Im Debattierclub 3*7*37 = 777
Natural numbers and primes meet in infinity
Will be continued
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$\sum_{i=1}^{\infty} \frac{1}{i^2} = \prod_{i=2}^{\infty} \frac{p_i^2}{p_i^2-1} = \frac{\pi^2}{6} \qquad [1,2,3]$$

$p_2=2,\ p_3=3,\ p_4=5,\ p_5=7,\ \ldots$ (1)

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Unit 999, Gl. (10)

$$\prod_{i=2}^{\infty} \frac{i^3+1}{i^3-1} = \prod_{i=2}^{\infty} \frac{p_i^2+1}{p_i^2-1} = \frac{3}{2} \qquad (2)$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
Finden Sie ähnliche Zusammenhänge zwischen den infiniten Kettenbrüchen

$$\frac{I_1(2)}{I_0(2)} = 0.667774657964007982006790592 5\ldots$$

$$= [0;1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots]$$

[4,5,6]
und
tanh(1) = 0.761594155955764888119458 2…

$$= [0;1,3,5,7,9,11,13,15,17,19,21,\ldots]$$

und
[0;1,3,5,7,11,13,17,19,23,29,31,37,41,43,…] = 0.761593314377600290742797245709 9…

……..

(3.1,2,3,…)

Find more.

** Untersuchen Sie die Konvergenz der einzelnen Summen, Produkte, Kettenbrüche, Determinanten, etc.

** Berechnen Sie die zugehörigen Differenzenquotienten als Grundlage für eine Konvergenzverbesserung (vgl. Units 1, 44, 50).

** Lateral 1 →

** Lateral 2 →

** Lateral 3 →

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Fortsetzung von Unit 999, S.11

Besun: $\prod_{i=2}^{\infty} \frac{i^3+1}{i^3-1} = \prod_{i=2}^{\infty} \frac{p_i^2+1}{p_i^2-1} = \frac{3}{2}$ (2)

** Interpretieren Sie diese Gleichung.

** Schreiben Sie eine Kurzgeschichte (vgl. Unit 15, part 78.4, S. 20-21), in der Gl. (2) kongenial kodiert ist.

** Untersuchen Sie mögliche Erweiterungen von Gl. (2), z.B.

** Vermutung:
Es existieren natürliche Zahlen m und n, so dass exakt oder mit hoher Genauigkeit gilt

$$\frac{\prod_{i=2}^{\infty} \frac{i^m+1}{i^m-1}}{\prod_{i=2}^{\infty} \frac{p_i^n+1}{p_i^n-1}} = \frac{\frac{u}{v}}{\frac{x}{y}} = \frac{a}{b} ;$$

$$u,v,x,y,a,b \in \mathbb{N} \text{ , } ggt(a,b) = 1 . \qquad (4)$$

Beispiel 1: m=3, n=2, $\frac{u}{v}=\frac{x}{y}=\frac{3}{2}$, a=b=1, exakt

Unterwegs:

$$\frac{\prod_{i=2}^{20000}\frac{i^3+1}{i^3-1}}{\prod_{i=2}^{20000}\frac{p_i^2+1}{p_i^2-1}}=\frac{1.499999962501875000\ldots}{1.499998940366516582\ldots}$$

$$=1.0000006681428053127 \quad (5.1,2)$$

Check more.

Heißer (m,n)-Kandidat:

(19,12) → a=b=1,

Unterwegs:

$$\frac{\prod_{i=2}^{10000}\frac{i^{19}+1}{i^{19}-1}}{\prod_{i=2}^{10000}\frac{p_i^{12}+1}{p_i^{12}-1}}=\frac{1.000003816432715673\ldots}{1.000003771697181927\ldots}$$

$$=1.000000044735365017\ldots \quad (6.1,2)$$

Auch im Blickfeld:

(7,5), (11,7), (13,8), (14,9), (15,11), (18,13), (20,13),…

Im Positivfalle: Bildungsgesetz, Herleitung und Beweis.

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Lateral 1

Determinant representations

$\frac{I_1(2)}{I_0(2)}$ = [0;1,2,3,4,5,6,7,8,9,10,11,12,13,14,…]

$$= -\begin{vmatrix} 0 & \mathbf{0*1} & \mathbf{1*2} & \mathbf{2*3} & \mathbf{3}*4 & \mathbf{4*5} & \rightarrow \\ 1 & \mathbf{0*0} & \mathbf{1*1} & \mathbf{2*2} & \mathbf{3*3} & \mathbf{4*4} & \rightarrow \\ 1 & 0 & \mathbf{0*0} & \mathbf{1*1} & \mathbf{2*2} & \mathbf{3*3} & \rightarrow \\ 1 & 0 & 0 & \mathbf{0*0} & \mathbf{1*1} & \mathbf{2*2} & \rightarrow \\ 1 & 0 & 0 & 0 & \mathbf{0*0} & \mathbf{1*1} & \rightarrow \\ 1 & 0 & 0 & 0 & 0 & \mathbf{0*0} & \rightarrow \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \searrow \end{vmatrix}$$

