Algo-snippets
** Kopplung und Verdrillung von Algorithmen
** Doppler-Effekt bei algorithmischen Prozessen
** Tetrapodenstabilisierung eines Algorithmus, https://en.wikipedia.org/wiki/Tetrapod_(structure)
** Algornado, Algorithmophagen

Quanta Magazine
https://www.quantamagazine.org/
https://www.quantamagazine.org/kaisa-matomaki-dreams-of-primes-20170720/

КВАНТ
http://www.kvant.info/current.htm

Tetrapodic metamaterial

Minimalistische Ontologie: Die Atome sind nichts, die Beziehungen zwischen ihnen alles.

M. Esfeld. Erkenntnistheorie. Wissenschaft, Erkenntnis und ihre Grenzen.
Spektrum der Wissenschaft 8.17, S. 12-18

18-7-Besun

18*987654321-1=17777777777
18*98765432+1=1777777777
18*9876543+3=177777777
18*987654+5=17777777
18*98765+7=1777777
18*9876+9=177777
18*987+11=17777
18*98+13=1777
18*9+15=177
18*0+17=17
18*87654321-1=1577777777
18*8765432+1=157777777
18*876543+3=15777777
18*87654+5=1577777
18*8765+7=157777
18*876+9=15777
18*87+11=1577
18*8+13=157
18*0+15=15
18*7654321-1=137777777
18*765432+1=13777777

18*76543+3=1377777
18*7654+5=137777
18*765+7=13777
18*76+9=1377
18*7+11=137
18*0+13=13
18*654321-1=11777777
18*65432+1=1177777
18*6543+3=117777
18*654+5=11777
18*65+7=11777
18*6+9=117
18*0+11=11
18*54321-1=977777
18*5432+1=97777
18*543+3=9777
18*54+5=977
18*5+7=97
18*0+9=9
18*4321-1= 77777
18*432+1= 7777
18*43+3= 777
18*4+5= 77
18*0 +7= 7
18*321-1=5777
18*32+1=577
18*3+3= 57
18*0+5=5

18*21-1=377
18*2+1=37
18*0+3=3
18*1-1=17
18*0+1=1

18*987654321-1=17777777777
18*87654321-1=1577777777
18*7654321-1=137777777
18*654321-1=11777777
18*54321-1=977777
18*4321-1=77777
18*321-1=5777
18*21-1=377
18*1-1=17

18*98765432 +1=1777777777
18*9876543+3=177777777
18*987654+5=17777777
18*98765+7=1777777
18*9876+9=177777
18*987+11=17777
18*98+13=1777
18*9+15=177
18*0+17=17

18*987654321-1=17777777777
18*9765431-1=175777757

18*96541-1=1737737
18*951-1=17117
18*987654321-1=17777777777
18*98764321-1=1777757777
18*987321-1=1737737
18*951-1=17117

** Find more “18 – 7”.
** Find the underlying “18 – 7” equation.
** Extend “18 – 7”, f.i. to “?? – 2”.
** What’s the use of it?

The simple continued fraction (scf) representation of $r \in \mathbb{R}_+$ and modified versions (m…scf)
Transformation

$$r \in \mathbb{R}_{+.} \rightarrow \operatorname{scf}(r) = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \ddots \cfrac{1}{a_n}}}}}$$

$$= [a_0; a_1, a_2, a_3, a_4, \ldots, a_n]$$

or

$$r \in \mathbb{R}_{+.} \rightarrow \operatorname{mscf}(r) = A_0 + \cfrac{A_0}{A_1 + \cfrac{A_1}{A_2 + \cfrac{A_2}{A_3 + \cfrac{A_3}{A_4 + \ddots \cfrac{A_{n-1}}{A_n}}}}}$$

$$= [[A_0; A_1, A_2, A_3, A_4, \ldots, A_n]]$$

Transformation-**alg**orithms

Input: $r \in \mathbb{R}_+$; length

$x:=r$; $i:=0$

$$\left.\begin{matrix} a_0 \\ A_0 \end{matrix}\right\} := \text{int}(x)$$

repeat until

$$x:=\begin{cases} \dfrac{1}{x-\text{int}(x)} \\ \dfrac{\text{int}(x)}{x-\text{int}(x)} \end{cases}$$

