Unit n° 53

Supplement 5

EA extended
Start

The simple continued fraction (scf) representation of $r \in \mathbb{Q}_+, \mathbb{R}_+$

$$scf(r) = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}} \quad \ldots + \cfrac{1}{a_n} = [a_0; a_1, a_2, a_3, \ldots, a_n]$$

$$= a_0 + (a_1 + (a_2 + (a_3 + \ldots + a_{n-1} + (a_n)^{-1})^{-1} \ldots)^{-1})^{-1})^{-1}$$

$n \to \infty$ infinite scf

is a very efficient tool to analyze the "inner discrete structure" of mathematical constants and functions via

Algorithm 1: transformation $r \in \mathbb{Q}_+, \mathbb{R}_+ \to scf(r)$

$r \in \mathbb{Q}_+, \mathbb{R}_+$; length((approx.)scf(r))	
x:=r ; i:=0	
a_0:=int(x)	
repeat until	i:=i+1
	$x:=(x - int(x))^{-1}$
	a_i:=int(x)
x=int(x) or i=length	
n:=i	
(approx.)scf(r)=$[a_0; a_1, a_2, \ldots a_n]$	

Example
r = e = 2.7182818284…= [2; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,…]

Algorithm 2.1,2: inverse transformation $[a_0;a_1,a_2,a_3,\ldots a_n] \rightarrow$ (approx.)r

x:=1; y:=0
i=n(1)0 (x,y):=((a_i*x+y),x)
a:=x ; b:=y (approx.)r:=$\frac{a}{b}$

x:=0
i=n(1)1 x:=$(a_i+x)^{-1}$
(approx.)r:=$a_0 + x$

Extension
It seems to be obvious to extend the x-recursion "x:=$(x - int(x))^{-1}$ in algorithm 1 to
algorithm 1ext : transformation $r \in \mathbb{Q}_+,\mathbb{R}_+ \rightarrow$ extscf(r)

Input: $r \in \mathbb{Q}_+,\mathbb{R}_+$; length(approx.extscf(r))
x:=r ; i:=0

a_0:=int(x)	
repeat until	i:=i+1
	x:=f(x)
	a_i:=int(x)
x=int(x) or i=length	
n:=i	
Output: approx.extscf(r)=[a_0;a_1,a_2,...a_n]	

with $f(x)\in$

$$\left\{\left(\frac{x-\text{int}(x)}{(i \bmod k)+1}\right)^{-1}, \left(\frac{x-\text{int}(x)}{\text{int}(x)}\right)^{-1}, \left(\frac{x-\text{int}(x)}{k}\right)^{-1}, \left(\frac{x-\text{int}(x)}{k}\right)^{-m}, \ldots\right\};$$

$i,k,m\in\infty$

Example 1

$r=2+\sqrt{7}=4.645751311\ldots$; $scf(2+\sqrt{7})=[4; 1, 1, 1, per]$

$$f(x)=\left(\frac{x-\text{int}(x)}{2}\right)^{-1}$$

$$extscf(2+\sqrt{7})=[4; 3, 20, 3, per]=scf(R)=scf\left(2+\sqrt{\frac{6*7}{31}}\right)$$

Find the relation between r and R and the corresponding inversion algorithm!

Example 2

$r=\pi$; $scf(\pi)=[3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, \ldots]$

$$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$$

extscf(π)=[3; k*7, 15, k*1, 292, μ(1,k), the following sequence is visually not connected with scf(r)] for k=2(1)15

k	μ(1,k) → formula?
1	1 = 2 – 1
2	3 = 4 – 1

3	4 = 6 – 2
4	6 = 8 – 2
5	7 = 10 – 3
6	9 = 12 – 3
7	11 = 14 – 3
8	12 = 16 – 4
9	14 = 18 – 4
10	15 = 20 – 5
11	17 = 22 – 5
12	18 = 24 – 6
13	20 = 26 – 6
14	22 = 28 – 6
15	23 = 30 – 7
16	25 = 32 – 7
17	26 = 34 – 8
19	28 = 36 – 8

↓

Discussion!

Example 3

r=3.19115936409461031 41…; scf(r)=[3; 5, 4, 3, 12, 3]

$$f(x)=\left(\frac{x-\mathrm{int}(x)}{k}\right)^{-1}$$

extscf(r)=[3; k*5, 4, k*3, 12, k*3] ; k=1(1)4

Inverse transformation: $[a_0;a_1,a_2,a_3,\ldots a_n] \rightarrow$ (approx.)r

x:=0
i=n(1)1
$x:=(a_i+k*x)^{-1}$
(approx.)r:=$a_0 + k*x$

Discussion!

Example 4

r=0.31303673643358290638395160264178247639668977180326…

scf(r)=[0;3,5,7,11,13,17,19,23,29,31,37,…] sequence of primes

$$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$$

extscf(r)=[0; k*3, 5, k*7, 11, k*13, 17, k*19, 23, k*29, 31, k*37, ad infinitum] → prime "immortality" for k=1(1)5

Open problems waiting for you:

** We insert 15 between 13 and 17 and calculate the resulting r'. What happens with extscf(r')?

** We delete 13 and 17 in the prime sequence and calculate the resulting r''. What happens with extscf(r'')?

** We double 17 in the prime sequence and calculate the resulting r'''. What happens with extscf(r''')?

** We substitute 37 in the prime sequence by a larger prime and calculate the resulting r''''. What happens with extscf(r'''')?

** We delete in the prime sequence the subsequence [31,37,41,43,47,53] and refill the gap with the sequence [9,9,9,9,9,9,9,9,9,9], - a strong shot -. What happens now? Robustness?