$$\text{mit } n = n, \ \mathbf{n} = \frac{1}{n!} \qquad (6)$$

tanh(1) = [0;1,3,5,7,9,11,13,15,17,19,21,23,…]

$$= -\begin{vmatrix} 0 & \mathbf{1} & \mathbf{3} & \mathbf{5} & \mathbf{7} & \mathbf{9} & \rightarrow \\ 1 & \mathbf{0} & \mathbf{2} & \mathbf{4} & \mathbf{6} & \mathbf{8} & \rightarrow \\ 1 & 0 & \mathbf{0} & \mathbf{2} & \mathbf{4} & \mathbf{6} & \rightarrow \\ 1 & 0 & 0 & \mathbf{0} & \mathbf{2} & \mathbf{4} & \rightarrow \\ 1 & 0 & 0 & 0 & \mathbf{0} & \mathbf{2} & \rightarrow \\ 1 & 0 & 0 & 0 & 0 & \mathbf{0} & \rightarrow \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \searrow & \searrow \end{vmatrix}$$

$$\text{mit } n = n, \ \mathbf{n} = \frac{1}{n!} \qquad (7)$$

Find a determinant representation of [0;1,3,5,7,11,13,17,19,23,29,31,37,41,43,…] . (8)

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Lateral 2

Infinite simple continued fractions with „bodyguards“

L2.1 [4,5,6]

$\frac{I_1(2)}{I_0(2)}$ = [0;1,2,3,4,5,6,7,8,9,10,11,12,13,14,…]

= 2*[0;2,**1**,6,**2**,10.,**3**,14,**4**,18,**5**,22,**6**,..]

[0;2,**1**,6,**1**,10.,**1**,14,**1**,18,**1**,22,**1**,..]

= $\frac{1}{2}$*[0;1,2,3,2, 5,2, 7,2, 9,2,11,2, ,…]

(9.1,2,3,4)

$\frac{J_1(2)}{J_0(2)}$ - 2 = [0;1,1,2,1,3,1,4,1,5,1,6,1,7,...]

[0;1,1,1,2,1,1,3,1,1,4,1,1,5,1,1,6,…] =

0.6322499074198799235126670655…

(10.1,2,3)

……

L2.2

tanh(1) = [0;1,3,5,7,9,11,13,15,17,19,21.,,,]

tan(1)-1 = [0;1,1,3,1,5,1,7,1,9,1,11,1,13,…]

[0;1,1,1,3,1,1,5,1,1,7,1,1,9,1,1,11,…] =

0.64026267605485331908438582 71=

2*[0;3,8,12,16,20,24,28,…] =

4*[0;6,4,24, 8,40,12,56,16,72,20,88,…]

(11.1,2,3,4,5,6)

……

L2.3

$\frac{I_1(1)}{I_0(1)}$ = [0;2,4,6,8,10,12,14,16,18,20,…]

$\frac{2J_1(1)-J_0(1)}{J_0(1)-J_1(1)}$ = [0;2,1,4,1,6,1,8,1,10,1,12,…]

= $\frac{1}{2}$*[0;1,2,2,2,3,2,4,2, 5,2, 6,…]

$\frac{3-e}{e-2}$ = [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,…]

= q*[0;5,10,14,18,22,26,30,…], q=2

(12.1,2,3,4,5)

** Vergleichen Sie Konvergenzverhalten und Differenzenquotienten der scf Gln (12.4) und (12.5)→

2*[0;5]=0.4

2*[0;5,10]=0.39256862…

2*[0;5,10,14]=0.392211404…

2*[0;5,10,14,18]=0.3922111900…

2*[0;5,10,14,18,22]=0.39221119…

(12.5.1)

** Gibt es Faktoren q∈$\mathbb{Q}_+$, sodass in $\frac{3-e}{e-2}$ = q*[?] der scf(q*[?]) noch besser konvergiert als in Gl. (12.2) für q=2?

q=4:

$\frac{3-e}{e-2}$ = 4*[0; 10, 5, 28, 9, 44, 13, 60, 17,
76, 21, 92, 25, 108, 29, 124, 33,
140, 37, 156, 41, 172, 45, 188,
49, 204, 53, 220, 57, 236, 61,

252, 65, 268, 69, 284, 73, 300,
77, 316, 81, 332, 85, 348, 89,
364, 93, 380, 97, 396, 101, 412,
105, 428, 109, 444, 113, 460,
117, 476, 121, 492, 125, 508,
129, 524, 133, 540, 137, 556,…]

