Unit 135

With all-wheel drive to π^2

Part 1

Character string representation

135.1.1

$$6\sum_{i=1}^{\infty}\frac{1}{i^2}=\pi^2 \qquad (1.1)$$

$$12\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{i^2}=\pi^2 \qquad (1.2)$$

$$8\sum_{i=1}^{\infty}\frac{1}{(2i-1)^2}=\pi^2 \qquad (1.3)$$

Lateral

$$\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{(2i-1)^2}=0.91596559417\ldots=? \qquad (1.4)$$

$$\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{i}=\ln 2 \qquad (1.5)$$

$$4\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{2i-1}=\pi \qquad (1.6)$$

135.1.2

$$8\prod_{i=1}^{\infty}\left(\frac{1-\frac{1}{(2i+1)^2}}{1-\frac{1}{(2i)^2}}\right)=8\prod_{i=1}^{\infty}\left(1+\frac{(4i+1)}{(2i-1)(2i+1)^3}\right)=\pi^2 \qquad (2.1)$$

Lateral

$$\prod_{i=2}^{\infty} \frac{1}{1+\frac{1}{i^2}} = \frac{\pi}{\sinh(\pi)} \quad (2.2)$$

$$\prod_{i=2}^{\infty} \frac{1}{1-\frac{1}{i^2}} = 2 \quad (2.3)$$

$$\prod_{i=2}^{\infty} \frac{1}{1+\frac{1}{(2i-1)^2}} = 0.7970737303109834 \ldots = ? \quad (2.4)$$

$$\prod_{i=2}^{\infty} \frac{1}{1-\frac{1}{(2i-1)^2}} = \frac{4}{\pi} \quad (2.5)$$

$$15\prod_{i=1}^{\infty} \frac{1}{1+\frac{1}{p_i^2}} = \pi^2 \quad (2.6)$$

$$6\prod_{i=1}^{\infty} \frac{1}{1-\frac{1}{p_i^2}} = \pi^2 \quad (2.7)$$

$p_1 = 2,\ p_2 = 3,\ p_3 = 5,$ ... prime sequence (2.8)

Lateral

$$\sum_{i=1}^{\infty} \frac{1-\frac{1}{p_{i+1}^2}}{1-\frac{1}{p_i^2}} = \frac{4}{3} \quad (2.9)$$

135.1.3

$$12\left(\frac{1}{|1} + \frac{1^4}{|3} + \frac{2^4}{|5} + \frac{3^4}{|7} + \frac{4^4}{|9} + \frac{5^4}{|11} + \frac{6^4}{|13} + \ldots\right) = \pi^2 \quad (3.1)$$

$$??\left(\frac{1}{|1}+\frac{1^4}{|3}-\frac{2^4}{|5}+\frac{3^4}{|7}-\frac{4^4}{|9}+\frac{5^4}{|11}-\frac{6^4}{|13}\pm\ldots\right)=\pi^2\,??$$

(3.2)

Lateral

$$4\left(\lceil\frac{1}{1}\rfloor+\lceil\frac{1^2}{3}\rfloor+\lceil\frac{2^2}{5}\rfloor+\lceil\frac{3^2}{7}\rfloor+\lceil\frac{4^2}{9}\rfloor+\cdots\right)=\pi$$

(3.3)

$$\left(\lceil\frac{1}{1}\rfloor+\lceil\frac{1^2}{3}\rfloor-\lceil\frac{2^2}{5}\rfloor+\lceil\frac{3^2}{7}\rfloor-\lceil\frac{4^2}{9}\rfloor+\cdots\right)=$$

0.705310651936126381 00… = ? (3.4)

$$\left(\lceil\frac{1}{1}\rfloor-\lceil\frac{1}{3}\rfloor+\lceil\frac{1}{5}\rfloor-\lceil\frac{1}{7}\rfloor+\lceil\frac{1}{9}\rfloor-\cdots\right)=$$

1.4533529634217465 56… = ? (3.5)

