Unit 53, Supplement 2, first draw
A construction site, free for your discussion
EA extended
Start

The simple continued fraction representation of $r \in \mathbb{Q}_+, \mathbb{R}_+$

$$r = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}} \quad \ldots + \cfrac{1}{a_n} = [a_0; a_1, a_2, a_3, \ldots, a_n]$$

$$= a_0 + (a_1 + (a_2 + (a_3 + \ldots + a_{n-1} + (a_n)^{-1})^{-1} \ldots)^{-1})^{-1})^{-1}$$

$n \to \infty$ infinite scf

is a very efficient tool to analyze the "inner discrete (wave) structure" of mathematical constants and functions.

Algorithm 1: transformation $r \in \mathbb{Q}_+, \mathbb{R}_+ \to$ scf(r)

$r \in \mathbb{Q}_+, \mathbb{R}_+$; length((approx.)scf(r))	
x:=r ; i:=0	
a_0:=int(x)	
repeat until	$x:=(x - \text{int}(x))^{-1}$
	i:=i+1
	a_i:=int(x)
x=int(x) or i=length	
n:=i	
(approx.)scf(r)=$[a_0; a_1, a_2, \ldots a_n]$	

Example
r = e = 2.7182818284…= [2; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,…]

Algorithm 2.1,2: inverse transformation $[a_0;a_1,a_2,a_3,\ldots a_n] \rightarrow$ (approx.)r

x:=1; y:=0
i=n(1)0
(x,y):=((a_i*x+y),x)
a:=x ; b:=y
(approx.)r:= $\frac{a}{b}$

x:=0
i=n(1)1
x:=(a_i +x)$^{-1}$
(approx.)r:=$a_0 + x$

Extension
It seems to be obvious to extend the exponent 1 to $m \in \mathbb{R}_+$
$r = a_0 + (a_1 + (a_2 + (a_3 + \ldots + a_{n-1} + (a_n)^{-m})^{-m} \ldots)^{-m})^{-m})^{-m}$
and the algorithms 1 to
algorithm 1ext : transformation $r \in \mathbb{Q}_+,\mathbb{R}_+ \rightarrow$ scf(r,k) ; $k=m^{-1}$

$r \in \mathbb{Q}_+,\mathbb{R}_+$; $k \in \mathbb{R}_+$
length(approx.scf(r))

x:=r ; i:=0	
a_0:=int(x)	
repeat until	x:=(x – int(x))$^{-k}$
	i:=i+1
	a_i(k):=int(x)
x=int(x) or i=length	
n:=i	
approx.scf(r,k)=[a_0;a_1,a_2,...a_n]	

What happens?

Experiment 1

Given: r = e, k = $\frac{1}{2}$

$\rightarrow$ scf(r,k) = [2; 1, 2, 1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 2, 2, 2, 2, 4, 1, 1, 1, 4, 1, 1, 1, 1, 1, 20, 1, 1, 1, 1, 1, 1, 7, 1, 1, 1, 1, 2, 1, 4, 1, 1, 2, 22, 2, 2, 4, 1, 1, 1, 3, 5, 2, 2, 10, 1, 2, 1, 2, 2, 3, 3, 1, 1, 1, 1, 1, 1, 3, 1, 3, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 3, 1, 3, 1, 1, 1, 2, 1, 1, ...]

Experiment 2

Given: r = e, k = $\frac{1}{2}(\sqrt{2}-1) = 0.207106781...$

$\rightarrow$ scf(r,k) = [2; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...]

Experiment 3

Given: r = e, k = 0.2 ; k = 0.201

$\rightarrow$ scf(r,k) = [2; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
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1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...]

Experiment 4

Given: r = e, k = 0.205

→ scf(r,k) = [2; 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
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1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...]

Experiment 5

Given: r = e, k = 0.208

→ scf(r,k) = [2; 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
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1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...]

Experiment 6

Given: r = e, k = 0.21

→ scf(r,k) = [2; 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 2, 2,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
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1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...]

