In the debating club n°23

Sati and divisibility rules
A naïve sandtable exercise

"The Abhidhammattha Sangaha, a key Abhidharma text from the Theravāda tradition, defines **sati** as follows:
The word sati derives from a root meaning 'to remember,' but as a mental factor it signifies presence of mind, attentiveness to the present, rather than the faculty of memory regarding the past. It has the characteristic of not wobbling, i.e. not floating away from the object. Its function is absence of confusion or non-forgetfulness. It is manifested as guardianship, or as the state of confronting an objective field. Its proximate cause is strong perception (thirasaññā) or the four foundations of mindfulness." http://en.wikipedia.org/wiki/Sati_(Buddhism)

Let's practice this "sati-technology" in the field of number theory.
Our subject is the number 19. Satiing 19, we find:
19 = 1*10 + 9 (1)
9 +1 + 9 (bodyguard model) =
1 +2*9 = 19 !!! (2)
So, from this "seed" we formulate light-footedly the hypothesis:
Given a natural number n.
** We decompose n into 2 parts
n = div(n,10) + rest(n,10) . (3)

** If n'= div(n,10) + 2*rest(n,10) ≤ n
divisible by 19
then is n divisible by 19
else not. (4)

** Of course, we can repeat this test
recursively. (5)

Test by sampling:

2*19 =38 → 3+2*8=19

3*19 =57 → 5+2*7=19

4*19 =76 → 7+2*6=19

5*19 =95 → 9+2*5=19

6*19 = 114 → 11+2*4=19

7*19 = 133 → 13+2*3=19

11*19 = 209 → 20+2*9=38 → 3+2*8=19

13*19 = 247 → 24+2*7=38 → 3+2*8=19

19*19 = 361 → 36+2*1=38 → 3+2*8=19

351 → 35+2*1=37 → 3+2*7=17 ≠19!!

7183*19 =136477 → 13647+2*7=13661 →

1366+2*1 =1368 → 136+2*8=152 →

15+2*2 =19

136471 → 13647+2*1=13649 →

1364+2*9 =1382 → 138+2*2=142 →

14+2*2 =18≠19!!

This test is successful, start the mathematical proof!

Now we check the bodyguard model approach with another subject, the prime 13.

$13 = 1*10 + 3$ (6)

$3 + 3 + 1 + 3 + 3 =$

$1 + 4*3 = 13$!!! (7)

→ Hypothesis:

Given a natural number n.

** We decompose n into 2 parts

$n = \text{div}(n,10) + \text{rest}(n,10)$. (8)

** If $n' = \text{div}(n,10) + 4*\text{rest}(n,10) \leq n$ (9.1)

divisible by 13

then is n divisible by 13

else not. (9.2)

** Of course, we can repeat this test

recursively with equ. (9.1) (10)

Test by sampling:

$2*13 = 26 \rightarrow 2+4*6 = 26$

$3*13 = 39 \rightarrow 3+4*9 = 39$

$4*13 = 52 \rightarrow 5+4*2 = 13$

$5*13 = 65 \rightarrow 6+4*5 = 26$

$6*13 = 78 \rightarrow 7+4*8 = 39$

$7*13 = 91 \rightarrow 9+4*1 = 13$

$11*13 = 143 \rightarrow 14+4*3 = 26$

$13*13 = 169 \rightarrow 16+4*9=52 \rightarrow 5+4*2=13$

$19*13 = 247 \rightarrow 24+4*7=52 \rightarrow 5+4*2=13$

$353 \rightarrow 35+4*3=47 \rightarrow 4+4*7=32 \neq 39$!!

7183*13 = 93379 → 9337+4*9=9373 →

937+4*3 = 949 → 94+4*9=130 →

13+4*0 = 13

136471 → 13647+4*1=13651 →

1365+4*1 = 1369 → 136+4*9=172 →

17+4*2 = 25≠26!!

This test is successful, start the mathematical proof!

Now we check the bodyguard model approach with other subjects, …

37 = 3*10 + 7 (11)

7 + 7 + 3 + 3 + 3 + 7 + 7 =

3*3 +4*7 = 37 !!! (12)

→ Hypothesis:

Given a natural number n.

** We decompose n into 2 parts

n = div(n,10) + rest(n,10) . (14)

** If n'= 3*div(n,10) + 4*rest(n,10) ≤ n (15.1)

divisible by 37

then is n divisible by 37

else not. (15.2)

** Of course, we can repeat this test recursively with equ. (15.1). (16)

Test by sampling:

17*37 **= 629** → 3*62+4*9=222 → 3*22+4*2

=74 → 3*7+4*4

= **37 ok**

37*37 **= 1369** → 3*136+4*9=444 →

3*44+4*4 =148 → 3*14+4*8=74 →

3*7+4*4 = **37 ok**

137731*37 = 5096047→

3*509604+4*7 = 1528840 →

3*152884+4*0 = 458652 →

3*45865+4*2 = 137603 →

3*13760+4*3 = 41292 →

3*4129+4*2 = 12395 →

3*1239+4*5 = 3737 →

3*373+4*7 = 1147 →

3*114+4*7 = 370 →

3*37+4*0 = 111 →

3*11+4*1 = **37 ok**

This test is successful, start the mathematical proof!

Our 3 special cases and the related examples inspire us to **find in this way a general divisibility test algorithm:**

Is the natural number n divisible by the prime p, p|n ?

Approach for a test formula (reduction $n \rightarrow n' \leq n$) = body in the test loop:

$$n' = a(p)*\mathrm{div}(n,10) + b(p)*\mathrm{rest}(n,10) \leq n \; . \quad (17)$$

Examples for a(p) and b(p) (found empirically)

p	a(p)	b(p)
3	1	1
7	1	-2
11	1	-1

13	1	4
19	1	2
37	3	4

How we can calculate a(p) and b(p) for a given p?
From equ. (17) we obtain for $n=n'=p$

$$a(p)*div(p,10) + b(p)*rest(p,10) = p\ . \qquad (18)$$

For given p, equ. (18) is a linear Diophantine equ. with the unknowns a(p) and b(p).
The solution is described in Unit 53.
We consider the example p=37.

Equ. (18) → $3a + 7b = 37$ →

$$a(37) = -74 + 7t\ ;\ b(37) = 37 - 3t\ ;\ t\in\mathbb{Z} \qquad (19.1)$$

Smallest values of a(37) and b(37) for t=11 →

$$a(37) = 3\ ;\ b(37) = 4 \rightarrow \qquad (19.2)$$

reduction equation

$$n' = 3div(n,10) + 4rest(n,10)\ . \qquad (19.3)$$

Your turn is to handle the case p=277.
And to continue our sandbox exercise with the extended reduction equation

$$n' = a(p,k)*div(n,10^k) + b(p,k)*rest(n,10^k)\ ;$$
$$k=1,2,3,\ldots\ . \qquad (20)$$

What is it good for?

References

http://de.wikipedia.org/wiki/Teilbarkeit
http://en.wikipedia.org/wiki/Divisibility_rule
http://mathworld.wolfram.com/DivisibilityTests.html