Part 2

Determinant quotient representations

135.2.1

Determinants of reciprocal factorials

135.2.1.1

$$f(n(\Rightarrow 5)) = 4\frac{\begin{vmatrix} 2. & 0. & 0 & 0 & 0 & \cdots \\ 4. & 2. & 0. & 0 & 0 & \\ 6. & 4. & 2. & 0. & 0 & \\ 8. & 6. & 4. & 2. & 0. & \\ 10. & 8. & 6. & 4. & 2. & \\ \vdots & & & & & \ddots \end{vmatrix}}{\begin{vmatrix} 2. & 0. & 0 & 0 & 0 & 0 & \cdots \\ 4. & 2. & 0. & 0 & 0 & 0 & \\ 6. & 4. & 2. & 0. & 0 & 0 & \\ 8. & 6. & 4. & 2. & 0. & 0 & \\ 10. & 8. & 6. & 4. & 2. & 0. & \\ 12. & 10. & 8. & 6. & 4. & 2. & \\ \vdots & & & & & & \ddots \end{vmatrix}}; \text{i.} = \frac{1}{\text{i!}}$$

(4.1)

$$df(n) = \frac{f(n+1)-f(n)}{f(n+2)-f(n+1)} \qquad (4.2)$$

$$\lim_{n\to\infty} f(n) = \pi^2 \; ; \; \lim_{n\to\infty} df(n) = df = 9$$

(4.3)

Acceleration

Acc1

$$f1(n) = \frac{df}{df-1} f(n+1) - \frac{1}{df-1} f(n) =$$

$$\frac{\begin{vmatrix} f(n) & f(n+1) \\ df & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ df & 1 \end{vmatrix}} \Rightarrow \frac{9}{8} f(n+1) - \frac{1}{8} f(n)$$

(4.4)

Acc2, triple accelerator

$$f1t(n) = \frac{\begin{vmatrix} f(n) & f(n+1) \\ f(n+1)-f(n) & f(n+2)-f(n+1) \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ f(n+1)-f(n) & f(n+2)-f(n+1) \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} f(n) & f(n+1) \\ f(n+1) & f(n+2) \end{vmatrix}}{f(n)-2f(n+1)+f(n+2)} \qquad (4.5)$$

More you can find in appendix 1.

