Im Debattierclub 21

A nat. number and prime asymmetry: nat. numbers and the primes p_=6k-1 and p_+=6k+1 generate different kinds of continued fractions.

All primes $\geqslant 5$ have the form

$$p_\pm = 6k \pm 1\,;\ k \in \mathbb{N}. \tag{1}$$

The infinite simple continued fractions

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ddots}}} = [a_0\,;\ a_1,\, a_2,\, a_3,\, ...]\,;\ a_i \in \mathbb{N}_0 \tag{2}$$

of the nat. number with (1) and of the primes function

$$f(p) = \frac{\sqrt[p]{e}}{3 - 2\sqrt[p]{e}} \tag{3}$$

are sequences of triplets of triplets both for p_ and p_+:

$$f(p_-) = [\ \underbrace{\underbrace{1\,;\ c_0,\, 1}_{\text{elementary triplet}}\ ,\ \underbrace{5,\, c_1,\, 1}\ ,\ \underbrace{5,\, c_2,\, 5}}_{\text{main triplet}}\ ,\ \underbrace{\underbrace{1,\, c_3,\, 1}_{\text{elementary triplet}}\ ,\ \underbrace{5,\, c_4,\, 1}\ ,\ \underbrace{5,\, c_5,\, 5}}_{\text{main triplet}}\ ,\ ...] \tag{4}$$

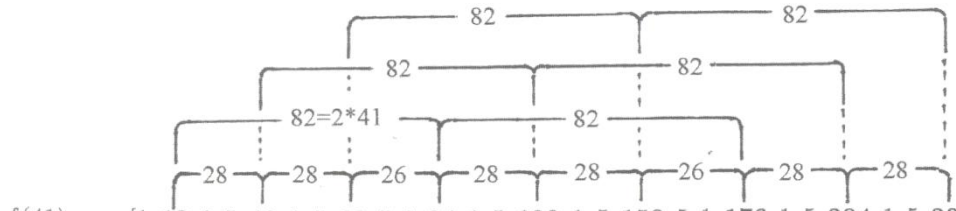
$$f(p_+) = [\underbrace{1\,;\ d_0,\, 1}, \underbrace{1,\, d_1,\, 1}, \underbrace{1,\, d_2,\, 5}, \underbrace{1,\, d_3,\, 1}, \underbrace{1,\, d_4,\, 1}, \underbrace{1,\, d_5,\, 5}, ...]\,, \tag{5}$$

but with different arrangements of the bodyguard elements (noise) 1 and 5 of the elementary triplets and with remarkably different structures of the variable elements (signals c_i and d_i, respectively:

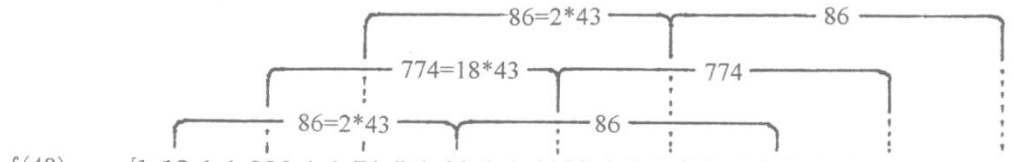
$$\begin{aligned} c_i &= \frac{p_- + 1}{3}\Big(2\,i + 1\Big) - 2\Big(\text{int}\Big(\frac{i}{3}\Big) + 1\Big) \\ &= 2k\Big(2\,i + 1\Big) - 2\Big(\text{int}\Big(\frac{i}{3}\Big) + 1\Big) \\ &= c_0 + (c_1 - c_0)i - 2\text{int}\Big(\frac{i}{3}\Big);\ c_0 = \frac{p_- - 5}{3} = 2(k-1)\,;\ c_1 = p_- - 1 = 2(3k-1) \end{aligned} \tag{6}$$

$$d_i = \begin{cases} d_0 + (d_3 - d_0)\frac{i}{3} & i\,\text{mod}3 = 0 \\ d_1 + (d_4 - d_1)\frac{i-1}{3} & i\,\text{mod}3 = 1 \\ d_2 + (d_5 - d_2)\frac{i-2}{3} & i\,\text{mod}3 = 2 \end{cases} \text{for} \quad ; \quad \begin{array}{c|c} i & d_i \\ \hline 0 & 2k-1 \\ 1 & 54k+8 \\ 2 & 10k+1 \\ 3 & 14k+1 \\ 4 & 162k+26 \\ 5 & 22k+3 \end{array} \quad ;\ 2k = \frac{p_+ - 1}{3} \tag{7}$$

So we have f.e. for the prime twins p_=41 and p_+=43 (k=7):



$$f(41) = [1; \mathbf{12}, 1, 5, \mathbf{40}, 1, 5, \mathbf{68}, 5, 1, \mathbf{94}, 1, 5, \mathbf{122}, 1, 5, \mathbf{150}, 5, 1, \mathbf{176}, 1, 5, \mathbf{204}, 1, 5, \mathbf{232}, 5, \ldots] \quad (8)$$



$$f(43) = [1; \mathbf{13}, 1, 1, \mathbf{386}, 1, 1, \mathbf{71}, 5, 1, \mathbf{99}, 1, 1, \mathbf{1160}, 1, 1, \mathbf{157}, 5, 1, \mathbf{185}, 1, 1, \mathbf{1934}, 1, 1, \mathbf{243}, 5, \ldots] \quad (9)$$

The continued fractions (8) and (9) illustrate the aforementioned difference between c_i and d_i. In the first case p_ have all successive c_i-differences inside a main triplet and all differences (c_{i+3}-c_i) the same values:

$$(\forall i)_{i \bmod 3=0}\ (c_{i+1} - c_i) = (\forall i)_{i \bmod 3=1}\ (c_{i+1} - c_i) = \frac{2}{3}\Big(p_- + 1\Big) \quad (10)$$

$$(\forall i)\ (c_{i+3} - c_i) = 2\,p_- \,. \quad (11)$$

In the case of p_+ the structure of the accompanying continued fraction (5) is more difficult:

$$(\forall i)_{i \bmod 3=0}\ (d_{i+1} - d_i) \neq (\forall i)_{i \bmod 3=1}\ (d_{i+1} - d_i) \quad (12)$$

$$(\forall i)_{i \bmod 3=0}\ (d_{i+3} - d_i) = (\forall i)_{i \bmod 3=2}\ (d_{i+3} - d_i) = 2\,p_+ \quad (13)$$

$$(\forall i)_{i \bmod 3=1}\ (d_{i+3} - d_i) = 18\,p_+ \,. \quad (14)$$

Discuss applications, esp. of this prime asymmetry.