In the debating club n °88

A collection of number curiosities

88.1 Pairs of reverse numbers

88.1.1

$13 \Leftrightarrow 31 \rightarrow 13^2 = 169 \Leftrightarrow 31^2 = 961$ (1)

Lateral

$169 = 13^2$ (2.1)

$196 = 14^2$ (2.2)

619 prim (2.3)

691 prim, Bernoulli marker [1] (2.4)

$916 = 4*229$ (2.5)

$961 = 31^2$ (2.6)

88.1.2

$12 \Leftrightarrow 21 \rightarrow 122 = 144 \Leftrightarrow 212 = 441$ (3.1)

$44 = 4*11$ (3.2)

88.1.3

$122 \Leftrightarrow 221 \rightarrow 122^2 = 14884 \Leftrightarrow 221^2 = 48841$ (4.1)

$4884 = 44*111$ (4.2)

88.1.4

Find more. Find a general representation of this reversion problem.

88.2 A lateral to equ. (3) and (4)

$1222^2 = 1493284 \Updownarrow 2221^2 = 4932841$ (5.1)

$493284 = 4* 123321 = 444*1111 = 4884*101$ (5.2)

$12222^2 = 149377284 \Updownarrow 22221^2 = 493772841$ (6.1)

$49377284 = 4*12344321 = 4444*11111$ (6.2)

88.3 **A palindromic lateral to 88.2**

$11 = 1*11$ (7.1)
$1221 = 11*111$ (7.2)
$123321 = 111*1111$ (7.3)
$12344321 = 1111*11111$ (7.4)
$1234554321 = 11111*111111$ (7.5)

$11*111*1111 = 1356531$ (8)

$11 = 11*1^2$ (9.1)
$1331 = 11*121 = 11*11^2$ (9.2)
$135531 = 11*12321 = 11*111^2$ (9.3)
$13577531 = 11*1234321 = 11*1111^2$ (9.4)
$1357997531 = 11*123454321 = 11*11111^2$ (9.5)

88.4

$11^0 = 1$ (10.1)
$11^1 = 11$ (10.2)
$11^2 = 121$ (10.3)
$11^3 = 1331$ (10.4)
$11^4 = 14641$ (10.5)

88.5 A leap of thought

Let's jump from the equs (7), (8), and (9) to the open problem
Representation of numbers by the infinite

** sum $$a_0 + \sum_{i=1}^{\infty} \frac{a_i}{1^i}$$ (11)

** product $$\prod_{i=1}^{\infty} (1^i)^{a_i}$$ (12)

** continued fraction $a_0 + \left| \frac{a_i}{1^i} \right|_{i=1}^{\infty}$ (13)

with $1^i = \underbrace{111...1}_{\text{i times}}$. (14)

** Find related expansion algorithms.
** Find convincing examples.

88.6 Wallis product variations

88.6.1 Wallis product

$$\frac{2*2}{1*3}*\frac{4*4}{3*5}*\frac{6*6}{5*7}*...=\prod_{i=1}^{\infty}\frac{(2i)*(2i)}{(2i-1)*(2i+1)} = \prod_{i=1}^{\infty}\frac{(2i)^2}{(2i)^2-1}=\frac{\pi}{2} \quad (16.1)$$

Related segment determinant

$$\begin{vmatrix} 2i & 2i+1 \\ 2i-1 & 2i \end{vmatrix} = 1 \quad (16.2)$$

Related segment simple continued fraction

$$\left[\frac{2*2}{1*3};\frac{4*4}{3*5},\frac{6*6}{5*7},\frac{8*8}{7*9},\frac{10*10}{9*11},...\right]=1.2...=? \quad (16.3)$$

88.6.2 Wallis product laterals

88.6.2.1

$$\frac{3*3}{2*4}*\frac{5*5}{4*6}*\frac{7*7}{6*8}*\ldots=$$

$$\prod_{i=1}^{\infty}\frac{(2i+1)*(2i+1)}{(2i-1+1)*(2i+1+1)}=$$

$$\prod_{i=1}^{\infty}\frac{(2i+1)^2}{(2i+1)^2-1}=1.27323954\ldots=\frac{4}{\pi} \qquad (17.1)$$

$$\underbrace{\left(\frac{3*3}{2*4}\right)^{\frac{1}{4}}}_{\text{segment 1}}*\underbrace{\left(\frac{5*5}{4*6}*\frac{7*7}{6*8}\right)^{\frac{1}{8}}}_{\text{segment 2}}*\underbrace{\left(\frac{9*9}{8*10}*\frac{11*11}{10*12}*\frac{13*13}{12*14}*\frac{15*15}{14*16}\right)^{\frac{1}{16}}}_{\text{segment 3}}*$$