f(n) – f1(n) – f1t(n) calculation record

```
f(n); n = 1,2,3,4,…
9.60000000000000000000000000000,
9.83606557377049180327868852459, 9.86570397111913357400722021661,
9.86916331822410482769541378833, 9.86955506675571128085497629280,
9.86959890611693970346183238334, 9.86960378999365549478142566999,
9.86960433316793884462137964118, 9.86960439354165480178770937011,
9.86960440025068961300049387597, 9.86960440099617176231646187365,
9.86960440107900446717826165738, 9.86960440108820815527922694766,
9.86960440108923078946015489538, 9.86960440108934441556705890105,
9.86960440108935704069352052674, 9.86960440108935844348548849316,
9.86960440108935859935126826763, 9.86960440108935861666968846481,
9.86960440108935861859395738450, 9.86960440108935861880776504260,
9.86960440108935861883152144907, 9.86960440108935861883416104979,
9.86960440108935861883445433876, 9.86960440108935861883448692642,
9.86960440108935861883449054727, 9.86960440108935861883449094959,
9.86960440108935861883449099429, 9.86960440108935861883449099926,
9.86960440108935861883449099981, 9.86960440108935861883449099987,
9.86960440108935861883449099988, 9.86960440108935861883449099988, 30 valid digits at n=33
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988

Acc1: f1(n); n = 1,2,3,4,…
9.86557377049180327868852459016,
9.86940877078771379534828667811, 9.86959573661222623440643798479,
9.86960403532216208749992160586, 9.86960438603709325628768939466,
9.86960440047824496869637483082, 9.86960440106472426335137388757,
9.86960440108836929643350058622, 9.86960440108931896440209193920,
9.86960440108935703098095787336, 9.86960440108935855528598663035,
9.86960440108935861629184760894, 9.86960440108935861873277088884,
9.86960440108935861883042190176, 9.86960440108935861883432822995,
9.86960440108935861883448448896, 9.86960440108935861883449073944,
9.86960440108935861883449098946, 9.86960440108935861883449099946,
9.86960440108935861883449099986, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988,                    n=22
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988,
9.86960440108935861883449099988, 9.86960440108935861883449099988

Acc2: f1t(n); n = 1,2,3,4,…
9.86995939238983305760264701459,
9.86962044261484036842136328148, 9.86960509511122960957587897904,
9.86960443023000297178494875103, 9.86960440228874239877130583411,
9.86960440113811341534504570767, 9.86960440109132608357815418002,
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9.869604401089437689703804712 60, 9.869604401089361789546302889 75,
9.869604401089358745827621612 61, 9.869604401089358623917630778 12,
9.869604401089358619037887027 11, 9.869604401089358618842628288 87,
9.869604401089358618834816521 13, 9.869604401089358618834504021 33,
9.869604401089358618834491520 75, 9.869604401089358618834491020 71,
9.869604401089358618834490999 91,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,                n=21
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88,
9.869604401089358618834490999 88, 9.869604401089358618834490999 88
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135.2.1.2

$$f(n(\Rightarrow 5)) = \frac{\begin{vmatrix} 2. & 1. & 0 & 0 & 0 & \cdots \\ 4. & 3. & 1. & 0 & 0 & \\ 6. & 5. & 3. & 1. & 0 & \\ 8. & 7. & 5. & 3. & 1. & \\ 10. & 9. & 7. & 5. & 3. & \\ \vdots & & & & & \ddots \end{vmatrix}}{\begin{vmatrix} 2. & 1. & 0 & 0 & 0 & 0 & \cdots \\ 4. & 3. & 1. & 0 & 0 & 0 & \\ 6. & 5. & 3. & 1. & 0 & 0 & \\ 8. & 7. & 5. & 3. & 0. & 0 & \\ 10. & 9. & 7. & 5. & 3. & 1. & \\ 12. & 11. & 9. & 7. & 5. & 3. & \\ \vdots & & & & & & \ddots \end{vmatrix}} ; i. = \frac{1}{i!}$$

