In the Debating Club 65.1
<u>From</u>
Hieronymus Brunschwig. Liber de arte distillandi de compositis, p. 54
Straßburg 1512



to
numerical exploration of the behavior patterns of the systems of lateral coupled Heron recursion equations
init x1(0), x2(0) (1)
rec
**1

$$x1(t+1) = \frac{1}{2}\left(x1(t) + \frac{a}{x2(t)} \right)$$

$$x2(t+1) = \frac{1}{2}\left(x2(t) + \frac{b}{x1(t)} \right) \quad (2.1)$$

t = 0(1)...
**2

$$x1(t+1) = \frac{1}{2}\left(x1(t) + \frac{x2(t)}{x1(t)} \right)$$

$$x2(t+1) = \frac{1}{2}\left(x2(t) + \frac{x1(t)}{x2(t)} \right) \quad (2.2)$$

t = 0(1)...
**more.

Example 1
**1, x1(0)=x2(0)=1, a=2, b=3

Execution record

t	x1(t)	x2(t)
0	1.0	1.0
1	1.5	2.0
2	1.25	2.0
3	1.125	2.2
4	1.017045455…	2.43per
5	0.9194816314…	2.691527002…
6	0.8312771218…	2.977117460…
7	0.7515339407…	3.293011133…
8	0.6794404046…	3.642423407…
9	0.6142626945…	4.028910848…
10	0.5553373855…	4.456407399…
11	0.5020646940…	4.929264422…
12	0.4539023727…	5.452294993…
13	0.4103601914…	6.030822885…
14	0.3709949470…	6.670736768…
15	0.3354059521…	7.378550136…
16	0.3032309567…	8.161467617…
17	0.2741424609…	9.027458300…
18	0.2478443813…	9.985336851…
19	0.2240690374…	11.04485324…
20	0.2025744271…	12.21679197…
21	0.1831417628…	13.51308186…
22	0.1655732452…	14.94691748…

23 0.1496900494… 16.53289342…
24 0.1353305050… 18.28715287…
30 0.0738945361… 33.49110452…
40 0.0269550434… 91.81248930…

x1(40)*x2(40) = 2.474809633632684120291035235819604080282689381240460695287192421639406763951 2...

Find a formula representation, cacc.

Solution

$$\frac{2+3+\sqrt{2^{2^2}+3^{2^2}}}{2*3} = 2.474809633632684120291035235819604080282689381240460695287192421639406763951 2...$$

$$\Rightarrow \lim_{t\to\infty} x1(t)*x2(t) = \frac{2+3+\sqrt{2^{2^2}+3^{2^2}}}{2*3}$$

$$= \text{solution of the equ. } z^2-(1+\frac{2}{3})z = 2$$

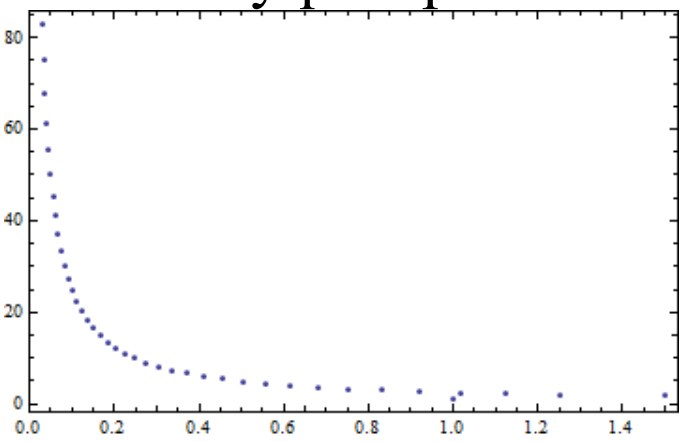
Related difference quotient

$$dx1*x2 = \lim_{t\to\infty} \frac{x1(t)*x2(t)-x1(t+1)*x2(t+1)}{x1(t+1)*x2(t+1)-x1(t+2)*x2(t+2)}$$

$$= 196.48857801796104722...$$

Find a formula representation for dx1*x2, cacc.

