Supplement 3 to Deb 20
With and without bodyguards

Part 1

$$\frac{J_1(2)}{J_0(2)} = 2.575920321368221956857496782 32....$$

$$= [2;1,1,2,1,3,1,4,1,5,1,6,1,7,1,8,1,9,1,10,$$
$$1,11,1,12,1,13,1,14,1,15,...]$$

$$\frac{I_1(2)}{I_0(2)} = 0.697774657964007982006790592552...$$

$$= [0;1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,...]$$

tan1 = 1.557407724654902230506974807 46…

= [1;1,1,3,1,5,1,7,1,9,1,11,1,13,1,15, 1,17,1, 19,1,21,1,23,1, 25,1,27,…]

tanh1 = 0.761594155955764888119458282605…

= [0;1,3,5,7,9,11,13,15,17,19,21,23, 25,27,…]

Part 2

$$\frac{J_1(1)}{J_0(1)} = 0.57508091500430596049944339531 9...$$

$$= [0;1,1,2,1,4,1,6,1,8,1,10,1,12,1,14,1,16,...]$$

$$\frac{I_1(1)}{I_0(1)} = 0.44638996589653450704768179519 3...$$

$$= [0;2,4,6,8,10,12,14,16,...]$$

$$\tan 0.5 = 0.54630248984379051325517946 5780\ldots$$

$$= [0;1,1,4,1,8,1,12,1,16,1,20,1,24,\ldots]$$

$$\tanh 0.5 = 0.46211715726000975850231848 3644\ldots$$

$$= [0;2,6,10,14,18,22,26,30,\ldots]$$

$$\frac{J_1(x)}{J_0(x)} = \frac{\sum_{i=0}^{\infty} \frac{(-1)^i}{i!(i+1)!}\left(\frac{x}{2}\right)^{2i+1}}{\sum_{i=0}^{\infty} \frac{(-1)^i}{i!i!}\left(\frac{x}{2}\right)^{2i}} =$$

$$\begin{vmatrix}
0 & \frac{1}{0!1!} & \frac{1}{1!2!} & \frac{1}{2!3!} & \frac{1}{3!4!} & \frac{1}{4!5!} & \rightarrow \\
-\left(\frac{x}{2}\right)^1 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \\
+\left(\frac{x}{2}\right)^3 & 0 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \\
-\left(\frac{x}{2}\right)^5 & 0 & 0 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \\
+\left(\frac{x}{2}\right)^7 & 0 & 0 & 0 & \frac{1}{0!0!} & \frac{1}{1!1!} & \\
-\left(\frac{x}{2}\right)^9 & 0 & 0 & 0 & 0 & \frac{1}{0!0!} & \\
\downarrow & & & & & & \searrow
\end{vmatrix}$$

$$\frac{I_1(x)}{I_0(x)} = \frac{\sum_{i=0}^{\infty} \frac{1}{i!(i+1)!}\left(\frac{x}{2}\right)^{2i+1}}{\sum_{i=0}^{\infty} \frac{1}{i!i!}\left(\frac{x}{2}\right)^{2i}} =$$

$$-\begin{vmatrix} 0 & \frac{1}{0!1!} & \frac{1}{1!2!} & \frac{1}{2!3!} & \frac{1}{3!4!} & \frac{1}{4!5!} & \rightarrow \\ \left(\frac{x}{2}\right)^{1} & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \frac{1}{4!4!} & \\ \left(\frac{x}{2}\right)^{3} & 0 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \frac{1}{3!3!} & \\ \left(\frac{x}{2}\right)^{5} & 0 & 0 & \frac{1}{0!0!} & \frac{1}{1!1!} & \frac{1}{2!2!} & \\ \left(\frac{x}{2}\right)^{7} & 0 & 0 & 0 & \frac{1}{0!0!} & \frac{1}{1!1!} & \\ \left(\frac{x}{2}\right)^{9} & 0 & 0 & 0 & 0 & \frac{1}{0!0!} & \\ \downarrow & & & & & & \searrow \end{vmatrix}$$

For comparison

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!} x^{2i+1}}{\sum_{i=0}^{\infty} \frac{(-1)^i}{i!i!} x^{2i}} =$$

$$\begin{vmatrix}
0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \rightarrow \\
-x^1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \\
+x^3 & 0 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \\
-x^5 & 0 & 0 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \\
+x^7 & 0 & 0 & 0 & \frac{1}{0!} & \frac{1}{2!} & \\
-x^9 & 0 & 0 & 0 & 0 & \frac{1}{0!} & \\
\downarrow & & & & & & \searrow
\end{vmatrix}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\sum_{i=0}^{\infty} \frac{1}{(2i+1)!} x^{2i+1}}{\sum_{i=0}^{\infty} \frac{1}{(2i)!} x^{2i}} =$$

$$-\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \rightarrow \\ x^1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \\ x^3 & 0 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \\ x^5 & 0 & 0 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \\ x^7 & 0 & 0 & 0 & \frac{1}{0!} & \frac{1}{2!} & \\ x^9 & 0 & 0 & 0 & 0 & \frac{1}{0!} & \\ \downarrow & & & & & & \searrow \end{vmatrix}$$

Part 3

Find more similar to part 1 and 2.

