Unit 135, part 3, appendix 2
Experimental material

DS's 26.04.2018 experiments

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) =$

$\underline{A}(m-1)\begin{pmatrix} (km-(q-1)) & (km-q) \\ (km-q) & (km-(q+1)) \end{pmatrix};$

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} q+1 & q \\ q & q-1 \end{pmatrix}$

(29.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (29.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (29.5)$$

$\lim_{m\to\infty} f(m) = ?$ (numerical value, formula and scf representation);
$\lim_{m\to\infty} df(m) = f = df = ?$

(29.6,7)

Experiments

k=5, q=1

$$\underline{A}(4) = \begin{pmatrix}2 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}10 & 9\\ 9 & 8\end{pmatrix}\begin{pmatrix}15 & 14\\ 14 & 13\end{pmatrix}\begin{pmatrix}20 & 19\\ 19 & 18\end{pmatrix}$$

$$= \begin{pmatrix}30116 & 28573\\ 10403 & 9870\end{pmatrix}$$

f(m); m=2(1)40 record

```
2.8944444444444444444, 2.8949345852365645971, 2.8949341487399248970,
2.8949341489693209995, 2.8949341489692406777, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978,
2.8949341489692406978, 2.8949341489692406978, 2.8949341489692406978
Quick convergence!
```

scf(f(40)) = [2; 1, 8, 1, 1, 13, 1, 1, 18, 1, 1, 23, 1, 1, 28, 1, 1, 33, 1, 1, 38, 1, 1, 43, 1, 1, 48, 1, 1, 53, 1, 1, 58, 1, 1, 63, 1, 1, 68, 1, 1, 73, 1, 1, 78, 1, 1, 83, 1, 1, 88, 1, 1, 93, 1, 1, 98, 1, 1, 103, 1, 1, 108, 1, 1, 113, 1, 1, 118, 1, 1, 123, 1, 1, 128, 1, 1, 133, 1, 1, 138, 1, 1, 143, 1, 1, 148, 1, 1, 153, 1, 1, 158, 1, 1, 163, 1, 1, 168, …]