$$\underbrace{\left(\frac{17*17}{16*18}*\frac{19*19}{18*20}*\frac{21*21}{20*22}*\frac{23*23}{22*24}*\frac{25*25}{24*26}*\frac{27*27}{26*28}*\frac{29*29}{28*30}*\frac{31*31}{30*32}\right)^{\frac{1}{32}}}_{\text{segment 4}}*\ldots=$$

$$\frac{2\sqrt{2}}{e} \qquad (28)$$

and

$$\underbrace{\left(\frac{3*3}{2*4}\right)^{\frac{1}{1}}}_{\text{segment 1}}*\underbrace{\left(\frac{5*5}{4*6}*\frac{7*7}{6*8}\right)^{\frac{1}{1}}}_{\text{segment 2}}*\underbrace{\left(\frac{9*9}{8*10}*\frac{11*11}{10*12}*\frac{13*13}{12*14}*\frac{15*15}{14*16}\right)^{\frac{1}{1}}}_{\text{segment 3}}*$$

$$\underbrace{\left(\frac{17*17}{16*18}*\frac{19*19}{18*20}*\frac{21*21}{20*22}*\frac{23*23}{22*24}*\frac{25*25}{24*26}*\frac{27*27}{26*28}*\frac{29*29}{28*30}*\frac{31*31}{30*32}\right)^{\frac{1}{1}}}_{\text{segment 4}}*\ldots=$$

$$\frac{4}{\pi} \qquad (17.1)$$

Related segment determinant

$$\begin{vmatrix} 2i+1 & 2i+2 \\ 2i & 2i+1 \end{vmatrix}=1 \qquad (17.2)$$

Related segment simple continued fraction

$$\left[\frac{3*3}{2*4};\frac{5*5}{4*6},\frac{7*7}{6*8},\frac{9*9}{8*10},\frac{11*11}{10*12},\ldots\right]=1.7\ldots=?\qquad(17.3)$$

88.6.2.2

$$\frac{3*3}{1*5}*\frac{5*5}{3*7}*\frac{7*7}{5*9}*\frac{9*9}{7*11}*\ldots=$$

$$\prod_{i=1}^{\infty}\frac{(2i+1)*(2i+1)}{(2i+1-2)*(2i+1+2)}=\qquad(18.1)$$

$$\prod_{i=1}^{\infty}\frac{(2i+1)^2}{(2i+1)^2-4}=3$$

Related segment determinant

$$\begin{vmatrix}2i+1 & 2i+3\\ 2i-1 & 2i+1\end{vmatrix}=-2\qquad(18.2)$$

Related segment simple continued fraction

$$\left[\frac{3*3}{1*5};\frac{5*5}{3*7},\frac{7*7}{5*9},\frac{9*9}{7*11},\frac{11*11}{9*13},\ldots\right]=2.3\ldots=?\qquad(18.3)$$

88.6.3 Wallis product extension

$$\frac{f(2)*f(2)}{f(1)*f(3)}*\frac{f(4)*f(4)}{f(3)*f(5)}*\frac{f(6)*f(6)}{f(5)*f(7)}*\frac{f(8)*f(8)}{f(7)*f(9)}*\ldots=$$

$$\prod_{i=1}^{\infty}\frac{f(2i)*f(2i)}{f(2i-1)*f(2i+1)}=P(f(i))$$

(19.1)

Related segment determinant

$$\begin{vmatrix} f(2i) & f(2i+1) \\ f(2i-1) & f(2i) \end{vmatrix} = D(f(i)) \qquad (19.2)$$

Related segment simple continued fraction

$$\left[\frac{f(2)*f(2)}{f(1)*f(3)}; \frac{f(4)*f(4)}{f(3)*f(5)}, \frac{f(6)*f(6)}{f(5)*f(7)}, \frac{f(8)*f(8)}{f(7)*f(9)}, \ldots \right] = C(f(i))$$

(19.3)

Special cases

** f(i) = id(i) = i nat. numbers, equ. (16)

P(f(i)) = P(id(i)) = P(i) = $\frac{\pi}{2}$

D(f(i)) = D(id(i)) = 1

C(f(i)) = 1.2… = ? (20)

** f(i) = fib(i) Fibonacci sequence

fib(0) = fib(1) = 1

fib(i+1) = fib(i) + fib(i-1)

P(f(i)) = P(fib(i)) = 1.3983…= ?