Kommentar zu den Experimenten 2 bis 6

** k liegt im Bereich um 0.2

** In den Kettenbrüchen dann nur die Elemente 1 und 2 (quasibinär)

** Die Elemente 2 treten dabei in der Minderzahl und nur im Anfangsteil der Kettenbrüche auf.

** Kryptocode-verdächtig, ausloten!

** Ist die Situation bei k=1 und r im Bereich um 0.5(sqr(5)-1) ähnlich?

Experiment 7

Given: r = e, k = $\frac{1}{2}(\sqrt{3}-1) = 0.366025403...$

$\rightarrow$ scf(r,k) = [2; 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1,
1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 1, 6, 1, 1, 2, 1,
2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 3, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2,
1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1, 2, 2, 1, 1, 5, 1, 1, 1, 1, 1, 2, 1, 1, 3,
1, 1, 1, 3, 1, 1, 1, 1, 1, 3, 2, 3, 1, 1, 1, 10, 1, 1, 1, 2, 1, 1, 1, 2, 1, 5,
1, 1, 1, 1, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1,
2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 11, 1, 1, 3, 1, 1, 1, 1, 1, 2, 1, 1,
2, 1, 1, 2, 1, 1, 1, 1, 1, 3, 1, 1, 1, 2, 2, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 3, 7, 1, 2, 2, 5, 1, 3, 1, 1, 3, 1, 1, 1, 1, 1, 1, 3, 1, 1, 1, 1,
1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2,
1, 1, 1, 1, 1, 2, 1, 1, 9, 1, 1, 1, 3, 1, 3, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 2,
1, 1, 1, 1, 3, 1, 1, 1, 2, 4, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 2, 1, 1, 3, 1, 1, 1, 2, 1, 1, 2, 5, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 2,
1, 1, 1, 1, 2, 2, 1, 2, 2, 1, 1, 3, 1, 3, 1, 1, 1, 3, 10, 2, 4, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 3, 3, 3, 1, 1, 1, 5, 1, 1, 1, 1, 2, 2, 1, 1, 4, 1, 2, 1, 1, 1, 1, 2,
1, 1, 1, ...]

Experiment 9

Given: r = e, k = $\frac{1}{2}(\sqrt{e}-1) = 0.324360635...$

$\rightarrow$ scf(r,k) = [2; 1, 2, 3, 1, 1, 1, 1, 2, 4, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1,
1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 2, 2, 1, 1, 3, 1, 1, 1, 1, 1, 1,1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 3, 1, 1, 2, 2, 1, 1, 1, 1,
1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 2, 2, 1, 1, 4, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1,
4, 2, 1, 1, 3, 11, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 3,
1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1,
1, 1, 1, 1, 1, 1, 1, 3, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, ...]