(5.1)

$$df(n) = \frac{f(n+1)-f(n)}{f(n+2)-f(n+1)} \quad (5.2)$$

$$\lim_{n\to\infty} f(n) = \pi^2 ; \lim_{n\to\infty} df(n) = df = 9 \quad (5.3)$$

135.2.1.3

$$f(n(\Rightarrow 5)) = 4\frac{\begin{vmatrix} 1. & 0. & 0 & 0 & 0 & \cdots \\ 3. & 2. & 0. & 0 & 0 & \\ 5. & 4. & 2. & 0. & 0 & \\ 7. & 6. & 4. & 2. & 0. & \\ 9. & 8. & 6. & 4. & 2. & \\ \vdots & & & & & \ddots \end{vmatrix}}{\begin{vmatrix} 1. & 0. & 0 & 0 & 0 & 0 & \cdots \\ 3. & 2. & 0. & 0 & 0 & 0 & \\ 5. & 4. & 2. & 0. & 0 & 0 & \\ 7. & 6. & 4. & 2. & 0. & 0 & \\ 9. & 8. & 6. & 4. & 2. & 0. & \\ 11. & 10. & 8. & 6. & 4. & 2. & \\ \vdots & & & & & & \ddots \end{vmatrix}}; \mathrm{i}.= \frac{1}{\mathrm{i}!}$$

(6.1)

$$\mathrm{df}(n) = \frac{f(n+1)-f(n)}{f(n+2)-f(n+1)} \tag{6.2}$$

$$\lim_{n\to\infty} f(n) = \pi^2 \; ; \lim_{n\to\infty} \mathrm{df}(n) = \mathrm{df} = 9$$

(6.3)

135.2.1.4

$$f(n(\Rightarrow 5)) = \frac{\begin{vmatrix} 3. & 1. & 0 & 0 & 0 & \cdots \\ 5. & 3. & 1. & 0 & 0 & \\ 7. & 5. & 3. & 1. & 0 & \\ 9. & 7. & 5. & 3. & 1. & \\ 11. & 9. & 7. & 5. & 3. & \\ \vdots & & & & & \ddots \end{vmatrix}}{\begin{vmatrix} 3. & 1. & 0 & 0 & 0 & 0 & \cdots \\ 5. & 3. & 1. & 0 & 0 & 0 & \\ 7. & 5. & 3. & 1. & 0 & 0 & \\ 9. & 7. & 5. & 3. & 0. & 0 & \\ 11. & 9. & 7. & 5. & 3. & 1. & \\ 13. & 11. & 9. & 7. & 5. & 3. & \\ \vdots & & & & & & \ddots \end{vmatrix}} ; \mathrm{i.} = \frac{1}{\mathrm{i!}}$$

(7.1)

$$df(n) = \frac{f(n+1)-f(n)}{f(n+2)-f(n+1)} \quad (7.2)$$

$$\lim_{n\to\infty} f(n) = \pi^2 \; ; \lim_{n\to\infty} df(n) = df = 4 \quad (7.3)$$

135.2.2

Pascal’s triangle determinants

135.2.2.1

f(n(⇒7)) =

$$(2n+4)(2n+3)\frac{\begin{vmatrix} \binom{1}{0} & \binom{1}{1} & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \binom{2}{0} & \binom{2}{2} & 0 & 0 & 0 & 0 & \\ 0 & 0 & \binom{3}{1} & \binom{3}{3} & 0 & 0 & 0 & \\ 0 & 0 & \binom{4}{0} & \binom{4}{2} & \binom{4}{4} & 0 & 0 & \\ 0 & 0 & 0 & \binom{5}{1} & \binom{5}{3} & \binom{5}{5} & 0 & \\ 0 & 0 & 0 & \binom{6}{0} & \binom{6}{2} & \binom{6}{4} & \binom{6}{6} & \\ 0 & 0 & 0 & 0 & \binom{7}{1} & \binom{7}{3} & \binom{7}{5} & \\ \vdots & & & & & & & \ddots \end{vmatrix}}{\begin{vmatrix} \binom{1}{0} & \binom{1}{1} & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \binom{2}{0} & \binom{2}{2} & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & \binom{3}{1} & \binom{3}{3} & 0 & 0 & 0 & 0 & \\ 0 & 0 & \binom{4}{0} & \binom{4}{2} & \binom{4}{4} & 0 & 0 & 0 & \\ 0 & 0 & 0 & \binom{5}{1} & \binom{5}{3} & \binom{5}{5} & 0 & 0 & \\ 0 & 0 & 0 & \binom{6}{0} & \binom{6}{2} & \binom{6}{4} & \binom{6}{6} & 0 & \\ 0 & 0 & 0 & 0 & \binom{7}{1} & \binom{7}{3} & \binom{7}{5} & \binom{7}{7} & \\ 0 & 0 & 0 & 0 & \binom{8}{0} & \binom{7}{1} & \binom{7}{1} & \binom{7}{1} & \\ \vdots & & & & & & & & \ddots \end{vmatrix}} \tag{8.1}$$

$$\mathrm{df}(n) = \frac{f(n+1)-f(n)}{f(n+2)-f(n+1)} \tag{8.2}$$

$$\lim_{n\to\infty} f(n) = \pi^2;\ \lim_{n\to\infty} \mathrm{df}(n) = \mathrm{df} = 9 \tag{8.3}$$

135.2.2.2

