In the debating club 280
Meetings via infinity
An integrated view as basis for discussions
280.1 $\frac{\pi}{2}$ and $\sqrt{3}$

$$\frac{\pi}{2} = 1+\frac{1}{3}\left(1+\frac{1}{3-\frac{1}{2}}\left(1+\frac{1}{3-\frac{1}{2-\frac{1}{2}}}\left(1+\frac{1}{3-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}\left(1+\ldots\right.\right.\right.\right. = 1.57079\,6326\ldots.$$

$$= 1+\frac{1}{3}\left(+\frac{2}{5}\left(1+\frac{3}{7}\left(1+\frac{4}{9}\left(1+\frac{5}{11}(1+\cdots)\ldots\right)\right)\right)\right)$$

(1.1,2)

$$\sqrt{3} = 1 + \frac{1}{3}\left(1 + \frac{1}{3\,\frac{1}{3\,\frac{1}{3}}}\left(1 + \frac{1}{3\,\frac{1}{3\,\frac{1}{2}}}\left(1 + \frac{1}{3\,\frac{1}{3\,\frac{1}{2\,\frac{1}{2}}}}\left(1 + \frac{1}{3\,\frac{1}{3\,\frac{1}{2\,\frac{1}{2\,\frac{1}{2}}}}}\left(1 + \ldots\right.\right.\right.\right.\right.$$

$$= 1 + \frac{1}{3*1}\left(1 + \frac{1}{3*(1-\frac{1}{3})}\left(1 + \frac{1}{3*(1-\frac{2}{5})}\left(1 + \frac{1}{3*(1-\frac{3}{7})}\left(1 + \right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{3*(1-\frac{4}{9})}\left(1 + \frac{1}{3*(1-\frac{5}{11})}(1 + \cdots)\ldots\right)\right)\right)\right)$$

$$= 1 + \frac{1}{3}\left(1 + \frac{3}{6}\left(1 + \frac{5}{9}\left(1 + \frac{7}{12}\left(1 + \frac{9}{15}(1 + \cdots)\ldots\right)\right)\right)\right)$$

(2.1,2,3)

280.2 e and 1

$$\frac{3-e}{e-2} = \frac{1}{2+\frac{2}{3+\frac{3}{4+\frac{4}{5+\frac{5}{6+\frac{6}{7+\cdots}}}}}} \qquad 1 = \frac{2}{1+\frac{3}{2+\frac{4}{3+\frac{5}{4+\frac{6}{5+\frac{7}{6+\cdots}}}}}}$$

(3.1,2)

$$\frac{2-\sqrt{e}}{\sqrt{e}-1}=\cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cdots}}}}} \qquad 1=\cfrac{3}{2+\cfrac{5}{4+\cfrac{7}{6+\cfrac{9}{8+\cfrac{11}{10+\cdots}}}}}$$

(4.1,2)

$$1=\cfrac{2}{1+\cfrac{4}{3+\cfrac{6}{5+\cfrac{8}{7+\cfrac{10}{9+\cdots}}}}} \qquad ???=\cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\cfrac{7}{8+\cfrac{9}{10+\cdots}}}}}$$

$$=0.37973195474\ldots$$

(5.1,2)

$$\frac{1}{e-1}=\cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cdots}}}}} \qquad ???=\cfrac{1}{1+\cfrac{3}{3+\cfrac{5}{5+\cfrac{7}{7+\cfrac{9}{9+\cdots}}}}}$$

$$=0.56201675\ldots$$

(6.1,2)

$$\cfrac{1}{-1+\cfrac{3}{1+\cfrac{5}{3+\cfrac{7}{5+\cfrac{9}{7+\cfrac{11}{9+\cdots}}}}}}+\cfrac{-1}{1+\cfrac{1}{3+\cfrac{3}{5+\cfrac{5}{7+\cfrac{7}{9+\cfrac{9}{11}}}}}}=2$$

$$\frac{1}{-1+\cfrac{3}{1+\cfrac{5}{3+\cfrac{7}{5+\cfrac{9}{7+\cfrac{11}{9+\cdots}}}}}} - \frac{-1}{1+\cfrac{1}{3+\cfrac{3}{5+\cfrac{5}{7+\cfrac{7}{9+\cfrac{9}{11+\cdots}}}}}} =$$

$$2*\left(1+\frac{3}{3+\cfrac{5}{5+\cfrac{7}{7+\cfrac{9}{9+\cfrac{11}{11+\cdots}}}}}\right) = 3.558612793\ldots$$

(7.1,2)

$$\frac{2}{1+\cfrac{3}{2+\cfrac{4}{3+\cfrac{5}{4+\cfrac{6}{5+\cfrac{7}{6+\cdots}}}}}} + \frac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cdots}}}}}} = \frac{1}{e-2}$$

$$\frac{2}{1+\cfrac{3}{2+\cfrac{4}{3+\cfrac{5}{4+\cfrac{6}{5+\cfrac{7}{6+\cdots}}}}}} - \frac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cdots}}}}}} =$$

$$\frac{1}{2}*\left(2-\frac{3}{3+\cfrac{5}{5+\cfrac{7}{7+\cfrac{9}{9+\cfrac{11}{11+\cdots}}}}}\right) = 0.610346801680099\ldots$$

(8.1,2)