Part 4

A prime-generated monster wave

[1;1,1,1,3,1,1,5,1,1,7,1,1,11,1,1,13,1,1,17,1,1,19,1,1,23,1,1,29,1,1,31,1,1,37,1, 1,41,1,1,43,1,1,47,…] =

1.64026267536047359544339429895239142789126128410271406599354253986769075485363136575411305114158676 2… = a

[1;1,1,1,3,1,1,5,1,1,7,1,1,9.1,1,11,1,1,13,1,1,15,1,1,17,1,1,19,1,1,21,1,1,23,1,1,25,1,1,27,1,1,29,1,1,31,…] =

1.64026267605485331908438582710555719493094320474075230211427846101472425506981375137083325056423510 9… = b

a/b =

0.99999999957666553425997103672161472860705887466641742031698282002506362055852561311125469206593990 97… =

[0; 1, (2362198404=2^2*3*61*751*4297), 1, 1, 2, 2, 6, 2, 1, 1, 39, 3, 2, 2, 39, 3, 1, 6, 1, 22,1, 1, 2, 1, 2, 1, 4, 10, 997, 21, 3, 42, 1, 14, 2, 25, 1, 3, 3, 2, 4, 1, 2, 10, 1,1, 11, 3, 52, 5, 5, 2, 2, 1, 5, 2, 1, 8, 4, 1, 7, 1, 1, 1, 2, 468, 12, 5, 1, 3, 1, 9,1, 7, 72, 1, 3, 1, 1, 1, 1, 34, 6, 2, 1, 11, 2, 12, 1, 1, 16, 5, 1, 5, 3, 2, 1, 1, 1,…]

Part 5

Continued fractions collection of
** natural numbers **A**
** odd n.n **B**
** even n.n. **C**

$$\text{scfA} = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{6+\cfrac{1}{7+\cfrac{1}{8+\cfrac{1}{9+\ldots}}}}}}}}}$$

$$= 0.697774657964007982006790592552\ldots$$

$$= \frac{I_1(2)}{I_0(2)}$$

$$\text{cfA} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cfrac{9}{9+\ldots}}}}}}}}}}$$

$$= 0.581976706...$$

$$= [0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,...]$$

$$= \frac{1}{e-1}$$

$$\text{scfB} = \cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{9+\cfrac{1}{11+\cfrac{1}{13+\cfrac{1}{15+\cfrac{1}{17+\ldots}}}}}}}}}$$

$$= 0.761594155955764888119458282605\ldots ...$$

$$= \tanh 1 = \frac{e^2-1}{e^2+1}$$

$$\mathrm{cfB} = \cfrac{1}{1+\cfrac{3}{3+\cfrac{5}{5+\cfrac{7}{7+\cfrac{9}{9+\cfrac{11}{11+\cfrac{13}{13+\cfrac{15}{15+\cfrac{17}{17+\ldots}}}}}}}}}}$$

$$= 0.562016750958962357308282181493428281335215578656 94...$$

$$= [0; 1, 1, 3, 1, 1, 7, 1, 1, 11, 1, 1, 15, 1, 1, 19, 1, 1, 23, 1, 1, 27, 1, 1, 31, 1, 1, 35, 1, 1, 39, 1, 1, 43, 1, 1, 47, 1, 1, 51, 1, 1, 55, 1, 1, 59, 1, 1, 63, 1, 1, 67, 1, 1, 71, 1, 1, 75, 1, 1, 79, 1, 1, 83, 1, 1, 87, 1, 1, 91, 1, 1, 95, 1, 1, 99, 1, 1, 103, 1, 1, 107, 1, 1, 111, 1, 1, 115, 1, 1, 119, 1, 1, 123, 1, 1, 127, 1, 1, 131, 1, 1, 135, 1, 1, 139, 1, 1, 143, 1, 1, 147, 1, 1, 151, 1, 1, 155, 1, 1, 159, 1, 1, 163, 1, 1, 167, 1, 1, 171, 1, 1, 175, 1, 1, 179, 1, 1, 183, 1, 1, 187, 1, 1, 191, 1, 1, 195, 1, 1, 199, 1, 1, 203, 1, 1, 207, 1, 1, 211, 1, 1, 215, 1, 1, 219, 1, 1, ...]$$