$$f(n(\Rightarrow 7)) = \frac{(2n+5)(n+2)(n+3)}{2} \cdot \frac{\begin{vmatrix} \binom{3}{0} & \binom{3}{3} & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \binom{4}{1} & \binom{4}{3} & 0 & 0 & 0 & 0 & \\ 0 & \binom{5}{0} & \binom{5}{2} & \binom{5}{4} & 0 & 0 & 0 & \\ 0 & 0 & \binom{6}{1} & \binom{6}{3} & \binom{6}{5} & 0 & 0 & \\ 0 & 0 & \binom{7}{0} & \binom{7}{2} & \binom{7}{4} & \binom{7}{6} & 0 & \\ 0 & 0 & 0 & \binom{8}{1} & \binom{8}{3} & \binom{8}{5} & \binom{8}{7} & \\ 0 & 0 & 0 & \binom{9}{0} & \binom{9}{2} & \binom{9}{4} & \binom{9}{6} & \\ \vdots & & & & & & & \ddots \end{vmatrix}}{\begin{vmatrix} \binom{3}{0} & \binom{3}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \binom{4}{1} & \binom{4}{3} & 0 & 0 & 0 & 0 & 0 & \\ 0 & \binom{5}{0} & \binom{5}{2} & \binom{5}{4} & 0 & 0 & 0 & 0 & \\ 0 & 0 & \binom{6}{1} & \binom{6}{3} & \binom{6}{5} & 0 & 0 & 0 & \\ 0 & 0 & \binom{7}{0} & \binom{7}{2} & \binom{7}{4} & \binom{7}{6} & 0 & 0 & \\ 0 & 0 & 0 & \binom{8}{1} & \binom{8}{3} & \binom{8}{5} & \binom{8}{7} & 0 & \\ 0 & 0 & 0 & \binom{9}{0} & \binom{9}{2} & \binom{9}{4} & \binom{9}{6} & \binom{9}{8} & \\ 0 & 0 & 0 & 0 & \binom{10}{1} & \binom{10}{3} & \binom{10}{5} & \binom{10}{7} & \\ \vdots & & & & & & & & \ddots \end{vmatrix}} \quad (9.1)$$

$$df(n) = \frac{f(n+1)-f(n)}{f(n+2)-f(n+1)} \quad (9.2)$$

$$\lim_{n\to\infty} f(n) = \pi^2 \; ; \; \lim_{n\to\infty} df(n) = df = ?$$

(9.3)

Part 3
2x2 matrix chain product representation
135.3.1
Launch pad >

$$\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$$

rec $\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} m & m \\ 2(m+1) & (m+1) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}$ (10.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \quad (10.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \quad (10.5)$$

$$\lim_{m\to\infty} f(m) = \frac{\pi}{4};\ \lim_{m\to\infty} df(m) = df = ?$$

(10.6,7)

> to find a similar representation for π^2.
Lateral
** Check $\lim_{m\to\infty} f(m)$ for other inits $\underline{A}(1)$.

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} 2m & (2m-1) \\ (2m-1) & (2m-2) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ (11.1,2,3)

$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right)$ (11.4)

$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)}$ (11.5)

$\lim_{m\to\infty} f(m) = e = [2;1,2,1,1,4,1,1,6,1,1,8,\ldots]$

; $\lim_{m\to\infty} df(m) = df = ?$ (11.6,7)

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} (2m+1) & (2m-1) \\ (2m-1) & (2m-3) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ (12.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (12.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (12.5)$$

$$\lim_{m\to\infty} f(m) = e^2 \; ; \; \lim_{m\to\infty} df(m) = df = ?$$

(12.6,7)

$$** \; \underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$$

$$\text{rec} \;\; \underline{A}(m) = \underline{A}(m\text{-}1)\begin{pmatrix} (4m-1) & (4m-2) \\ (4m-2) & (4m-3) \end{pmatrix};$$

$$m = 2,3,4,\ldots$$

$$\text{init} \;\; \underline{A}(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \qquad (14.1,2,3)$$

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (14.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (14.5)$$

$$\lim_{m\to\infty} f(m) = \sqrt{e} = [1;1,1,1,5,1,1,9,1,1,13,\ldots]$$

$$\lim_{m\to\infty} df(m) = df = ? \qquad (14.6,7)$$

$$** \; \underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$$

rec $\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} 4m-2 & (4m-3) \\ (4m-3) & (4m-4) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ (15.1,2,3)

$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right)$ (15.4)

$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)}$ (15.5)

$\lim_{m\to\infty} f(m) = ?$ (numerical value,
formula and scf representation)
$= 2.819093541239003584 8\ldots$
$= [2;1,4,1,1,8,1,1,12,1,1,16,1,1,20,1,1,\ldots]$
$= ?$

$\lim_{m\to\infty} df(m) = df = ?$ (15.6,7)