(12.6)

L2.4

$\mathrm{scf}\left(\frac{1}{2^k} * \frac{3-e}{e-2}\right) =$

für **k=0**

[0; 2,1,1,
4,1,1,
6,1,1,
8,1,1,
10,1,1,
12,1,1,…]

für **k=1**

[0;5,10,
14,
18,
22,
26,
30,…]

für **k=2**

[0; 10, 5, 28,
9, 44,

13, 60,
17, 76,
21, 92,
25, 108,
29, 124,
33, 140,
37, 156,
41, 172,
45, 188,
49, 204,
53, 220,
57, 236,
61, 252, …]

für **k=3**

[0, 20, 2, 1, 1, 13, 1, 1, 4, 88, 6,
1, 1, 29, 1, 1, 8, 152, 10,
1, 1, 45, 1, 1, 12, 216, 14,
1, 1, 61, 1, 1, 16, 280, 18,
1, 1, 77, 1, 1, 20, 344, 22,
1, 1, 93, 1, 1, 24, 408, 26,
1, 1, 109, 1, 1, 28, 472, 30,
1, 1, 125, 1, 1, 32, 536, 34,
1, 1, 141, 1, 1, 36, 600, 38,
1, 1, 157, 1, 1, 40, 664, 42,
1, 1, 173, 1, 1, 44, 728, 46,
1, 1, 189, 1, 1, 48, 792, 50,
1, 1, 205, 1, 1, 52, 856, 54,
1, 1, 221, 1, 1, 56, 920, 58,
1, 1, 237, 1, 1, 60, 984, 62,

1, 1, 253, 1, 1, 64, 1048, 66,
1, 1, 269, 1, 1, 68, 1112, 70,
1, 1, 285, 1, 1, 72, 1176, 74,
1, 1, 301, 1, 1, 76, 1240, 78,
1, 1, 317, 1, 1, 80, 1304, 82,
1, 1, 333, 1, 1, 84, 1368, 86, …]

für **k=4**

[0; 40, 1, 3, 1, 6, 3, 1, 1, 1, 1, 43, 1, 1, 2, 1, 3, 14, 1, 3, 4, 304, 5,
3, 1, 22, 3, 1, 5, 1, 1, 107, 1 1 6, 1, 3, 30, 1, 3, 8, 560, 9,
3, 1, 38, 3, 1, 9, 1, 1, 171, 1, 1, 10, 1, 3, 46, 1, 3, 12, 816, 13,
3, 1, 54, 3, 1, 13, 1, 1, 235, 1, 1, 14, 1, 3, 62, 1, 3, 16, 1072, 17,
3, 1, 70, 3, 1, 17, 1, 1, 299, 1, 1, 18, 1, 3, 78, 1, 3, 20, 1328, 21,
3, 1, 86, 3, 1, 21, 1, 1, 363, 1, 1, 22, 1, 3, 94, 1, 3, 24, 1584, 25,
3, 1, 102, 3, 1, 25, 1, 1, 427, 1, 1, 26, 1, 3, 110, 1, 3, 28, 1840, 29,
3, 1, 118, 3, 1, 29, 1, 1, 491, 1, 1, 30, 1, 3, 126, 1, 3, 32, 2096, 33,
3, 1, 134, 3, 1, 33, 1, 1, 555, 1, 1, 34, 1, 3, 142, 1, 3, 36, 2352, 37,
3, 1, 150, 3, 1, 37, 1, 1, 619, 1, 1, 38, 1, 3, 158, 1, 3, 40, 2608, 41,
3, 1, 166, 3, 1, 41, 1, 1, 683, 1, 1, 42, 1, 3, 174, 1, 3, 44, 2864, 45,
3, 1, 182, 3, 1, 45, 1, 1, 747, 1, 1, 46, 1, 3, 190, 1, 3, 48, 3120, 49,
3, 1, 198, 3, 1, 49, 1, 1, 811, 1, 1, 50, 1, 3, 206, 1, 3, 52, 3376, 53,
3, 1, 214, 3, 1, 53, 1, 1, 875, 1, 1, 54, 1, 3, 222, 1, 3, 56, 3632, 57,
3, 1, 230, 3, 1, 57, 1, 1, 939, 1, 1, 58, 1, 3, 238, 1, 3, 60, 3888, 61,
3, 1, 246, 3, 1, 61, 1, 1, 1003, 1, 1, 62, 1, 3, 254, 1, 3, 64, 4144, 65.
3, 1, 262, 3, 1, 65, 1, 1, 1067, 1, 1, 66, 1, 3, 270, 1, 3, 68, 4400, 69,…]

(12.7.1,2,3,4,5)

L2.5

Transform the prime scf

[0;1,3,5,7,11,13,17,19,23,29,31,37,41,43,…]

(14)

with bodyguards?