Experiment 10

Given: $r = \sqrt{2}$, $k = \frac{1}{2}$

$\rightarrow$ scf(r,k) = [1; 1, 1, 1, 1, 2, 1, 1, 3, 1, 1, 1, 1, 1, 2, 2, 3, 1, 2, 3,
1, 1, 1, 1, 1, 1, 5, 1, 3, 1, 1, 5, 1, 2, 5, 2, 1, 1, 2, 3, 1, 6, 5, 1, 1, 2,
11, 2, 1, 4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 4, 1, 1, 1, 2, 1, 1, 3, 1,
2, 1, 2, 3, 1, 1, 3, 3, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1, 2, 1, 2, 1, 2, 5, 2, 1,
1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 4, 1, 1, 2, 4, 3, 1, 1, 1, 3, 1, 2, 1, 1, 1,
1, 2, 2, 1, 3, 1, 2, 2, 2, 2, 1, 3, 4, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 5,
2, 1, 5, 1, 1, 2, 19, 1, 1, 2, 2, 1, 1, 1, 2, 2, 1, 1, 8, 2, 3, 1, 3, 1, 5,
3, 1, 2, 4, 1, 1, 1, 1, 3, 1, 1, 1, 4, 4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1,
1, 1, 1, 1, 1, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 5, 1, 1, 1, 28, 2, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 3, 1, 2, 13, 1, 3, 2, 1, 2, 2, 1, 1, 12, 4,
1, 1, 1, 2, 1, 1, 2, 11, 1, 2, 2, 5, 2, 2, 5, 1, 1, 1, 1, 3, 1, 1, 1, 5, 2,
2, 5, 2, 2, 2, 2, 1, 1, 2, 4, 2, 1, 1, 2, 1, 1, 4, 1, 3, 2, 1, 5, 1, 1, 1, 1,
1, 2, 1, 5, 1, 3, 1, 1, 1, 1, 4, 6, 1, 6, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1,
1, 1, 4, 3, 1, 1, 2, 8, 1, 1, 2, 1, 1, 10, 1, 1, 1, 1, 2, 1, 2, 3, 1, 1, 2,
1, 2, 1, 8, 2, 1, 1, 2, 2, 1, 1, 5, 1, 8, 1, 1, 1, 2, 2, 1, 3, 1, 2, 1, 2, 2,
2, 1, 2, 2, 1, 1, 1, 1, 1, 2, 1, 3, 2, 1, 5, 1, 1, 1, 1, 1, 1, 1, ...]

Experiment 11

Given: r = $\sqrt{2}$, k = $\frac{1}{2}(\sqrt{5}-1)$

$\rightarrow$ scf(r,k) = [1;1,1,1,1,2,1,1,3,1,1,1,1,1,2,2,3,1,2,3,1,1,1,1,1,1,
5,1,3,1,1,5,1,2,5,2,1,1,2,3,1,6,5,1,1,2,11,2,1,4,2,1,1,1,1,1,1,1,1,
1,1,3,4,1,1,1,2,1,1,3,1,2,1,2,3,1,1,3,3,1,1,1,1,1,4,1,1,1,1,2,1,2,
1,2,5,2,1,1,1,1,2,1,1,1,1,1,1,2,4,1,1,2,4,3,1,1,1,3,1,2,1,1,1,1,2,
2,1,3,1,2,2,2,2,1,3,4,1,1,1,1,1,2,1,1,1,1,2,1,5,2,1,5,1,1,2,19,1,1,
2,2,1,1,1,2,2,1,1,8,2,3,1,3,1,5,3,1,2,4,1,1,1,1,3,1,1,1,4,4,1,1,1,
1,1,1,1,1,1,1,2,1,1,1,1,1,1,3,2,1,1,1,1,1,1,1,1,2,2,5,1,1,1,28,2,1,
1,1,1,1,1,1,1,1,1,1,2,1,3,1,2,13,1,3,2,1,2,2,1,1,12,4,1,1,1,2,1,1,
2,11,1,2,2,5,2,2,5,1,1,1,1,3,1,1,1,5,2,2,5,2,2,2,2,1,1,2,4,2,1,1,2,
1,1,4,1,3,2,1,5,1,1,1,1,1,2,1,5,1,3,1,1,1,1,4,6,1,6,2,2,1,1,1,1,1,
1,1,1,2,1,1,1,4,3,1,1,2,8,1,1,2,1,1,10,1,1,1,1,2,1,2,3,1,1,2,1,2,1,
8,2,1,1,2,2,1,1,5,1,8,1,1,1,2,2,1,3,1,2,1,2,2,2,1,2,2,1,1,1,1,1,2,
1,3,2,1,5,1,1,1,1,1,1,1,...]