k=6,q=2

```
f(m); m=1(1)40 record
1.7500000000000000000, 1.6557112068965517241,
1.6557544920467306951, 1.6557544638128281270,
1.6557544638237805742, 1.6557544638237777741, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
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1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746,
1.6557544638237777746, 1.6557544638237777746, 1.6557544638237777746

scf(f(40))=[1, 1, 1, 1, 9, 1, 1, 15, 1, 1, 21, 1, 1, 27, 1, 1, 33, 1, 1, 39, 1, 1,
45, 1, 1, 51, 1, 1, 57, 1, 1, 63, 1, 1, 69, 1, 1, 75, 1, 1, 81, 1, 1, 87, 1, 1, 93,
1, 1, 99, 1, 1, 105, 1, 1, 111, 1, 1, 117, 1, 1, 123, 1, 1, 129, 1, 1, 135, 1, 1,
141, 1, 1, 147, 1, 1, 153, 1, 1, 159, 1, 1, 165, 1, 1, 171, 1, 1, 177, 1, 1, 183,
1, 1, 189, 1, 1, 195, 1, 1, …]

k=7, q=3
scf(f(40))=[1, 2, 1, 1, 10, 1, 1, 17, 1, 1, 24, 1, 1, 31, 1, 1, 38, 1, 1, 45, 1,
1, 52, 1, 1, 59, 1, 1, 66, 1, 1, 73, 1, 1, 80, 1, 1, 87, 1, 1, 94, 1, 1, 101, 1,
1, 108, 1, 1, 115, 1, 1, 122, 1, 1, 129, 1, 1, 136, 1, 1, 143, 1, 1, 150, 1, 1,
157, 1, 1, 164, 1, 1, 171, 1, 1, 178, 1, 1, 185, 1, 1, 192, 1, 1, 199, 1, 1, 206,
1, 1, 213, 1, 1, 220, 1, 1, 227, 1, 1, …]

k=8, q=4
scf(f(40))=[1, 3, 1, 1, 11, 1, 1, 19, 1, 1, 27, 1, 1, 35, 1, 1, 43, 1, 1, 51, 1,
1, 59, 1, 1, 67, 1, 1, 7 5, 1, 1, 83, 1, 1, 91, 1, 1, 99, 1, 1, 107, 1, 1, 115, 1,
1, 123, 1, 1, 131, 1, 1, 139, 1, 1, 147, 1, 1, 155, 1, 1, 163, 1, 1, 171, 1, 1,
179, 1, 1, 187, 1, 1, 195, 1, 1, 203, 1, 1, 211, 1, 1, 219, 1, 1, 227, 1, 1, 235,
1, 1, 243, 1, 1, 251, 1, 1, 259, 1, 1, …]

k=13, q=9
scf(f(40))=[1, 8, 1, 1, 16, 1, 1, 29, 1, 1, 42, 1, 1, 55, 1, 1, 68, 1, 1, 81, 1,
1, 94, 1, 1, 107, 1, 1, 120, 1, 1, 133, 1, 1, 146, 1, 1, 159, 1, 1, 172, 1, 1,
185, 1, 1, 198, 1, 1, 211, 1, 1, 224, 1, 1, 237, 1, 1, 250, 1, 1, 263, 1, 1, 276,
1, 1, 289, 1, 1, 302, 1, 1, 315, 1, 1, 328, 1, 1, 341, 1, 1, 354, 1, 1, 367, 1, 1,
380, 1, 1, 393, 1, 1, 406, 1, 1, 419, 1, 1, …]

k=5, q=2
scf(f(40))=[1, 1, 1, 1, 7, 1, 1, 12, 1, 1, 17, 1, 1, 22, 1, 1, 27, 1, 1, 32, 1,
1, 37, 1, 1, 42, 1, 1, 47, 1, 1, 52, 1, 1, 57, 1, 1, 62, 1, 1, 67, 1, 1, 72, 1, 1,
77, 1, 1, 82, 1, 1, 87, 1, 1, 92, 1, 1, 97, 1, 1, 102, 1, 1, 107, 1, 1, 112, 1, 1,
117, 1, 1, 122, 1, 1, 127, 1, 1, 132, 1, 1, 137, 1, 1, 142, 1, 1, 147, 1, 1, 152,

k=6, q=3
scf(f(40))=[1, 1, 1, 1, 8, 1, 1, 14, 1, 1, 20, 1, 1, 26, 1, 1, 32, 1, 1, 38, 1, 1,
44, 1, 1, 50, 1, 1, 56, 1, 1, 62, 1, 1, 68, 1, 1, 74, 1, 1, 80, 1, 1, 86, 1, 1,
92, 1, 1, 98, 1, 1, 104, 1, 1, 110, 1, 1, 116, 1, 1, 122, 1, 1, 128, 1, 1, 134, 1,
1, 140, 1, 1, 146, 1, 1, 152, 1, 1, 158, 1, 1, 164, 1, 1, 170, 1, 1, 176, 1, 1,
182, 1, 1, 188, 1, 1, 194, 1, 1, …]

k=12, q=10