D(f(i)) = D(fib(i)) = $(-1)^i$

C(f(i)) = 1.93… = ? (21)

** f(i) = p_i prime sequence

p_1=2, p_2=3, p_3=5, p_4=7, p_5=11, …

P(f(i)) = P(p_i) =

D(f(i)) = D(p_i) =

C(f(i)) = (22)

88.6.4 Lateral 88.6.2.1 extension

$$\frac{f(3)*f(3)}{f(2)*f(4)}*\frac{f(5)*f(5)}{f(4)*f(6)}*\frac{f(7)*f(7)}{f(6)*f(8)}*\frac{f(9)*f(9)}{f(8)*f(10)}*\ldots=$$

$$\prod_{i=1}^{\infty}\frac{f(2i+1)*f(2i+1)}{f(2i)*f(2i+2)}=\Pi(f(i))$$

(23.1)

Related segment determinant

$$\begin{vmatrix} f(2i+1) & f(2i+2) \\ f(2i) & f(2i+1) \end{vmatrix}=\Delta(f(i)) \qquad (23.2)$$

Related segment simple continued fraction

$$\left[\frac{f(3)*f(3)}{f(2)*f(4)};\frac{f(4)*f(4)}{f(3)*f(5)},\frac{f(6)*f(6)}{f(5)*f(7)},\frac{f(8)*f(8)}{f(7)*f(9)},\ldots\right]=\Gamma(f(i))$$

(23.3)

Special cases

** f(i) = id(i) = i nat. numbers, equ. (17)

$\Pi(f(i)) = \Pi(id(i)) = \Pi(i) = 1.27323954\ldots = \frac{4}{\pi}$

$\Delta(f(i)) = \Delta(id(i)) = 1$

$\Gamma(f(i)) = 1.7\ldots = ?$ (24)

** f(i) = fib(i) Fibonacci sequence

fib(0) = fib(1) = 1

fib(i+1) = fib(i) + fib(i-1)

$\Pi(f(i)) = \Pi(fib(i)) = \ldots = ?$

$\Delta(f(i)) = \Delta(fib(i)) = -1$

$\Gamma(f(i)) = \ldots = ?$ (25)

** $f(i) = p_i$ prime sequence

$p_1=2, p_2=3, p_3=5, p_4=7, p_5=11, \ldots$

$\Pi(f(i)) = \Pi(p_i) =$

$\Delta(f(i)) = \Delta(p_i) =$

$\Gamma(f(i)) =$ (26)

88.6.5 How about

the mixed segment simple continued fraction

$$\left[\frac{f(2)*f(2)}{f(1)*f(3)};\frac{f(3)*f(3)}{f(2)*f(4)},\frac{f(4)*f(4)}{f(3)*f(5)},\frac{f(5)*f(5)}{f(4)*f(6)},\ldots\right] = M(f(i))$$

? (27)

Complete Deb 88.6.
Discuss your results.

26.04.2016 EPS

88.6.6 Wallis product variations, sandbox

88.6.6.1 Wallis lateral equ. (17) → π^{-1}, Wallis lateral equ. (17) binary segmented → e^{-1} [5]

$$\left(\frac{3*3}{2*4}\right)^{\frac{1}{4}}*\left(\frac{5*5}{4*6}*\frac{7*7}{6*8}\right)^{\frac{1}{8}}*\left(\frac{9*9}{8*10}*\frac{11*11}{10*12}*\frac{13*13}{12*14}*\frac{15*15}{14*16}\right)^{\frac{1}{16}}*$$

$$\left(\frac{17*17}{16*18}*\frac{19*19}{18*20}*\frac{21*21}{20*22}*\frac{23*23}{22*24}*\frac{25*25}{24*26}*\frac{27*27}{26*28}*\frac{29*29}{28*30}*\frac{31*31}{30*32}\right)^{\frac{1}{32}}*\ldots =$$

$$\frac{2\sqrt{2}}{e}$$ (28)

Compression of equ. (28) to the right wing

$$\left(\frac{3}{4}\right)^{\frac{1}{2}} * \left(\frac{5}{6} * \frac{7}{8}\right)^{\frac{1}{4}} * \left(\frac{9}{10} * \frac{11}{12} * \frac{13}{14} * \frac{15}{16}\right)^{\frac{1}{8}} * \left(\frac{17}{18} * \frac{19}{20} * \frac{21}{22} * \frac{23}{24} * \frac{25}{26} * \frac{27}{28} * \frac{29}{30} * \frac{31}{32}\right)^{\frac{1}{16}} * \ldots = \frac{2}{e} \quad (29)$$