Experiment 12

Given: $r = \sqrt{77},\ k = \frac{1}{2}$

$\rightarrow$ scf(r,k) = [8;1,2,1,2,1,1,2,1,1,1,2,1,1,1,2,1,2,1,2,4,2,1,1,1,1,1,
2,1,1,1,1,1,1,1,1,2,1,1,1,1,1,2,1,1,1,1,2,7,1,1,1,2,3,1,1,1,1,1,1,1,
3,1,3,1,1,1,2,1,14,3,1,3,1,1,1,1,1,1,2,2,1,1,1,1,1,2,1,1,1,1,1,2,2,
2,1,2,3,1,1,1,1,1,1,1,1,1,2,1,1,1,1,2,1,1,1,2,1,1,3,3,1,1,1,2,2,1,2,
1,1,1,1,2,10,2,1,1,2,2,1,4,2,13,1,1,1,2,1,1,1,4,1,1,1,1,6,1,1,1,1,
1,3,2,1,1,2,1,1,1,1,2,1,1,4,1,1,1,1,1,2,1,1,1,1,1,2,3,1,1,4,1,3,1,1,
1,1,2,1,1,3,1,1,1,1,3,1,1,2,1,2,1,1,1,1,1,1,1,2,4,2,1,1,1,1,1,1,12,
2,1,2,1,2,2,16,1,1,2,3,1,1,1,1,3,13,2,1,1,1,1,1,1,1,7,3,1,1,1,1,2,
1,1,1,1,3,1,2,2,2,1,1,2,1,1,1,1,3,1,2,2,2,1,2,1,3,3,1,1,4,1,1,2,1,1,
1,2,1,7,2,1,1,3,1,1,1,1,2,3,1,1,1,3,1,1,4,1,2,1,1,1,2,1,2,6,1,1,1,1,
2,2,1,1,1,2,1,1,1,1,2,1,1,1,1,1,1,2,1,1,1,1,1,1,3,1,1,1,1,1,3,2,1,1,
1,9,1,1,17,1,1,2,2,1,2,5,1,1,1,1,1,1,1,1,2,1,1,1,2,1,1,58,1,1,1,1,6,
2,1,1,1,2,2,...]

Algorithm 2ext.1,2 : inverse transformation $[a_0;a_1,a_2,a_3,\ldots a_n] \rightarrow$ (approx.)r

x:=1; y:=0
i=n(1)0
(x,y):=$((a_i*x+y)^m,x)$
a:=x ; b:=y (approx.)r:=$\frac{a}{b}$

x:=0
i=n(1)1 x:=$(a_i+x)^{-m}$
(approx.)r:=a_0+x

Krypto capability?
Coding experiment 1 with k=m=1
Given an infinite scf with the elements 1 (background), 2 (≙ 0), and 3 (≙ 1),
carrying a binary message, f.e.

[1; 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 2, 2, 2, 1, 1, 1, 1, 1, 1, …]
background message 000111000 background →
≙ SOS in Morse code

= [1; 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 2, 2, 2, $\frac{1}{2}(\sqrt{5}+1)$]

The corresponding value r (calculated via algorithmus 2.1 with pocket calculator CASIO fx-95MS = encoding) = code text is

r = 1.613285505167…

near to $\Phi=\frac{1}{2}(\sqrt{5}+1)=1.6180339865...$
The evaluation of r into a scf via algorithmus 1 with fx-95MS = decoding delivers the message again
[1; 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 2, 2, 2, 1, 1, 1, 1, (4), …]
"stop codon"

For short messages, this coding principle already works using a simple calculator.

Coding experiment 2 with k=m=1
Given an infinite scf with the elements 1 (background), 2 (≙ 0), and 3 (≙ 1),
carrying a binary message, f.e.