$i:=i+1$

$$\left.\begin{matrix} a_i \\ A_i \end{matrix}\right\} := \text{int}(x) \rightarrow$$

$x=\text{int}(x)$ or $i=\text{length}$

$n:=i$

Output: $\begin{cases} \text{scf}(r)=[a_0;a_1,a_2,a_3,\ldots,a_n] \\ \text{mscf}(r)=[[A_0;A_1,A_2,A_3,\ldots,A_n]] \end{cases}$

Exampels

$$\operatorname{scf}\left(\frac{1}{e-2}\right)=[1;2,1,1,4,1,1,6,1,1,8,1,1,10,\ldots]$$

$$\operatorname{mscf}\left(\frac{1}{e-2}\right)=[[1;2,3,4,5,6,\ldots]]$$

$$\operatorname{scf}\left(\frac{I_0(2)}{I_1(2)}\right)=[1;2,3,4,5,6,\ldots]$$

$$\operatorname{mscf}\left(\sqrt{a^2\pm1}\right)=\begin{cases}[[a;2a^2,\overline{4a^2}]] \\ [[a\text{-}1;a\text{-}1,\overline{2a\text{-}2}]]\end{cases};\ a\in\mathbb{N}$$

$$\operatorname{scf}\left(\frac{1}{2}(1+\sqrt{5})\right)=\operatorname{mscf}\left(\frac{1}{2}(1+\sqrt{5})\right)$$