How about the left wing?

$$\left(\frac{3}{2}\right)^{\frac{1}{2}} * \left(\frac{5}{4} * \frac{7}{6}\right)^{\frac{1}{4}} * \left(\frac{9}{8} * \frac{11}{10} * \frac{13}{12} * \frac{15}{14}\right)^{\frac{1}{8}} * \left(\frac{17}{16} * \frac{19}{18} * \frac{21}{20} * \frac{23}{22} * \frac{25}{24} * \frac{27}{26} * \frac{29}{28} * \frac{31}{30}\right)^{\frac{1}{16}} * \ldots = ? \quad (30)$$

88.6.6.2 Wallis equ. (16) → π, Wallis equ. (16) binary segmented → ?

$$\left(\frac{2*2}{1*3}\right)^{\frac{1}{4}} * \left(\frac{4*4}{3*5} * \frac{6*6}{5*7}\right)^{\frac{1}{8}} * \left(\frac{8*8}{7*9} * \frac{10*10}{9*11} * \frac{12*12}{11*13} * \frac{14*14}{13*15}\right)^{\frac{1}{16}} * \left(\frac{16*16}{15*17} * \frac{18*18}{17*19} * \frac{20*20}{19*21} * \frac{22*22}{21*23} * \frac{24*24}{23*25} * \frac{26*26}{25*27} * \frac{28*28}{27*29} * \frac{30*30}{29*31}\right)^{\frac{1}{32}} * \ldots = ? \quad (31)$$

Compression of equ. (31) to the right wing

$$\left(\frac{2}{3}\right)^{\frac{1}{2}} * \left(\frac{4}{5} * \frac{6}{7}\right)^{\frac{1}{4}} * \left(\frac{8}{9} * \frac{10}{11} * \frac{12}{13} * \frac{14}{15}\right)^{\frac{1}{8}} * \left(\frac{16}{17} * \frac{18}{19} * \frac{20}{21} * \frac{22}{23} * \frac{24}{25} * \frac{26}{27} * \frac{28}{29} * \frac{30}{31}\right)^{\frac{1}{16}} * \ldots = ? \quad (32)$$

How about the left wing?

$$\left(\frac{2}{1}\right)^{\frac{1}{2}} * \left(\frac{4}{3} * \frac{6}{5}\right)^{\frac{1}{4}} * \left(\frac{8}{7} * \frac{10}{9} * \frac{12}{11} * \frac{14}{13}\right)^{\frac{1}{8}} * \left(\frac{16}{15} * \frac{18}{17} * \frac{20}{19} * \frac{22}{21} * \frac{24}{23} * \frac{26}{25} * \frac{28}{27} * \frac{30}{29}\right)^{\frac{1}{16}} * \ldots = ? \quad (33)$$

88.6.6.3 Extensions

Now, it's your turn to extend 88.6.6.1, and 88.6.6.2 by analogy to 88.6.3, 88.6.4, and 88.6.5.

88.6.6.4 More Wallis-likes

$$\prod_{i=1}^{\infty} \frac{(2i)^2}{(2i)^2-1} = \frac{\pi}{2}$$

$$\prod_{i=1}^{\infty} \frac{(2i+1)^2}{(2i-1)^2-1} = \frac{4}{\pi}$$

$$\prod_{i=2}^{\infty} \frac{p_i^2}{p_i^2-1} = \frac{\pi^2}{8}$$

(34.1,2,3,4)

$$\prod_{i=2}^{\infty} \frac{i^2}{i^2-1} = 2$$

$$\prod_{i=1}^{\infty} \frac{(2i)^2}{(2i)^2+1} = \frac{\pi}{2\sinh\frac{\pi}{2}}$$

$$\prod_{i=1}^{\infty} \frac{(2i+1)^2}{(2i-1)^2+1} = \frac{2}{\cosh\frac{\pi}{2}}$$

$$\prod_{i=2}^{\infty} \frac{p_i^2}{p_i^2+1} = \frac{\pi^2}{12}$$

$$\prod_{i=2}^{\infty} \frac{i^2}{i^2+1} = \frac{2\pi}{\sinh\pi}$$

(35.1-7)

$$\sinh\frac{\pi}{2}=\prod_{i=1}^{\infty}\frac{(2i)^2+1}{(2i)^2-1}$$

$$\left.\begin{matrix}e^{\pi/2}\\ e^{-\pi/2}\end{matrix}\right\}=\prod_{i=1}^{\infty}\frac{(2i-1)^2+1}{(2i-1)^2}\pm\prod_{i=1}^{\infty}\frac{(2i)^2+1}{(2i)^2-1}$$