[1; 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 2, 2, 2, 1, 1, 1, 1, 1, 1, …]
background message 000111000 background →
≙ SOS in Morse code
≈ [1; 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 2, 2, 2, 1, 1, 1, 1, 1, 1]

The corresponding values a and b (calculated via algorithmus 2.2 with pocket calculator CASIO fx-95MS = encoding) = "safe key a and b" are

a = 611653 ; b = 379135

a und b können nach der Berechnung getrennt transportiert, getrennt gespeichert und getrennt hantiert werden. „Der klassische Safe kann nur geöffnet werden, wenn die Schlüssel a und b physisch präsent sind" = decoding via following algorithm

x = a ; y = b ; i = 0
a_i:=div(x,y) ⇒
(x,y):=(y,rest(x,y))
i:=i+1
y=0
gcd(a,b):=x; n:=i-1

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{a_n}}}$$

$=[a_0;a_1,a_2,..,a_n]=scf(a,b)$

delivers the message again:

[1; 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 2, 2, 2, 1, 1, 1, 1, 1, 1]

Coding experiment 3

Given a finite "bodyguarded" message = scf string with the elements 1 (background, separator), 2 (A 0), and 3 (A 1), f.e.

[1; 1, 1, 1, 1, 2, 2, 2, 1, 3, 3, 3, 1, 2, 2, 2, 1, 1, 1, 1, 1]

background message 000111000 background

A SOS in Morse code

↑ ↑

separators

We use now a more sophisticated procedure for the generation of the keys a and b:

1)

[1; 1, 1, 1, 1, 2, 2, 2, 1, 3, 3, 3, 1, 2, 2, 2, 1, 1, 1, 1, 1] = $[a_0; a_1, \ldots, a_{n=20}]$

2)
Encoding of the given scf string via

x:=0
i=n(1)1 x:=$(a_i + x)^{-m}$
(approx.)r:=$a_0 + x$

with $m \neq 1$,
f.e. $m = 2 \rightarrow r = 1.416741662\ldots$

3)
Decoding of r via

$r \in \mathbb{Q}_+, \mathbb{R}_+$; $k \in \mathbb{R}_+$ length(approx.scf(r))	
x:=r ; i:=0	
a_0:=int(x)	
repeat until	x:=$(x - int(x))^{-k}$
	i:=i+1
	$a_i(k)$:=int(x)
x=int(x) or i=length	
n:=i	
approx.scf(r,k)=$[a_0;a_1,a_2,\ldots a_n]$	

with k = 1 →

approx.scf(r=1.416741662...,k=1)=$[b_0;b_1,b_2,...b_p]$

$=[1;2,2,1,1,92,66,7]$

4)

Encoding of approx.scf(r=1.416741662…,k=1) via

x:=1; y:=0
i=n(1)0 (x,y):=$((a_i*x+y)^m,x)$
a:=x ; b:=y (approx.)r:=$\frac{a}{b}$

with m = 1 → a = 728881 ; b = 514477

These are the two separate keys for the classical safe containing the hidden message

[1; 1, 1, 1, 1, 2, 2, 2, 1, 3, 3, 3, 1, 2, 2, 2, 1, 1, 1, 1, 1] .

Now let's open the safe with the both keys.

1)

$$(a,b) \rightarrow r=\frac{a}{b}=\frac{728881}{514477}=1.416741662...$$

2)
Decoding of r via

$r \in \mathbb{Q}_+, \mathbb{R}_+$; $k \in \mathbb{R}_+$ length(approx.scf(r))	
x:=r ; i:=0	
a_0:=int(x)	
repeat until	$x:=(x - int(x))^{-k}$
	i:=i+1
	$a_i(k)$:=int(x)
x=int(x) or i=length	
n:=i	
approx.scf(r,k)=$[a_0;a_1,a_2,...a_n]$	

with k = 1/2 →

approx.scf(r=1.416741662...,k=1/2) ≈

[1;1,1,1,1,2,2,2,1,3,3,3,1,2,2,2,1,1,(2),…)

"stop codon"

The safe is opened!

Fragen

** Wie effizient und wie sicher sind vorstehende und auf dieser Basis weiter ausgebaute Verfahren zur Kryptocodierung?

** Eine Sicherheitsgarantie ist ja wohl das zwei-Schlüssel-Prinzip. Oder ist es möglich auf irgendeine Weise aus der alleinigen Kenntnis von Schlüssel a den Schlüssel b zu berechnen?