Challenge to go n° 60

A New Year 2015 encoded message from ACDG to Πλατων

60.1 Self-similar infinitely 2D-nested expressions

$$2 = 1+4\frac{1}{1+4(3)}\Big(3+4\cdot 5\frac{1}{1+4(3+4\cdot 5(5))}\Big(5+6\cdot 7\frac{1}{1+4(3+4\cdot 5(5+6\cdot 7(7)))}\Big(7+8\cdot 9\frac{1}{1+4(3+4\cdot 5(5+6\cdot 7(7+8\cdot 9(9))))}\Big(9+10\cdot 11\frac{1}{1+4(3+4\cdot 5(5+6\cdot 7(7+8\cdot 9(9+10\cdot 11(11)))))}\Big(11+\ldots$$

$$\frac{e}{2} = 1+\frac{1}{1\cdot4-1}\left(1+\frac{1\cdot4}{(1\cdot4-1)3\cdot6-1}\left(1+\frac{3\cdot6}{(1\cdot4-1)3\cdot6-1)5\cdot8-1}\left(1+\frac{5\cdot8}{(1\cdot4-1)3\cdot6-1)5\cdot8-1)7\cdot10-1}\left(1+\frac{7\cdot10}{(1\cdot4-1)3\cdot6-1)5\cdot8-1)7\cdot10-1)9\cdot12-1}\left(1+\frac{9\cdot12}{\cdots\cdots}\ \ldots\right.\right.\right.\right.$$

$$\frac{\sqrt{k^2+l}+k}{2k} =$$

$$1+\cfrac{1}{a}\left(1+\cfrac{1}{b}\left(1+\cfrac{1}{c-\cfrac{1}{a}}\left(1+\cfrac{1}{c-\cfrac{1}{b}}\left(1+\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{a}}}\left(1+\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{b}}}\left(1+\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{a}}}}\left(1+\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{b}}}}\left(1+\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{a}}}}}\left(1+\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{c-\cfrac{1}{b}}}}}\left(1+\ldots\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$k,l \in \mathbb{N};\ a = \dfrac{4k^2+l}{l};\ b = \begin{vmatrix} a & 1 \\ 1 & a+1 \end{vmatrix};\ c = a+b = \begin{vmatrix} a & 1 \\ 1 & a+2 \end{vmatrix}$

$x_1 = \dfrac{\sqrt{k^2+l}+k}{2k}$ and $x_2 = 1-x_1$ are roots of the quadratic equation $x^2 - x - \dfrac{l}{4k^2} = 0$.

60.2 Self-similar infinite mixed (nested expression, continued fraction) branched 2D expressions

$$\frac{\pi}{2} = 1+\frac{1}{1+2}\left(1+\frac{1}{1+2-\frac{1}{2}}\left(1+\frac{1}{1+2-\frac{1}{2-\frac{1}{2}}}\left(1+\frac{1}{1+2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}\left(1+\frac{1}{1+2-\frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}}\left(1+\cdots\right.\right.\right.\right.\right.\right.$$

$$\Phi = \frac{1}{2}\left(\sqrt{5}+1\right)$$

$$= 1+\frac{1}{2}\left(1+\frac{1}{5}\left(1+\frac{1}{7-\frac{1}{2}}\left(1+\frac{1}{7-\frac{1}{5}}\left(1+\frac{1}{7-\frac{1}{7-\frac{1}{2}}}\left(1+\frac{1}{7-\frac{1}{7-\frac{1}{5}}}\left(1+\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{2}}}}\left(1+\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{5}}}}\left(1+\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{2}}}}}\left(1+\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{7-\frac{1}{5}}}}}\left(1+\cdots\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\sqrt[k]{n} = 1+\frac{n-1}{nk}\left(1+\frac{n-1}{nk\left(1-\frac{k-1}{k+1}\right)}\left(1+\frac{n-1}{nk\left(1-\frac{k-1}{k+1-\frac{1}{2}}\right)}\left(1+\frac{n-1}{nk\left(1-\frac{k-1}{k+1-\frac{1}{2-\frac{1}{2}}}\right)}\left(1+\frac{n-1}{nk\left(1-\frac{k-1}{k+1-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}\right)}\left(1+\cdots\right.\right.\right.\right.\right.\right.$$

$$k, n \in \mathbb{N}$$

60.3 Exercise

Collect further systematically structured infinite branched 2D expressions for mathematical constants [1] (especially for the Feigenbaum constant δ [2,3,4]) and for elementary functions.

Ein solcher adäquater **Ausdruck für δ ist möglicherweise mehrstufig verzweigt** (vgl. L-System [7]). Die Tatsache, dass bisher kein systematisch aufgebauter infiniter 1D Zeichenkettenausdruck (Potenzreihe, nested expression, Kettenbruch, etc.) für δ gefunden wurde, könnte darauf hinweisen.

60.4 Find the used code algorithm and the **hidden message** to Πλατων.

References

[1] http://en.wikipedia.org/wiki/Mathematical_constants_and_functions

[2] H.-O. Peitgen, H. Jürgens, D. Saupe. Chaos and Fractals. New Frontiers of Science. 1992. ISBN 0-387-97903-4

[3] Weisstein, Eric W. "Feigenbaum Constant." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/FeigenbaumConstant.html

[4] http://de.wikipedia.org/wiki/Feigenbaum-Konstante

[5] http://en.wikipedia.org/wiki/Kryptos

[6] http://en.wikipedia.org/wiki/Plato

[7] http://en.wikipedia.org/wiki/L-system