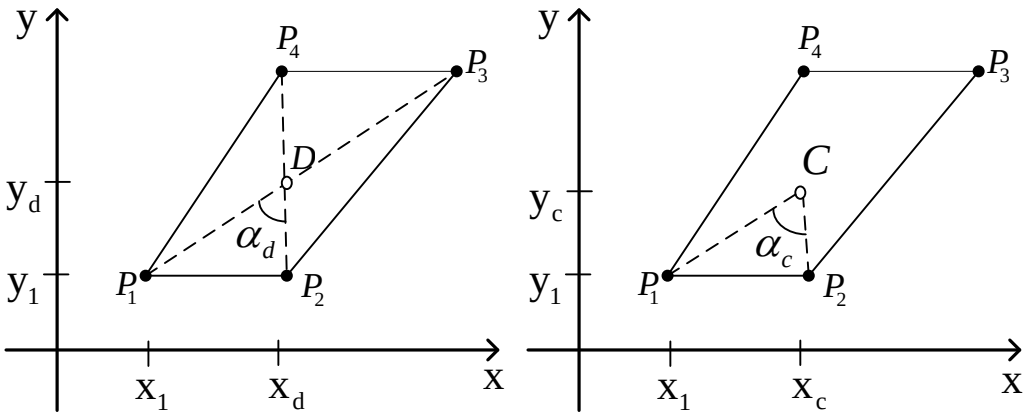
In the debating club 5

Complex numbers, mxn determinants, rule of Sarrus, and computational geometry

Area, angles, diagonal intersection point $D=(x_d,y_d)$, and centroid $C=(x_c,y_c)$ of a 2D simple convex quadrangle $\square P_1P_2P_3P_4$ given by the coordinates of its vertices $P_i=(x_i,y_i)$; i=1(1)4 represented by the complex numbers $z_i=x_i+jy_i$



1)

The complex number

$$z = x+jy = \begin{vmatrix} \overline{z_1} & 0 & \overline{z_3} & 0 \\ 0 & z_2 & 0 & z_4 \end{vmatrix}$$

is separable into

$$z = x+jy = \begin{vmatrix} x_1 & y_1 & x_3 & y_3 \\ y_4 & x_2 & y_2 & x_4 \end{vmatrix} + j \begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix} .$$

Then we obtain

$$\text{area} \not\frown P_1P_2P_3P_4 = \frac{y}{2}$$

$$\tan \sphericalangle P_1 \underline{D} P_2 = \tan \arg(z) = \frac{y}{x} .$$

** Find a geometric interpretation of x.
** Discussion and extension.

2)
For the coordinates of D and C in complex number representation we have the beautiful and extensible mxn determinant notations

$$z_d = x_d + jy_d = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ z_1 & z_1 & z_1 & z_1 \\ y_1 & y_1 & y_1 & y_1 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}$$

$$z_c = x_c + jy_c$$

$$= \frac{1}{3} \frac{\begin{vmatrix} x_1 & x_1 & x_2 & x_2 & x_3 & x_3 & x_4 & x_4 \\ z_1 & z_1 & z_2 & z_2 & z_3 & z_3 & z_4 & z_4 \\ y_1 & y_1 & y_2 & y_2 & y_3 & y_3 & y_4 & y_4 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}}$$

$z_d + 3z_c = z_1 + z_2 + z_3 + z_4$.

** Discussion and extension.

3)

** Find mxn determinant notations for $\tan\angle P_1 \underline{C} P_2$ and $\tan\angle P_1 \underline{P_3} P_2$.

** Find connections between $\tan\angle P_1 \underline{D} P_2$, $\tan\angle P_1 \underline{C} P_2$, and $\tan\angle P_1 \underline{P_3} P_2$.

** Discussion and extension.

5.1 Eadem mutata resurgo

Problem corresponding to Unit 1, Babylonian sqr(2) problem
Find a simple and fast algorithm for the numeric calculation of
$x = sqr(2) + sqr(3) = 3.1462643\ldots$. (1)

Solution

$x = sqr(2) + sqr(3) \rightarrow x^2 = 2 + 2sqr(6) + 3 \rightarrow (x^2 - 5)^2 = 24 \rightarrow$
$x^4 - 10x^2 + 1 = 0 \rightarrow x = (10x^2 + 1)/x^3$

$\rightarrow$ "Babylon" recursion approach:

$$x := \frac{1}{2}\left(x + \frac{10x^2 - 1}{x^3}\right) \rightarrow \quad (2)$$

Execution record for the initial value x(0)=5

t	x(t)
0	5.0
1	3.496
2	3.166504067
3	3.146532320
4	3.146267115
5	3.146264398
6	3.146264370
7	3.146264370

Associated **difference quotient**

$$\lim_{t\to\infty} dx(t) = \frac{x(t)-x(t+1)}{x(t+1)-x(t+2)} = \lim_{t\to\infty} (x(t))^4 = 97.98979485566..$$
$$= (sqr(2) + sqr(3))^4 \quad (3.1)$$

Your **homework**

Check $\lim_{t\to\infty} d^{(k)}x(t)$ with

$$d^{(k)}x(t) = \frac{d^{(k-1)}x(t) - d^{(k-1)}x(t+1)}{d^{(k-1)}x(t+1) - d^{(k-1)}x(t+2)} \; ; \quad (3.2)$$

$k=2,3,4,5,...$

$d^{(0)}x(t) = x(t)$

$d^{(1)}x(t) = dx(t)$

Diskussion!

Comment

The foregoing problem is an example for a converging recursion equation

$$x := \varphi(x) \tag{4}$$

with

$$\lim_{t\to\infty} x(t) = a \tag{5}$$

and

$$\lim_{t\to\infty} dx(t) = \frac{x(t) - x(t+1)}{x(t+1) - x(t+2)} = a^n \;;\; n \in \mathbb{N} \tag{6}$$

** Find more examples for the function $\varphi(x)$ that fulfils in the recursion equation (4) the conditions (5) and(6).

** Find a general solution for this problem.

** Are there possible applications in science and engineering?