280.3 π, π², ln2, and more

$$\frac{4}{\pi} = 1+\cfrac{1^2}{3+\cfrac{2^2}{5+\cfrac{3^2}{7+\cfrac{4^2}{9+\cfrac{5^2}{11+\cfrac{6^2}{13+\cdots}}}}}} \qquad (9)$$

$$\frac{1}{\ln 2} = 1+\cfrac{1^2}{1+\cfrac{2^2}{1+\cfrac{3^2}{1+\cfrac{4^2}{1+\cfrac{5^2}{1+\cfrac{6^2}{1+\cdots}}}}}} \qquad (10)$$

$$\frac{12}{\pi^2} = 1+\cfrac{1^4}{3+\cfrac{2^4}{5+\cfrac{3^4}{7+\cfrac{4^4}{9+\cfrac{5^4}{11+\cfrac{6^4}{13+\cdots}}}}}} \qquad (11)$$

$$1+\cfrac{1^4}{1+\cfrac{2^4}{1+\cfrac{3^4}{1+\cfrac{4^4}{1+\cfrac{5^4}{1+\cfrac{6^4}{1+\cdots}}}}}} =$$

1.689020203784129318406295676041021321741…= ?

(12)

280.4 π, Φ, and more

$$\frac{\pi}{2} = 1.570796327\ldots$$

$$= 1 + \frac{1}{3}*\left(1 + \frac{2}{5} * \left(1 + \frac{3}{7} * \left(1 + \frac{4}{9} * \left(1 + \frac{5}{11} * \left(1 + \frac{6}{13} * (1 + \cdots)\right)\right)\right)\right)\right) \quad (14)$$

$$\sqrt{\frac{5}{2}} * \ln(\Phi) = 0.926259282\ldots$$

$$= 1 - \frac{1}{3}*\left(1 - \frac{2}{5} * \left(1 - \frac{3}{7} * \left(1 - \frac{4}{9} * \left(1 - \frac{5}{11} * \left(1 - \frac{6}{13} * (1 - \cdots)\right)\right)\right)\right)\right) \quad (15)$$

What about other sign sequences, f.e.

$$1+\frac{1}{3}*\left(1-\frac{2}{5}*\left(1+\frac{3}{7}*\left(1-\frac{4}{9}*\left(1+\frac{5}{11}*\left(1-\frac{6}{13}*\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.(1+\cdots)\right)\right)\right)\right)\right)\right) = 1.173390570286992706933282605414112199760\ldots$$

(16)

$$1-\frac{1}{3}*\left(1+\frac{2}{5}*\left(1-\frac{3}{7}*\left(1+\frac{4}{9}*\left(1-\frac{5}{11}*\left(1+\frac{6}{13}*\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.(1-\cdots)\right)\right)\right)\right)\right)\right) = 0.601991051135538503998525457742123930405 2\ldots$$

(17)

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$$\left(\frac{\pi}{3}\right)^2 =$$

$$1+\frac{1}{3}*\frac{1}{4}*\left(1+\frac{2}{5}*\frac{2}{6}*\left(1+\frac{3}{7}*\frac{3}{8}*\left(1+\frac{4}{9}*\frac{4}{10}\left(1+\frac{5}{11}*\frac{5}{12}*\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left(1+\frac{6}{13}*\frac{6}{14}*(1+\cdots)\right)\right)\right)\right)\right) \qquad (18)$$

$$1-\frac{1}{3}*\frac{1}{4}*\left(1-\frac{2}{5}*\frac{2}{6}*\left(1-\frac{3}{7}*\frac{3}{8}*\left(1-\frac{4}{9}*\frac{4}{10}\left(1-\frac{5}{11}*\frac{5}{12}*\left(1-\frac{6}{13}*\frac{6}{14}*(1-\cdots)\right)\right)\right)\right)\right)=0.926259282\ldots=$$

$$(\ln(\Phi^2))^2 \qquad (19)$$

280.5 Natural numbers and primes

$$\prod_{n=2}^{\infty}\frac{n^3+1}{n^3-1}=\prod_{n=2}^{\infty}\frac{p_n^2+1}{p_n^2-1}=\frac{3}{2} \qquad (20)$$

Extension!

280.6 2 and more

$$2=1+2*3*\frac{1}{a_1}*(3+4*5*\frac{a_0}{a_2}*(5+6*7*\frac{a_1}{a_3}*(7+8*9*\frac{a_2}{a_4}*(9+10*11*\frac{a_3}{a_5}*(\ldots)\ldots)$$

$a_0 = 1$

$a_1 = 1+2*3*(3) = 19$

$a_2 = 1+2*3*(3+4*5*(5)) = 619$

$a_3 = 1+2*3*(3+4*5*(5+6*7*(7))) = 35899$

$a_4 = 1+2*3*(3+4*5*(5+6*7*(7+8*9*(9)))) = 3301819$

$a_5 = 442386619$

$a_6 = 81393657019$

$a_7 = 19696509177019$

$a_8 = 6066382786809019$

(21)

Check other sign sequences, see Unit 555.3.1, equ. (110) and Deb 280.4 .

280.7 e, π, and related difference quotients

64.7

$$x(n) = \left(\frac{2^{n+1} + 1}{2^{n+1} - 1} \right)^{2^n}$$

$$\lim_{n \to \infty} x(n) = e$$

$$\lim_{n \to \infty} \frac{x(n) - x(n+1)}{x(n+1) - x(n+2)} = \mathbf{4}$$

64.8

Input: $x := 0\,;\ y := 1$
$i = 0(1)n$
$z := y - x$
$x := \frac{1}{2}(x + y)$
$y := \sqrt{x\,y}$
$d := \frac{z}{y - x}$
Output: $y(n)\,;\ d(n)$

$$\lim_{n \to \infty} y(n) = \frac{2}{\pi}$$

$$\lim_{n \to \infty} d(n) = \mathbf{4}$$