$$\mathrm{scfC} = \cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{6+\cfrac{1}{8+\cfrac{1}{10+\cfrac{1}{12+\cfrac{1}{14+\cfrac{1}{16+\cfrac{1}{18+\ldots}}}}}}}}}$$

$$= 0.44638996589653450704768 17\ldots$$

$$= \frac{I_1(1)}{I_0(1)}$$

$$\mathrm{cfC} = \cfrac{2}{2+\cfrac{4}{4+\cfrac{6}{6+\cfrac{8}{8+\cfrac{10}{10+\cfrac{12}{12+\cfrac{14}{14+\cfrac{16}{16+\cfrac{18}{18+\ldots}}}}}}}}}$$

$$= 0.708874905\ldots$$

$$= [0;1,1,3,1,1,5,1,1,7,1,1,9,1,1,11,1,1,13,1,1,\ldots]$$

- A -

$$\mathrm{cfA} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cfrac{9}{9+\ldots}}}}}}}}}}$$

$= 0.581976706\ldots$

$= [0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,\ldots]$

$= \dfrac{1}{e-1}$

$$\mathrm{cfAo} = \cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cfrac{7}{8+\cfrac{8}{9+\cfrac{9}{10+\ldots}}}}}}}}}}$$

$= 0.392211191\ldots$

$= [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,\ldots]$

$= \dfrac{3-e}{e-2}$

$$\frac{1}{\text{cfA}}-\frac{1}{\text{cfAo}+1}=1\ ;\ \text{cfAo}=\frac{2\text{cfA}-1}{1-\text{cfA}}$$

$$\text{cfAo*}=\cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\cfrac{7}{8+\cfrac{9}{10+\cfrac{11}{12+\cfrac{13}{14+\cfrac{15}{16+\cfrac{17}{18+\ldots}}}}}}}}}$$

$$= 0.379731954740995632802106253634755$$
$$3816125941603590 8...$$

$$= [0; 2, 1, 1, 1, 2, 1, 2, 10, 2, 2, 66, 1, 1, 13, 66,$$
$$9, 5, 8, 1, 9, 1, 1, 1, 5, 2, 2, 3, 1, 1, 1, 16, 99,$$
$$6, 1, 5, 1, 2, 1, 55, 2, 2, 1, 1, 6, 4, 1, 1, 1, 40,$$
$$1, 1, 1, 6, 14, 7, 9, 1, 1, 2, 3, 2, 2, 2, 1, 1, 2, 7,$$
$$12, 1, 2, 2, 1, 4, 2, 4, 2, 1, 3, 2, 1, 10, 7, 1, 4,$$
$$1, 119, 1, 1, 1, 3, 5, 2, 12, 1, 1, 1, 3, 2, 6, 1, 2,$$
$$1, 1, 1, 1, 1, 4, 4, 1, 3, 1, 17, 1, 2, 1, 1, 13, 1,$$
$$1, 1, 2, 5, 1, 1, 1, 3, 2, 1, 1, 1, 1, 1, 2, 1, 4, 5,$$
$$1, 1, 1, 6, 1, 14, 1, 1, 1, 5, 1, 1, 2, 301, 1, 1, 4,$$
$$77, 4, 1, 136, 1, 2, 1, 1, 1, 1, 3, 1, 3, 1, 16, 1,$$
$$2, 2, 22, 1, 1, 1, 1, 146, 1, 6, 1, 203, 3, 15, 23,$$
$$3, 1, 3, 32, 1, 1, 2, 6, 1, 289, 1, 14, 1, 2, 1, ...]?$$

$$\text{cfAo**} = \cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cfrac{12}{13+\cfrac{14}{15+\cfrac{16}{17+\cfrac{18}{19+\ldots}}}}}}}}}}$$

$$= 0.541494082\ldots$$

$$= [0;1,1,5,1,1,9,1,1,13,1,1,17,1,1,21,1,1,\ldots]$$

$$= \frac{2-\sqrt{e}}{\sqrt{e}-1}$$

$$\text{cfAu} = \cfrac{2}{1+\cfrac{3}{2+\cfrac{4}{3+\cfrac{5}{4+\cfrac{6}{5+\cfrac{7}{6+\cfrac{8}{7+\cfrac{8}{8+\cfrac{9}{9+\ldots}}}}}}}}}$$

