Unit n °105

Variations on „e^{π} algorithmics“, Units 101 and 103

105.1

Variation on Unit 101, equs (1- 4)

** The extended recursion equation

$$z(t+1) = (z(t))^2 + \sqrt{\left((z(t))^2 + \sqrt{(z(t))^4 - 1}\right)^2 - 1} \quad ; t=0,1,2,3,... \qquad (1)$$

** with the initial value

$$z(0) = \frac{\sqrt[4]{2}+1}{\sqrt[4]{2}-1} = 11.5704270... \qquad (2)$$

** and the downstream output transformation

$$y(t) = \sqrt[2^t]{2z(t)} \qquad (3)$$

gives the following execution records for z(t)

11.570427015766490403162879294473207567885424985109,
401.618741646829713023325382998598673937539452690 50,
483 892.840921299064361829829394264859317987101764 83,

7.024568444846569270067044373920446486592570920196 9 10_{11},

1.480336855090024460911884806274013746767911779905 3 10_{24},

6.574191613613472239060533128139173094799799532346 9 10_{48},

1.296599861175171296011695719062210680644588433090 8 10_{98},

5.043513599998420431451782091101395886854018983340 5 10_{196},

7.631108830010708054726959180934837036369100327153 9 10_{393},

1.747014659264021926858744726553965220356605208592 4 10_{788},

9.156180659050159902327039891358955817329981999966 0 10_{1576},

2.515069327834926616082725785232983694321587097487 8 10_{3154},

1.897672117144808872352471846467885515721984444862910^6309,

1.080347839256658362320789694295365113016798935174210^12 619,

3.501454361359591605764885570880379242596830010369210^25 238,

3.678054793405231654049462630576017770714773524624510^50 477,

4.058426118987360391220430140511805316418829036815910^100 955,

4.941246768983642497257079109113800608240187450449210^201 911,

7.324775889597385933753660376540374824606923717320410^403 823,

1.609570254984815294679899866110057906137526231326510^807 648,

7.772149217195649974915651151042365404208132893475510^1 615 296,

1.812189103630648640631953231457678988347470757149510^3 230 594,

9.852088041952961394443085201712247487104540391902110^6 461 188,

2.911909163591776103920888276834965654031054854721910^12 922 378,

2.543764493102927126476064567400546064470669165664710^25 844 757,

1.941221338911357536520978154566734570062600378188910^51 689 515,

1.130502085993441091314154014778160184667550800408010^103 379 031,

3.834104899306565028292734628041357293779458966204910^206 758 062,

4.410108113665981546431743101059824019704472547829010^413 516 125,…

and y(t)

28.341444622560428439446552322632992174926358135576517342796351258026552421806063898-60465482033927948,

31.364964341900183753492813082862402437816059119576137804283518655410656434509174970-20332481071982590,

32.995613328331829978016952309141056585135688570007818231280441437580214124962435723-09763407009542936,

33.842458826056570205481600307616302238584005423959709803153474362506747809586475740-29162742624717737,

34.273997526533647372157148499682236488754488876823514326979745289962942891726844652-47227045555335708,

34.491826032977623076548612477529831416370118986194578048947084737857009357461358906-17895224375143654,

34.601258891268992274286032147511354126183173290753254561067179471389603475593337230-07861321664580805,

34.656105451523233199136591963276542566960614431531488590603706417247382309179086934-23920483457260006,

34.683561324585951509925520562818849409553087839424785903787031610067507196460011354-23475760807218588,

34.697297416882189064571506152362642921538201889700647352283021056226076709703604172-43158042827858652,

34.704167502913598890689373164503566242510651858247554314618978451257621392955322749-88048617969166475,

34.707603056017929216505580290731993459049144956106962959477335787422695328465119274-27064969790833221,

34.709320960106978797226151360088136243663410195623768542602571437332077398006684873-18523940566138126,

34.710179944037565870975118321888901532933625911231919659550345853937161973648951430-82782842210984686,

34.710609443974605136743141417052184250723577138178556707144366398379629002317881596-81025322818219610,

34.710824195936089972273823609358290505378706995585773472248920211400608785151868806-80898616973398713,

34.710931572415077287045461983468852114453897733394367140887081780606024185496029333-36614008784909730,

34.710985260779132618228541648310914940651811624811720423514038194950125199429802094-88581702771122838,

34.711012104992300758462770336759698789248767295124315644676478395089025960751822141-61648471804011330,

34.711025527106669954264738797512707363325170158096724191489548893299973271752308222-38689774729967874,

34.711032238165800834464959931427903119759981789461273115163179518631081552288905722-73331878395344378,

34.711035593695852845249632685857972875679702050963592973953996384379875636657314015-29830820625323040,

34.711037271461000493326828732626287710001772253321690886205133481434747864308724035-
76961349551846618,
34.711038110343604728038356563889828332323286888101146857573044106783311953089767328-
77852611050931443,
34.711038529784914448062567292812540970492073460082421931307194726498936352766211518-
70964123006819998,
34.711038739505571208741811155762377038547777653734140403101176507324716160517305337-
34329153589166649,
34.711038844365900064248221061255170974565616418529257998995096894670976049431165421-
90144685615840142,
34.711038896796064610793123426180508785155746168918220419679011344284335720299961722-
07799406610338056,…

These records show

1* “105.1”-y(t) converges very slowly compared with “101”-y(t).

$$\lim_{t\to\infty} \text{"105.1"} - y(t) = \frac{3}{2} e^{\pi} \ . \qquad (4)$$

2* the present algorithm equs (1,2,3) has the same serious disadvantage as “101”: its huge intrinsic numerical disbalance, z(t) increases exponentially (stronger than “101”) and equ. (4) reduces these values exponentially to 34.71…

105.2

Variation on Unit 103, equs (1- 3)

Transmutation trigonometric → hyperbolic

** in the recursion equation

$$\sinh\alpha(t+1) = \tanh^2 \frac{\alpha(t)}{2} \ ; \ t=0,1,2,3,... \qquad (5)$$

with the initial value

$$\alpha(0) = \ ? \qquad (6)$$

** and in the downstream output transformation

$$y(t) = \sqrt[2^{(t-1)}]{\frac{2}{\tanh\frac{\alpha(t)}{2}}} \quad . \qquad (7)$$

The execution records for $\alpha(0) = \frac{1}{2}$

$\longrightarrow \alpha(t)$

0.50000000000000000000000000000, 0.059949236038279567121336673801O,
0.00089793970378719944603811667283O, 2.01573900821330324664623283565 10 7,
1.01580093730818097440000801237 10 14, 2.57962886059044753554294454126 10 29,
1.66362126459779265158838482706 10 58, 6.91908928005489707412029200428 10 117,
1.19684491163426484785252923408 10 233, 3.58109435626207808039817420451 10 467

and y(t)

y(4) = 66.74310743949647783326295850991973876140731676627949434757O868663551
8194045651421190990268737097609S…

y(5) = 66.74310743949647783326295850991973876140731676627949434757 1562633881
8299137723172813340137096577554l…

** Compare the records of Unit 103 and Unit 105.1 .

** Find a formula representation for $\alpha(0) = 0.5 \longrightarrow$ $y(\infty) =$

66.74310743949647783326295850991973876140731676627949434757…. .