Example 5

r=0.697774657964007982006790592..

scf(r)=[0;1,2,3,4,5,6,7,8,9,…] sequence of natural numbers

$$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$$

extscf(r)=[0; k*1, 2, k*3, 4, k*5, 6, k*7, 8, k*9, ad inf.]; k=1(1)2

Example 6

r=1.433127426722311758317183455…

scf(r)=[1;2,3,4,5,6,7,8,9,…]

$$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$$

extscf(r)=[1; k*2, 3, k*4, 5, k*6 ,7, k*8, 9, ad inf.]; k=1(1)3

Example 7

r= 0.462117157260009758502318483 64…=(e-1)/(e+1)

scf(r)=[0;2,6,10,14,18,22,26,30,34,…]

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[0; k*2, 6, k*10, 14, k*18, 22, k*26, ad inf]; k=1(1)6

Example 8

r=1.698156279…

scf(r)=[1;1,2,3,5,8,13,21,34,55,89,…]

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[1; k*1, 2, k*3, 5, k*8, 13, k*21, 34, ad inf.]; k=1(1)2

But!

Example 9

$r=\sqrt{e^{\pi}}=4.810477381...$

scf(r)=[4;1,4,3,1,1,1,1,1,1,1,1,7,1,20,1,3,6,10,3,2,1,1,7,2,…

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[4; k*1, 4, the following sequence is visually not connected with scf(r)] for k=2(1)4

Example 10

$r=\pi^2=9.869604401…$

scf(r)=[9;1,6,1,2,47,1,8,1,1,2,2,1,1,8,3,1,10,5,1,3,1,2,1,1,3,15,1,…]

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[9; k*1, 6, the following sequence is visually not connected with scf(r)] for k=2(1)6

Example 11

$r=\sqrt{\pi}=1.772453851…$

scf(r)=[1;1,3,2,1,1,6,1,28,13,1,1,2,18,1,1,1,83,1,4,1,2,4,1,288,1,…]

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[1; k*1, 3, the following sequence is visually not connected with scf(r)] for k=2(1)6

Example 12

$r=\sqrt{e*\pi}=2.922282365\ldots$

scf(r)=[2;1,11,1,6,1,1,9,1,9,5,8,10,5,1, 4,2,1,1,1,3,2,2,3,1,26,11,…]

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[2; k*1, 11, the following sequence is visually not connected with scf(r)] for k=2(1)11

Example 14

$r=\sqrt{\frac{\pi}{e}}=1.075047684\ldots$

scf(r)=[1;13,3,12,1,4,9,7,1,1,1,2,7,2,1,1,1,1,4,4,3,1,85,2,1,6,1,..]

$f(x)=\left(\frac{x-\text{int}(x)}{k}\right)^{-1}$

extscf(r)=[1; k*13, 3, the following sequence is visually not connected with scf(r)] for k=2(1)3

Obvious conjecture

Basis

The simple continued fraction (scf) representation of $r\in\mathbb{Q}_+,\mathbb{R}_+$

$$\text{scf}(r) = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots \cfrac{}{\ldots + \cfrac{1}{a_n}}}}} = [a_0; a_1, a_2, a_3, \ldots, a_n]$$

$$= a_0 + (a_1 + (a_2 + (a_3 + \ldots + a_{n-1} + (a_n)^{-1})^{-1} \ldots)^{-1})^{-1})^{-1}$$

$n\to\infty$ infinite scf

is a very efficient tool to analyze the “inner discrete structure” of mathematical constants and functions via

Algorithm 1: transformation $r\in\mathbb{Q}_+,\mathbb{R}_+ \to$ scf(r)

$r \in \mathbb{Q}_+, \mathbb{R}_+$; length((approx.)scf(r))	
x:=r ; i:=0	
a_0:=int(x)	
repeat until	i:=i+1
	$x:=(x - int(x))^{-1}$
	a_i:=int(x)
x=int(x) or i=length	
n:=i	
(approx.)scf(r)=$[a_0;a_1,a_2,...a_n]$	

Conjecture

There are $r \in \mathbb{R}_+$ so, that the extended scf representation via Algorithm 2: transformation $r \in \mathbb{R}_+ \rightarrow$ extscf(r)

Input: $r \in \mathbb{R}_+$; $k \in \mathbb{N}$ length(approx.extscf(r))	
x:=r ; i:=0	
b_0:=int(x)	
repeat until	i:=i+1
	$x:=\left(\frac{x - int(x)}{k}\right)^{-1}$
	b_i:=int(x)
x=int(x) or i=length	
n:=i	
Output: approx.extscf(r)=$[b_0;b_1,b_2,...b_n]$	

delivers the accompanying

extscf(r)=[a_0; k*a_1, a_2, k*a_3, a_4, k*a_5, a_6, k*a_7, a_8, ad infinitum]

with $k=1(1)a_2$.

Problems

** Suchen Sie hierfür weitere positive Beispiele.
Find more positive example of this.

** Was haben diese und die vorstehenden Beispiele gemeinsam?
What do these and the above examples all have in common?

** Entwickeln Sie auf dieser Grundlage einen Beweis für die vorliegende Vermutung.
How about a proof for this conjecture?

** Diskutieren Sie denkbare zahlentheoretische und kryptographische Aspekte der vorliegenden Vermutung.
Discuss conceivable number-theoretic and cryptographic aspects of this conjecture.

29.09.2016 EPS(inv.comp), DS(calc), ShL(foto)



Foto “Large area search” by Shen Li