In the debating club n°33

The magic number $\overbrace{11\ldots1}^{\text{n times}} = 1^{\underline{n}}$ **etc.**

** The first trace

For what odd numbers n is the relation

" $n \mid 1^{\underline{n-1}} \quad 1^{\underline{n-1}}$ is divisible by n" (1)

true? (See [1])

Or in modulo representation:

For what odd numbers n is

$1^{\underline{n-1}} \equiv 0 \pmod{n}$? (2)

Hint

$n \in \{7,11,13,17,19,23,29,31,37,41,43,47,53,$
$59,61,67,71,73,79,83,89,91,97,101,$
$103,107,109,113,127,\ldots,239,241,251,$
$259,263,269,271,277,\ldots\}$ (3)

Full sequence of primes ≥ 11 (black)?

and a small number of nonprimes (red).

Check more → full sequence of primes?

and nonprimes.

Reduced:
For what odd numbers n is the relation

$$\text{“}n \mid (10)^{\frac{n-3}{2}} 1\text{”} \qquad (4)$$

true?
Or in modulo representation:
For what odd numbers n is

$$(10)^{\frac{n-3}{2}} 1 \equiv 0 \pmod{n} \ ? \qquad (5)$$

Hint

$$n \in \{7,13,17,19,23,29,31,37,41,43,47,53,$$
$$59,61,67,71,73,79,83,89,91,97,101,$$
$$103,107,109,113,127,\ldots,239,241,251,$$
$$259,263,269,271,277,\ldots\} \qquad (6.1)$$

Full sequence of primes $\geqq 13$ (black)?

and a small number of nonprimes (red).

Check more.

There is more

$n \in \{7,13,17,19,23,29,31,37,41,43,47,53,$

59,61,67,71,73,79,83,89,91,97,101,
103,107,109,113,127,…,239,241,251,
259,263,269,271,277,…, 479,481,487,
…,701,703,709,…,..,1729,..,2821,..,
3367,..,4141,..,4187,..,5461,..,6533,..,
6541,..,6601,..,7471,..,8149,..,8401,..,
8911,..,10001,..,11111,..,12403,..,
14701,..,14911,..,15211,..,15841,..,
19201,..,21931,..,22321,..,24661,..,
27613,..,29341,..,35113,..,38503,..,
45527,..,46657,..,50851,..,50881,..,
52633,..,54913,..,57181,..,63139,..,
63973,..,65311,..,66991,..,67861,..,
79003,..,82513,..,83119,..,94139,..,
97673,97681,97687,…}

(6.2)

Full sequence of primes from 13 to 97687 (black)

and 49 nonprimes (red) ($\approx$ 0.5%).

Check more $\rightarrow$ full sequence of primes?

and nonprimes.

Discuss the result.

Discuss further reductions, esp. for large n.

** The second trace

Is there a connection between the equs (2), (3), (5), and (6) and Wilson's theorem [2,3]

Iff p is prime

then is $(p-1)! + 1 \equiv 0 \pmod{p}$. **?** (7)

Reduced Wilson's theorem [4]

Iff p is prime

then is $\left(\frac{p-1}{2}!\right)^2 + 1 \equiv 0 \pmod{p}$. (8)

Discuss further reductions.

** The third trace

For what odd numbers n is

$(10)^{\frac{n-3}{2}} 1 \equiv 1 \pmod{n}$? (9)

Hint

$n \in \{15, 21, 25, 39, 51, 57, 69, 75, 87, 93, 105,$

$111, 123, 129, 141, 159, 177, 183, 195,$

$201, 213, 219, 237, 249, 267, 273, 291,$

303, 309, 321, 327, 339, 381, 393, 399,
411, 417, 447, 453, 471, 489, 501, 519,
525, 537, 543, 555, 573, 575, 579, 591,
597, 633, 669, 681, 687, 699, 717, 723,
741, 753, 771, 777, 789, 807, 813, 831,
843, 849, 879, 921, 933, 939, 951, 975,
993, 1011, 1041, 1047, 1059, 1077, 1101,
1119, 1137, 1149, 1167, 1191, 1203, 1227,
1257, 1263, 1293, 1299, 1317, 1329, 1347,
1365, 1371, 1383, 1389, 1401, 1425, 1437,
1443, 1461, 1473, 1497, 1509, 1515, 1527,
1563, 1569, 1623, 1641, 1671, 1689, 1707,
1713, 1731, 1761, 1779, 1797, 1803, 1821,
1839, 1851, 1857, 1893, 1923, 1929, 1941,
1959, 1977, 1983, …}

(10)

Discussion!

**** The fourth trace**

For what odd numbers n is

$$(10)^{\frac{n-3}{2}} 1 \equiv k \pmod{n} \; ; \; k < n \; ? \qquad (11)$$

Discussion! Krypto and stego applications?

To be continued.

References

[1] http://scienceblogs.de/mathlog/2015/01/02/kias-kalender-januar-2015/

[2] Weisstein, Eric W. "Wilson's Theorem." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/WilsonsTheorem.html

[3] Number of digits in n! http://oeis.org/A034886

[4] Chia-Long Wu et al. Computational Reduction of Wilson's Primality Test for Modern Cryptosystems. Informatica 33 (2009) 453-458