$$=[1;1,1,1,1,1,\ldots]=[[1;1,1,1,1,1,\ldots]]$$

$$\operatorname{scf}\left(\sqrt{2}\right)=[1;\overline{2}]$$

$$\operatorname{scf}\left(\sqrt{3}\right)=[1;\overline{1,2}]$$

$$\operatorname{scf}\left(\sqrt{5}\right)=[2;\overline{4}]$$

$$\operatorname{scf}\left(\sqrt{6}\right)=[2;\overline{2,4}]$$

$$\mathrm{mscf}\left(\sqrt{2}\right) = [[1;2,\overline{4}]]$$

$$\mathrm{mscf}\left(\sqrt{3}\right) = [[1;1,\overline{2}]]$$

$$\mathrm{mscf}\left(\sqrt{5}\right) = [[2;8,\overline{16}]]$$

$$\mathrm{mscf}\left(\sqrt{6}\right) = [[2;4,\overline{8}]]$$

$$\mathrm{scf}\left(\sqrt{7}\right) = [2;\overline{1,1,1,4}]$$

$$\mathrm{scf}\left(\sqrt{8}\right) = [2;\overline{1,4}]$$

$$\mathrm{scf}\left(\sqrt{10}\right) = [3;\overline{6}]$$

$$\mathrm{scf}\left(\sqrt{11}\right) = [3;3,\overline{6}]$$

$$\mathrm{scf}\left(\sqrt{12}\right) = [3;2,\overline{6}]$$

$$\text{mscf}\left(\sqrt{7}\right) = [[2;3,30,34,111,235,2720,\ldots]]$$

$$\text{mscf}\left(\sqrt{8}\right) = [[2;2,\overline{4}]]$$

$$\text{mscf}\left(\sqrt{10}\right) = [[3;18,\overline{36}]]$$

$$\text{mscf}\left(\sqrt{11}\right) = [[3;9,\overline{18}]]$$

$$\text{mscf}\left(\sqrt{12}\right) = [[3;6,\overline{12}]]$$

$$\text{scf}\left(\sqrt{13}\right) = [3;\overline{1,1,1,1}]$$

$$\text{scf}\left(\sqrt{14}\right) = [3;\overline{1,1,1,1,6}]$$

$$\text{scf}\left(\sqrt{15}\right) = [3;\overline{1,6}]$$

$$\text{scf}\left(\sqrt{17}\right) = [4;\overline{8}]$$

$$\text{scf}\left(\sqrt{18}\right) = [4;\overline{4,8}]$$

$$\mathrm{mscf}\left(\sqrt{13}\right)=[[3;4,4,20,24,48,49,53,75,32811,\ldots]]$$

$$\mathrm{mscf}\left(\sqrt{14}\right)=[[3;4,88,97,122,152,405,444,522,\ldots]]$$

$$\mathrm{mscf}\left(\sqrt{15}\right)=[[3;3,\overline{6}]]$$

$$\mathrm{mscf}\left(\sqrt{17}\right)=[[4;32,\overline{64}]]$$

$$\mathrm{mscf}\left(\sqrt{18}\right)=[[4;16,\overline{32}]]$$

$$\mathrm{scf}(\tan 1=1.557497725\ldots)=$$
$$[1;1,1,3,1,5,1,7,1,9,1,\ldots]$$
$$\mathrm{mscf}(\tan 1)=[[1;1,1,3,3,5,5,7,7,9,9,\ldots]]$$

$$\mathrm{scf}\left(\frac{e^2+1}{e^2-1}\right)=[1;3,5,7,9,11,13,\ldots]$$

$$\mathrm{mscf}\left(\frac{e^2+1}{e^2-1}\right)=[1;1*3,3*5,5*7,7*9,9*11,\ldots]$$

$$\mathrm{scf}\left(\frac{e^{\frac{2}{n}}+1}{e^{\frac{2}{n}}-1}\right)=[n;3n,5n,7n,9n,11n,\ldots]$$

$$\mathrm{mscf}\left(\frac{e^{\frac{2}{n}}+1}{e^{\frac{2}{n}}-1}\right)=[[n;3n^2,15n^2,35n^2,63n^2,99n^2,\ldots]]$$

Find more, see also [1,2,3,4].

Problems for you

** What connection is there between
  ** scf(r) and mscf(r)
  ** the a_i and the A_i ?

** Discuss other scf(r)-modifications (algorithms, explorativity, applications) f.i.

$$m'scf(r) = \alpha_0 + \cfrac{\alpha_1}{\alpha_1 + \cfrac{\alpha_2}{\alpha_2 + \cfrac{\alpha_3}{\alpha_3 + \cfrac{\alpha_4}{\alpha_4 + \ddots \cfrac{\alpha_n}{\alpha_n}}}}}$$

** The recurrence equations

loop body (scf) = "x := $\dfrac{1}{x - int(x)}$"

or

loop body (mscf) = "x := $\dfrac{int(x)}{x - int(x)}$"

resp., form the main part of the considered transformation.

Discuss other loop bodies.

** Find inversion formulas and algorithms

$$\left.\begin{matrix}\text{scf(r)}\\ \text{mscf(r)}\end{matrix}\right\} \rightarrow \text{r.}$$

Hints

** See Unit 7, special 1

** scf, mscf 2by2 matrix chain product representation

scf

$$\begin{pmatrix}0 & 1\\ 1 & a_0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & a_1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & a_2\end{pmatrix}\cdots\begin{pmatrix}0 & 1\\ 1 & a_n\end{pmatrix}=\begin{pmatrix}\cdots & p(n)\\ \cdots & q(n)\end{pmatrix}$$

$$\frac{q(n)}{p(n)}=r$$

mscf

$$\begin{pmatrix}0 & 1\\ 1 & A_0\end{pmatrix}\begin{pmatrix}0 & A_0\\ 1 & A_1\end{pmatrix}\begin{pmatrix}0 & A_1\\ 1 & A_2\end{pmatrix}\cdots\begin{pmatrix}0 & A_{n-1}\\ 1 & A_n\end{pmatrix}=\begin{pmatrix}\cdots & P(n)\\ \cdots & Q(n)\end{pmatrix}$$

$$\frac{Q(n)}{P(n)}=r$$

References

[1] https://en.wikipedia.org/wiki/Continued_fraction

[2] https://en.wikipedia.org/wiki/Engel_expansion

[3] https://en.wikipedia.org/wiki/Mathematical_constants_by_continued_fraction_representation

[4] https://de.wikibooks.org/wiki/Formelsammlung_Mathematik:_Kettenbr%C3%BCche

28.07.2017 EPSnoct