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} 5m-3 & (5m-4) \\ (5m-4) & (5m-5) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ (16.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (16.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (16.5)$$

$\lim_{m\to\infty} f(m) = ?$ (numerical value,
formula and scf representation)
= 2.846689138324612621 6…
= [2;1,5,1,1,10,1,1,15,1,1,20,1,1,25,1,1,…]
= ?

$$\lim_{m\to\infty} df(m) = df = ? \qquad (16.6,7)$$

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m\text{-}1)\begin{pmatrix} 5m-2 & (5m-3) \\ (5m-3) & (5m-4) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ (17.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (17.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (17.5)$$

$\lim_{m\to\infty} f(m) = ?$ (numerical value,
formula and scf representation)
= 1.651207674659799342 0…
= [1;1,1,1,6,1,1,11,1,1,16,1,1,21,1,1,…]
= ?

$\lim_{m\to\infty} df(m) = df = ?$ (17.6,7)

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m-1)\begin{pmatrix} 9m-2 & (9m-3) \\ (9m-3) & (9m-4) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ (18.1,2,3)

$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right)$ (18.4)

$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)}$ (18.5)

$\lim_{m\to\infty} f(m) = ?$ (numerical value,
formula and scf representation)
= 1.659345685900409534 2…
= [1,1,1,1,14,1,1,23,1,1,32,1,1,41,1,1,50,…]
= ?

$$\lim_{m\to\infty} df(m) = df = ? \qquad (18.6,7)$$

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m\text{-}1)\begin{pmatrix} 8m-2 & (8m-3) \\ (8m-3) & (8m-4) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ (19.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (19.4)$$

$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (19.5)$$

$\lim_{m\to\infty} f(m) = ?$ (numerical value,
formula and scf representation)
= 1.658235468119068812 2…
= [1;1,1,1,12,1,1,20,1,1,28,1,1,36,1,1,…]
= ?

$$\lim_{m\to\infty} df(m) = df = ? \qquad (19.6,7)$$

** $\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m\text{-}1)\begin{pmatrix} 7m-2 & (7m-3) \\ (7m-3) & (7m-4) \end{pmatrix}$;

$m = 2,3,4,\ldots$

init $\underline{A}(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ (20.1,2,3)

$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right)$ (20.4)

$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)}$ (20.5)

$\lim_{m\to\infty} f(m) = ?$ (numerical value,
formula and scf representation)
= 1.656728771615406136 0…
= [1;1,1,1,10,1,1,17,1,1,24,1,1,31,1,1,…]
= ?

$\lim_{m\to\infty} df(m) = df = ?$ (20.6,7)

** Problems

Comprime the equs (11), (12), and (14).

Solution

$\underline{A}(m) = \begin{pmatrix} A_{11}(m) & A_{12}(m) \\ A_{21}(m) & A_{22}(m) \end{pmatrix}$

rec $\underline{A}(m) = \underline{A}(m\text{-}1)\begin{pmatrix} (a+p) & a \\ a & (a-p) \end{pmatrix}$;

$$a = (2m-1)q \; ; \; m = 2,3,4,\ldots$$

$$\text{init } \underline{A}(1) = \begin{pmatrix} q+p & q \\ q & q-p \end{pmatrix}$$

(21.1,2,3)

$$f(m) = \frac{1}{2}\left(\frac{A_{11}(m)}{A_{21}(m)} + \frac{A_{12}(m)}{A_{22}(m)}\right) \qquad (21.4)$$

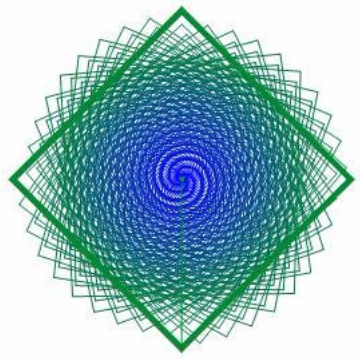
$$df(m) = \frac{f(m+1)-f(m)}{f(m+2)-f(m+1)} \qquad (22.5)$$

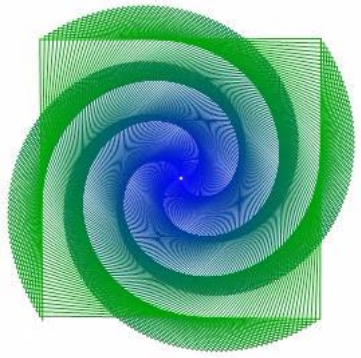
$$\lim_{m\to\infty} f(m) = e^{\frac{p}{q}} = \sqrt[q]{e^p} \; ; p, q \in \mathbb{N}$$

$$\lim_{m\to\infty} df(m) = df = ?$$

(22.6,7)

Find a direct way to equ. (22) and go then to 2x2 matrix chain product representations of $\pi^{\frac{p}{q}}, \frac{\pi}{e}$, and e^{π}.





23.04.2018 EPSnoct DScalc

To be continued.