$$= 1$$

$$\mathrm{cfAu}^* = \cfrac{2}{1+\cfrac{4}{3+\cfrac{6}{5+\cfrac{8}{7+\cfrac{10}{9+\cfrac{12}{11+\cfrac{14}{13+\cfrac{16}{15+\cfrac{18}{17+\ldots}}}}}}}}}$$

$$= 1$$

$$\mathrm{cfAu}^{**} = \cfrac{3}{2+\cfrac{5}{4+\cfrac{7}{6+\cfrac{9}{8+\cfrac{11}{10+\cfrac{13}{12+\cfrac{15}{14+\cfrac{17}{16+\cfrac{19}{18+\ldots}}}}}}}}}$$

$$= 1$$

$$\text{cfa.}1,1 = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\ldots}}}} = [0;1,1,2,1,1,4,1,1,6,1,1,8,\ldots]$$

$$= \frac{1}{e-1} = 0.581976706\ldots$$

$$\text{cfa.}1,2 = \cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\ldots}}}} = [0;2,1,1,4,1,1,6,1,1,8,1,1,\ldots]$$

$$= \frac{3-e}{e-2} = 0.392211191\ldots$$

$$\text{cfa.}1,3 = \cfrac{1}{3+\cfrac{2}{4+\cfrac{3}{5+\cfrac{4}{6+\ldots}}}} = 0.290616693\ldots$$

$$= [0;3,2,3,1,2,1,3,4,3,1,4,1,3,6,3,1,6,1,\ldots]$$

$$\text{cfa.1,4} = \cfrac{1}{4+\cfrac{2}{5+\cfrac{3}{6+\cfrac{4}{7+\ldots}}}} = 0.... = [0;...]$$

$$\text{cfa.1,5} = \cfrac{1}{5+\cfrac{2}{6+\cfrac{3}{7+\cfrac{4}{8+\ldots}}}} = 0.... = [0;...]$$

$$\text{cfa.1,6} = \cfrac{1}{6+\cfrac{2}{7+\cfrac{3}{8+\cfrac{4}{9+\ldots}}}} = 0.... = [0;...]$$

cfa.1,4 = 0.22902536539711983767523123
81033260539220324278767 7…
=
[0; 4, 2, 1, 2, 1, 2, 2, 1, 11, 2, 11, 2, 11, 1, 1, 1, 1, 2, 2, 2,
2, 2, 1, 1, 2, 1, 1, 2, 2, 3, 11, 1, 2, 1, 11, 4, 11, 1, 3, 1, 1,
2, 2, 4, 2, 2, 1, 1, 4, 1, 1, 2, 2, 5, 11, 1, 4, 1, 11, 6, 11, 1,
5, 1, 1, 2, 2, 6, 2, 2, 1, 1, 6, 1, 1, 2, 2, 7, 11, 1, 6, 1, 11,
8, 11, 1, 7, 1, 1, 2, 2, 8, 2, 2, 1, 1, 8, 1, 1, 2, 2, 9, 11, 1,
8, 1, 11, 10, 11, 1, 9, 1, 1, 2, 2, 10, 2, 2, 1, 1, 10, 1, 1, 2,

2, 11, 11, 1, 10, 1, 11, 12, 11, 1, 11, 1, 1, 2, 2, 12, 2, 2, 1, 1, 12, 1, 1, 2, 2, 13, 11, 1, 12, 1, 11, 14, 11, 1, 13, 1, 1, 2, 2, 14, 2, 2, 1, 1, 14, 1, 1, 2, 2, 15, 11, 1, 14, 1, 11, 16, 11, 1, 15, 1, 1, 2, 2, 16, 2, 2, 1, 1, 16, 1, 1, 2, 2, 17, 11, 1, 16, 1, 11, 18, 11, 1, …]

cfa.1,6 = 0.153193128536240689855310492315471782291570396755 61…

=

[0; 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1, 16, 1, 1, 18, 1, 1, 20, 1, 1, 22, 1, 1, 24, 1, 1, 26, 1, 1, 28, 1, 1, 30, 1, 1, 32, 1, 1, 34, 1, 1, 36, 1, 1, 38, 1, 1, 40, 1, 1, 42, 1, 1, 44, 1, 1, 46, 1, 1, 48, 1, 1, 50, 1, 1, 52, 1, 1, 54, 1, 1, 56, 1, 1, 58, 1, 1, 60, 1, 1, 62, 1, 1, 64, 1, 1, 66, 1, 1, 68, 1, 1, 70, 1, 1, 72, 1, 1, 74, 1, 1, 76, 1, 1, 78, 1, 1, 80, 1, 1, 82, 1, 1, 84, 1, 1, 86, 1, 1, 88, 1, 1, 90, 1, 1, 92, 1, 1, 94, 1, 1, 96, 1, 1, 98, 1, 1, 100, 1, 1, 102, 1, 1, 104, 1, 1, 106, 1, 1, 108, 1, 1, 110, 1, 1, 112, 1, 1, 114, 1, 1, 116, 1, 1, 118, 1, 1, 120, 1, 1, 122, 1, 1, 124, 1, 1, 126, 1, 1, 128, 1, 1, 130, 1, 1, 132, 1, 1, 134, 1, 1, 136, 1, 1, 138, 1, 1, 140, 1, 1, 142, 1, …]