Lateral 3

Kettenbruchdarstellungen für e

L3.1

e = [2;1,**2**,1,1,**4**,1,1,**6**,1,1,**8**,1,1,**10**,1,1,**12**,…]

L3.2

e = 2 + $\frac{1}{2}$*[0; $\frac{1}{2}$,5,10,14,18,22,26,30,34,38,…]

L3.3

e = 2*[1; 2, **1**, 3, 1, **1**, 1, 3, **3**, 3, 1, **3**, 1, 3, **5**,
3, 1, **5**, 1, 3, **7**, 3, 1, **7**, 1, 3, **9**, 3, 1, **9**, 1, 3, **11**,
3, 1, **11**, 1, 3, **13**, 3, 1, **13**, 1, 3, **15**, 3, 1, **15**,
1, 3, **17**, 3, 1, **17**, 1, 3, **19**, 3, 1, **19**, 1, 3, **21**,
3, 1, **21**, 1, 3, **23**, 3, 1, **23**, 1, 3, **25**, 3, 1, **25**,
1, 3, **27**, 3, 1, **27**, 1, 3, **29**, 3, 1, **29**, 1, 3, **31**,
3, 1, **31**, 1, 3, **33**, 3, 1, **33**, 1, 3, **35**, 3, 1, **35**,
1, 3, **37**, 3, 1, **37**, 1, 3, **39**, 3, 1, **39**, 1, 3, …]

→ **L3.4**

[1; 2, **1**, 1, 3, **1**, 3, 1, **3**, 1, 3, **3**,
3, 1, **5**, 1, 3, **5**, 3, 1, **7**, 1, 3, **7**, 3, 1, **9**, 1, 3, **9**, 3,
1, **11**, 1, 3, **11**, 3, 1, **13**, 1, 3, **13**, 3, 1, **15**,
1, 3, **15**, 3, 1, **17**, 1, 3, **17**, 3, 1, **19**, 1, 3, **19**,
3, 1, **21**, 1, 3, **21**, 3, 1, **23**, 1, 3, **23**, 3, 1, **25**,
1, 3, **25**, 3, 1, **27**, 1, 3, **27**, 3, 1, **29**, 1, 3, **29**,
3, 1, **31**, 1, 3, **31**, 3, 1, **33**, 1, 3, **33**, 3, 1, **35**,
1, 3, **35**, 3, 1, **37**, 1, 3, **37**, 3, 1, **39**, 1, 3, …] = ?

[1; 2, **1**, 1, **1**, 3, **3**, 1, **3**, 3, **5**, 1 , **5**, 3, **7**, 1, **7**, 3, **9**, 1, **9**, 3, **11**, 1, **11**, 3, **13**, 1, **13**, 3, **15**, 1, **15**, 3, **17**, 1, **17**, 3, **19**, 1, **19**, 3, **21**, 1, **21**, 3, **23**, 1, **23**, 3, **25**, 1, **25**, 3, **27**, 1, **27**, 3, **29**, 1, **29**, 3, **31**, 1, **31**, 3, **33**, 1, **33**, 3, **35**, 1, **35**, 3, **37**, 1, **37**, 3, **39**, 1, **39**, 1, …] = ?

[1; 2, **1**, 3 ,**1**, 1, **3**, 3, **3**, 1, **5**, 3, **5**, 1, **7**, 3, **7**, 1, 3, **9**, 3, **9**, 1, **11**, 3, **11**, 1, **13**, 3, **13**, 1, **15**, 3, **15**, 1, **17**, 3, **17**, 1, **19**, 3, **19**, 1, **21**, 3, **21**, 1, **23**, 3, **23**, 1, 3, **25**, 3, 1, **25**, 1, **27**, 3, **27**, 1, **29**, 3, **29**, 1, **31**, 3, **31**, 1, 3, **33**, 3, 1, **33**, 1, 3, **35**, 3, **35**, 1, **37**, 3, **37**, 1, **39**, 3, **39**, 1, …] = ?

[1; 2, **1**, **1**, **3**, **3**, **5**, **5**, **7**, **7**, **9**, **9**, **11**, **11**, **13**, **13**, **15**, **15**, **17**, **17**, **19**, **19**, **21**, **21**, **23**, **23**, **25**, **25**, **27**, **27**, **29**, **29**, **31**, **31**, **33**, **33**, **35**, **35**, **37**, **37**, **39**, **39**, …] = ?

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Links to
Unit15, part 78.4, S. 20-21
Link zu Unit 75, …
Unit 999, S.11

Potenzielles Ideeneinzugsgebiet:
https://www.spektrum.de/news/auf-der-jagd-nach-dem-supermetall/1702088?utm_source=pocket-newtab

10.02.2020 EPSalg DScalc