scf(f(40))=[1, 1, 1, 1, 13, 1, 1, 25, 1, 1, 37, 1, 1, 49, 1, 1, 61, 1, 1, 73, 1,
1, 85, 1, 1, 97, 1, 1, 109, 1, 1, 121, 1, 1, 133, 1, 1, 145, 1, 1, 157, 1, 1, 169,
1, 1, 181, 1, 1, 193, 1, 1, 205, 1, 1, 217, 1, 1, 229, 1, 1, 241, 1, 1, 253, 1, 1,
265, 1, 1, 277, 1, 1, 289, 1, 1, 301, 1, 1, 313, 1, 1, 325, 1, 1, 337, 1, 1, 349,
1, 1, 361, 1, 1, 373, 1, 1, 385, 1, 1, …]

k=10, q=2
scf(f(40))=[1, 1, 1, 1, 16, 1, 1, 26, 1, 1, 36, 1, 1, 46, 1, 1, 56, 1, 1, 66, 1,
1, 76, 1, 1, 86, 1, 1, 96, 1, 1, 106, 1, 1, 116, 1, 1, 126, 1, 1, 136, 1, 1, 146,
1, 1, 156, 1, 1, 166, 1, 1, 176, 1, 1, 186, 1, 1, 196, 1, 1, 206, 1, 1, 216, 1, 1,
226, 1, 1, 236, 1, 1, 246, 1, 1, 256, 1, 1, 266, 1, 1, 276, 1, 1, 286, 1, 1, 296,
1, 1, 306, 1, 1, 316, 1, 1, 326, 1, 1, …]

→ Result 1: …,1,1, a_m, 1,1, a_{m+1}, 1,1, … → $a_{m+1} - a_m = k$

→ Result 2: …,1,1, a_1, 1,1, a_2, 1,1, … → $a_1 = g(k,q) = ???$

In this special case „q=k-4“ we obtain

$a_1 = g(k,q) = \frac{1}{4}(7k - 3q)$ ➡ generalization ???

Sandbox „Towards a Cf Compound Formula“

The components

** f(a,b) =
[0;1,1,a,1,1,a+b,1,1,a+2b,1,1,a+3b,1,1,a+4b,1,1,...] =
$\frac{1}{2}$[1; 2(a+1), 2(a+b+1),2(a+2b+1),2(a+3b+1),2(a+4b+1),...]

(97)

** $\ulcorner\frac{1}{1}\lrcorner + \ulcorner\frac{n+1}{n+1}\lrcorner + \ulcorner\frac{2n+1}{2n+1}\lrcorner + \ulcorner\frac{3n+1}{3n+1}\lrcorner + \ulcorner\frac{4n+1}{4n+1}\lrcorner + \cdots =$
[0;1,1,n+1,1,1,3n+1,1,1,5n+1,1,1,7n+1,1,1,…=
$\frac{1}{2}$ [1;2(n+2),2(3n+2),2(5n+2),2(7n+2),…]

(98)

** See Unit 12C, Unit 193.4, Deb 18, Deb 20

**

$$\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$$

init

$$\underline{A}(1) = \begin{pmatrix} p-q & p-q-1 \\ p-q-1 & p-q-2 \end{pmatrix}$$

rec

$$\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} pm-q & (pm-(q+1)) \\ (pm-(q+1)) & (pm-(q+2)) \end{pmatrix};$$

$$m = 2,3,4,\ldots\ ;\ p,q \in \mathbb{N};\ p-q \geq 2$$

prod

$$\underline{A}(m) = \prod_{i=1}^{m} \begin{pmatrix} pi - q & (pi - (q+1)) \\ (pi - (q+1)) & (pi - (q+2)) \end{pmatrix}$$

(99.1,2,3,4)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (99.5)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (99.6)$$

$\lim_{m\to\infty} f(m) = ?$ (numerical value,formula and scf representation)

Example 1

p=7, q=4

$\underline{A}(5)$

$$= \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 10 & 9 \\ 9 & 8 \end{pmatrix}\begin{pmatrix} 17 & 16 \\ 16 & 15 \end{pmatrix}\begin{pmatrix} 24 & 23 \\ 23 & 22 \end{pmatrix}\begin{pmatrix} 31 & 30 \\ 30 & 29 \end{pmatrix}$$

$$= \begin{pmatrix} 68595 & 65678 \\ 41458 & 39695 \end{pmatrix}$$

f(5) = 1.654566067…

scf(f(5)) = [1;1,1,1,8,1,1,15,1,1,22,1,1,…]