Interpret your results.
x1-x2-“Nyquist plot”



Example 2

**1, x1(0),x2(0)$\in\mathbb{R}$!!!, a=3, b=7

$$\lim_{t\to\infty} x1(t)*x2(t) = 4.7936105065489530363770867$$
$$42362977520392152235607824$$
$$76907046792734468308850 21\ldots$$

$$dx1*x2 = \lim_{t\to\infty} \frac{x1(t)*x2(t) - x1(t+1)*x2(t+1)}{x1(t+1)*x2(t+1) - x1(t+2)*x2(t+2)}$$
$$= 46.452078799117147773...$$

Find formula representation for both, cacc.
Interpret your results.

Example 3

**1, x1(0),x2(0)$\in\mathbb{R}$!!!, a=7, b=11

$$\lim_{t\to\infty} x1(t)*x2(t) = 8.8878405775518979031760284$$
$$8463950264278309298728761 1$$

$$\mathrm{dx1*x2} = \lim_{t\to\infty} \frac{\begin{matrix}88414013531503809329215 69\ldots\\ \mathrm{x1(t)*x2(t)} - \mathrm{x1(t+1)*x2(t+1)}\end{matrix}}{\mathrm{x1(t+1)*x2(t+1)} - \mathrm{x1(t+2)*x2(t+2)}}$$

$$= 158.48584779695062169...$$

Find formula representation for both, cacc.
Interpret your results.

Example 4

**1, x1(0),x2(0)$\in\mathbb{R}$!!!, a=7, b=13

$$\lim_{t\to\infty} \mathrm{x1(t)*x2(t)} = 9.771069305275988612862491$$
$$695924734548420294447921621791591502005580 98\ldots$$

$$\mathrm{dx1*x2} = \lim_{t\to\infty} \frac{\mathrm{x1(t)*x2(t)} - \mathrm{x1(t+1)*x2(t+1)}}{\mathrm{x1(t+1)*x2(t+1)} - \mathrm{x1(t+2)*x2(t+2)}}$$

$$= 85.36268425739547964 1...$$

Find formula representation for both, cacc.
Interpret your results.

Example 5

**1, x1(0),x2(0)$\in\mathbb{R}$!!!, a=17, b=13

$$\lim_{t\to\infty} \mathrm{x1(t)*x2(t)} = 14.933109617167559812887877364082634093489681108120831558704935341285956716\ldots$$

$$dx1*x2 = \lim_{t\to\infty} \frac{x1(t)*x2(t) - x1(t+1)*x2(t+1)}{x1(t+1)*x2(t+1) - x1(t+2)*x2(t+2)}$$

$$= 446.49496638627009579...$$

Find formula representation for both, cacc.
Interpret your results.

Example 6

**1, x1(0),x2(0)$\in\mathbb{R}$!!!, a=37, b=43

$$\lim_{t\to\infty} x1(t)*x2(t) = 39.943690548356041688511469$$
$$476247869828940765760032781$$
$$229260710742442417874356\ldots$$

$$dx1*x2 = \lim_{t\to\infty} \frac{x1(t)*x2(t) - x1(t+1)*x2(t+1)}{x1(t+1)*x2(t+1) - x1(t+2)*x2(t+2)}$$

$$= 1418.7206368450500006...$$

Find formula representation for both, cacc.
Interpret your results.

Example 7

**1, x1(0),x2(0)$\in\mathbb{R}$!!!, a=127, b=13

$$\lim_{t\to\infty} x1(t)*x2(t) = 56.420759612477164283744819$$
$$25710800775177\ldots$$

$$dx1*x2 = \lim_{t\to\infty} \frac{x1(t)*x2(t) - x1(t+1)*x2(t+1)}{x1(t+1)*x2(t+1) - x1(t+2)*x2(t+2)}$$

$$= 8.309855043779750384 5...$$

Find formula representation for both, cacc.
Interpret your results.

Example 8
**2, x1=1, x(2)=2
→ x1(∞)=x2(∞)=1
More!

Dear visitor,
If you find in the previous examples some nuggets
then **extension** to chain (1D) and to array (2D) systems of lateral coupled Heron recursion equations.
Systemize the observed behavior patterns.
Discuss possible technical implementations and fields of application in science and technology.
See also Unit 15, part 55.1, Kopplung von Algorithmen.
Go to static/dynamic nonlinear lateral coupling.

Systemize the now observable new emerging behavior patterns.

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