Unit 11

Places of interest near Benford`s Law

Benford's law (first digit law) is the mathematical representation of the phenomenological observation that in large data sets of decimal numbers describing a wide variety of similar real objects, systems, and processes, like

**length of rivers or surfaces of lakes in a country

**population number of towns and cities in a country and other demographic data

**electricity and water bills

the leading digit $k \in \{1,2,3,\ldots,9\}$ is non-uniform distributed:

the frequency p(k) of k is given approximately by

$$p(k) = \log_{10}(1+\frac{1}{k}) \neq \frac{1}{9}$$

k	p(k)
1	0.301029995…
2	0.176091259…
3	0.124938736…
4	0.096910013…
5	0.079181246…
6	0.066946789…
7	0.057991946…
8	0.051152522…
9	0.045757490…

Application: check of large numeric data sets to detect frauds or systematic errors.

Surprisingly, we find similar non-uniform distributions of the numbers 0, 1, 2, …, 9 also in pure mathematical objects and problems like

$\langle \pi \rangle_{t=0}^{10000} = 3\ \ 1\ \ 4\ \ 1\ \ 5\ \ 9\ \ 2\ \ 6\ \ 5\ \ 3\ \ 5\ \ 8\ \ 9\ \ 2\ \ \ldots$

$\langle D1\pi \rangle = \ \ 2\ \ \ 3\ \ 3\ \ 4\ \ 4\ \ 7\ \ 4\ \ 1\ \ 2\ \ 2\ \ 3\ \ 1\ \ 7\ \ \ldots$

$\langle D2\pi \rangle = \ \ \ \ 1\ \ 0\ \ 1\ \ 0\ \ 3\ \ 3\ \ 3\ \ 1\ \ 0\ \ 1\ \ 2\ \ 6\ \ \ldots$

k	$p(\pi,k) \to 0.1$	$p(D1\pi,k)$	$p(D2\pi,k)$
0	0.0968...	$0.9450... \to 0.10$	0.1397...
1	0.1026...	$0.1838... \to 0.18$	0.2471...
2	0.1021...	$0.1574... \to 0.16$	0.1961...
3	0.0975...	$0.1448... \to 0.14$	0.1486...
4	0,1012...	$0.1226... \to 0.12$	0.1073...
5	0.1046...	$0.0975... \to 0.10$	0.0717...
6	0.1021...	$0.0810... \to 0.08$	0.0481...
7	0.0969...	$0.0571... \to 0.06$	0.0268...
8	0.0948...	$0.0386... \to 0.04$	0.0106...
9	0.1014...	$0.0227... \to 0.02$	0.0040...

Discussion!

References

http://en.wikipedia.org/wiki/Benford%27s_law
http://mathworld.wolfram.com/BenfordsLaw.html

To be continued.