→ scf(f(∞)) = [1;1,1,1,8,1,1,15,1,1,22,1,1,…]

↑←7→↑ ←7→ ↑

↓

p

Example 2

p=7, q=5

$\underline{A}(5) = \begin{pmatrix}2 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}9 & 8\\ 8 & 7\end{pmatrix}\begin{pmatrix}16 & 15\\ 15 & 14\end{pmatrix}\begin{pmatrix}23 & 22\\ 22 & 21\end{pmatrix}\begin{pmatrix}30 & 29\\ 29 & 28\end{pmatrix}$

$= \begin{pmatrix}1914136 & 1849275\\ 664035 & 641534\end{pmatrix}$

$f(5) = 2.882582996\ldots$

$scf(f(5)) = [2;1,7,1,1,14,1,1,21,1,1,\ldots]$

$\rightarrow scf(f(\infty)) = [2;1,7,1,1,14,1,1,21,1,1,\ldots]$

$\uparrow\leftarrow 7\rightarrow\uparrow \leftarrow 7\rightarrow \uparrow$

$\downarrow$

p

Example 3

p=5, q=2

$\underline{A}(5) = \begin{pmatrix}3 & 2\\ 2 & 1\end{pmatrix}\begin{pmatrix}8 & 7\\ 7 & 6\end{pmatrix}\begin{pmatrix}13 & 12\\ 12 & 11\end{pmatrix}\begin{pmatrix}18 & 17\\ 17 & 16\end{pmatrix}\begin{pmatrix}23 & 22\\ 22 & 21\end{pmatrix}$

$= \begin{pmatrix}1309837 & 1251660\\ 793260 & 758027\end{pmatrix}$

$f(5) = 1.651207675\ldots$

$scf(f(5)) = [1;1,1,1,6,1,1,11,1,1,16,1,1,21,1,1,\ldots]$

$\rightarrow scf(f(\infty)) = [1;1,1,1,6,1,1,11,1,1,16,1,1,\ldots]$

$\uparrow\leftarrow 5\rightarrow\uparrow \leftarrow 5\rightarrow \uparrow$

$\downarrow$

p

Example 4

p=4, q=2

$\underline{A}(5) = \begin{pmatrix}2 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}6 & 5\\ 5 & 4\end{pmatrix}\begin{pmatrix}10 & 9\\ 9 & 8\end{pmatrix}\begin{pmatrix}14 & 13\\ 13 & 12\end{pmatrix}\begin{pmatrix}18 & 17\\ 17 & 16\end{pmatrix}$

$= \begin{pmatrix}256078 & 241461\\ 90837 & 85652\end{pmatrix}$

$f(5) = 2.819093541\ldots$

$scf(f(5)) = [2;1,4,1,1,8,1,1,12,1,1,16,1,1,\ldots]$

$\rightarrow scf(f(\infty)) = [2;1,4,1,1,8,1,1,12,1,1,16,1,1,\ldots]$

↑←4→↑←4→↑
↓
p

Example 5

p=4, q=1

$$\underline{A}(5) = \begin{pmatrix}3 & 2\\ 2 & 1\end{pmatrix}\begin{pmatrix}7 & 6\\ 6 & 5\end{pmatrix}\begin{pmatrix}11 & 10\\ 10 & 9\end{pmatrix}\begin{pmatrix}15 & 14\\ 14 & 13\end{pmatrix}\begin{pmatrix}19 & 18\\ 18 & 17\end{pmatrix}$$

$$= \begin{pmatrix}636291 & 601930\\ 385930 & 365089\end{pmatrix}$$

f(5) =1.648721271…

scf(f(5)) = [1;1,1,1,5,1,1,9,1,1,13,1,1,17,1,1,…]

→ scf(f(∞)) = [1;1,1,1,5,1,1,9,1,1,13,1,1,17,1,1,…]

↑←4→↑←4→↑
↓
p

An attempt of a generalization

##

scf(f(∞)) = [… ,1,1, a_m, 1,1, a_{m+1}, 1,1, …]

→ $a_{m+1} - a_m = p$; m = 1,2,3,4,… (99.7)

##

p-q=1 →

scf(f(∞)) =

- [1;p-1,1,1,2p-1,1,1,3p-1,1,1,4p-1,1,1,...]

(DS.99.8)

p-q=2 →

scf(f(∞)) = [2;1,p,1,1,2p,1,1,3p,1,1,4p,1,1,…]

(99.9)

$p-q>2 \rightarrow$

$$scf(f(\infty)) = [1;(p-(q+2)),1,1,(2p-(q+2)),1,1,(3p-(q+2)),1,1,(4p-(q+2)),1,1,...]$$

(DS.99.10)

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