$$\prod_{i=2}^{\infty}\frac{p_i^2+1}{p_i^2-1}=\frac{3}{2}$$

88.6.6.5 Extensions of 88.6.6.4 by analogy to 88.6.6.3

88.6.6.6 Extensions of the equs (17.1) and (28)

$$\underbrace{\left(\frac{3*3}{2*4}\right)^{\frac{1}{4}}}_{\text{segment 1}}*\underbrace{\left(\frac{5*5}{4*6}*\frac{7*7}{6*8}\right)^{\frac{1}{8}}}_{\text{segment 2}}*\underbrace{\left(\frac{9*9}{8*10}*\frac{11*11}{10*12}*\frac{13*13}{12*14}*\frac{15*15}{14*16}\right)^{\frac{1}{16}}}_{\text{segment 3}}*$$

$$\underbrace{\left(\frac{17*17}{16*18}*\frac{19*19}{18*20}*\frac{21*21}{20*22}*\frac{23*23}{22*24}*\frac{25*25}{24*26}*\frac{27*27}{26*28}*\frac{29*29}{28*30}*\frac{31*31}{30*32}\right)^{\frac{1}{32}}}_{\text{segment 4}}*\ldots=$$

$$\frac{2\sqrt{2}}{e}\qquad(28)$$

and

$$\underbrace{\left(\frac{3*3}{2*4}\right)^{\frac{1}{1}}}_{\text{segment 1}}*\underbrace{\left(\frac{5*5}{4*6}*\frac{7*7}{6*8}\right)^{\frac{1}{1}}}_{\text{segment 2}}*\underbrace{\left(\frac{9*9}{8*10}*\frac{11*11}{10*12}*\frac{13*13}{12*14}*\frac{15*15}{14*16}\right)^{\frac{1}{1}}}_{\text{segment 3}}*$$

$$\underbrace{\left(\frac{17*17}{16*18}*\frac{19*19}{18*20}*\frac{21*21}{20*22}*\frac{23*23}{22*24}*\frac{25*25}{24*26}*\frac{27*27}{26*28}*\frac{29*29}{28*30}*\frac{31*31}{30*32}\right)^{\frac{1}{1}}}_{\text{segment 4}}*\ldots=$$

$$\frac{4}{\pi}\qquad(17.1)$$

are special cases of

$$\underbrace{\left(\frac{3*3}{2*4}\right)^{s(1)}}_{\text{segment 1}} * \underbrace{\left(\frac{5*5}{4*6}*\frac{7*7}{6*8}\right)^{s(2)}}_{\text{segment 2}} * \underbrace{\left(\frac{9*9}{8*10}*\frac{11*11}{10*12}*\frac{13*13}{12*14}*\frac{15*15}{14*16}\right)^{s(3)}}_{\text{segment 3}} *$$

$$\underbrace{\left(\frac{17*17}{16*18}*\frac{19*19}{18*20}*\frac{21*21}{20*22}*\frac{23*23}{22*24}*\frac{25*25}{24*26}*\frac{27*27}{26*28}*\frac{29*29}{28*30}*\frac{31*31}{30*32}\right)^{s(4)}}_{\text{segment 4}} *\ldots =$$

$$F(s(k)) \qquad (36)$$

Complete the table

$\frac{1}{s(k)}$	$\frac{1}{F(s(k))}$
1	$\frac{\pi}{4}$
2^k	$\frac{e}{2\sqrt{2}}$
k	?
k^2	?
fib(k)	?
λ(k)	?

$$\langle \lambda(k) \rangle_{k=1}^{\infty} = 1,5,7,11,13,17,19,23,25,29,31,35,37,$$
$$41,43,47,49,53,55,59,\ldots \qquad (37)$$

88.6.6.7 Extensions of the equs (16.1) and (31)

Complete Deb 88.6.
Discuss your results:
** convergence, possibilities to accelerate the convergence
** 88.6.6 related recurrence equations
** π and e
** …..

References

[1] http://en.wikipedia.org/wiki/691_%28number%29
"In number theory, 691 is a "marker": whenever it appears in a computation, one can be sure that Bernoulli numbers are involved."
[2] Weisstein, Eric W. "Infinite Product." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/InfiniteProduct.html
[3] Sondow, Jonathan and Weisstein, Eric W. "Wallis Formula." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/WallisFormula.html
[4] https://en.wikipedia.org/wiki/Wallis_product
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To be continued.