$$= \frac{193 - 71e}{39e - 106}$$

$$\mathrm{cfa}.2,1 = \cfrac{2}{1+\cfrac{3}{2+\cfrac{4}{3+\cfrac{5}{4+\ldots}}}} = 1$$

$$\mathrm{cfa}.3,1 = \cfrac{3}{1+\cfrac{4}{2+\cfrac{5}{3+\cfrac{6}{4+\ldots}}}} = \frac{4}{3}$$

$$\mathrm{cfa}.4,1 = \cfrac{4}{1+\cfrac{5}{2+\cfrac{6}{3+\cfrac{7}{4+\ldots}}}} = \frac{21}{13}$$

- B -

$$\mathrm{cfB} = \cfrac{1}{1+\cfrac{3}{3+\cfrac{5}{5+\cfrac{7}{7+\cfrac{9}{9+\cfrac{11}{11+\cfrac{13}{13+\cfrac{15}{15+\cfrac{17}{17+\ldots}}}}}}}}}$$

$= 0.562016750958962357308282181493428281335215578656 94...$

$= [0; 1, 1, 3, 1, 1, 7, 1, 1, 11, 1, 1, 15, 1, 1, 19, 1, 1, 23, 1, 1, 27, 1, 1, 31, 1, 1, 35, 1, 1, 39, 1, 1, 43, 1, 1, 47, 1, 1, 51, 1, 1, 55, 1, 1, 59, 1, 1, 63, 1, 1, 67, 1, 1, 71, 1, 1, 75, 1, 1, 79, 1, 1, 83, 1, 1, 87, 1, 1, 91, 1, 1, 95, 1, 1, 99, 1, 1, 103, 1, 1, 107, 1, 1, 111, 1, 1, 115, 1, 1, 119, 1, 1, 123, 1, 1, 127, 1, 1, 131, 1, 1, 135, 1, 1, 139, 1, 1, 143, 1, 1, 147, 1, 1, 151, 1, 1, 155, 1, 1, 159, 1, 1, 163, 1, 1, 167, 1, 1, 171, 1, 1, 175, 1, 1, 179, 1, 1, 183, 1, 1, 187, 1, 1, 191, 1, 1, 195, 1, 1, 199, 1, 1, 203, 1, 1, 207, 1, 1, 211, 1, 1, 215, 1, 1, 219, 1, 1, ...]$

$$\mathrm{cfBo} = \cfrac{1}{3+\cfrac{3}{5+\cfrac{5}{7+\cfrac{7}{9+\cfrac{9}{11+\cfrac{11}{13+\cfrac{13}{15+\cfrac{15}{17+\cfrac{17}{19+\ldots}}}}}}}}}}$$

$= 0.28319234169228049286689887344036139$
$57353949653724 7...$

$= [0; 3, 1, 1, 7, 1, 1, 11, 1, 1, 15, 1, 1, 19, 1, 1, 23,$
$1, 1, 27, 1, 1, 31, 1, 1, 35, 1, 1, 39, 1, 1, 43, 1, 1,$
$47, 1, 1, 51, 1, 1, 55, 1, 1, 59, 1, 1, 63, 1, 1, 67,$
$1, 1, 71, 1, 1, 75, 1, 1, 79, 1, 1, 83, 1, 1, 87, 1, 1,$
$91, 1, 1, 95, 1, 1, 99, 1, 1, 103, 1, 1, 107, 1, 1,$
$111, 1, 1, 115, 1, 1, 119, 1, 1, 123, 1, 1, 127, 1,$
$1, 131, 1, 1, 135, 1, 1, 139, 1, 1, 143, 1, 1, 147,$
$1, 1, 151, 1, 1, 155, 1, 1, 159, 1, 1, 163, 1, 1, 167,$
$1, 1, 171, 1, 1, 175, 1, 1, 179, 1, 1, 183, 1, 1, ...]$

$$\mathrm{cfBo}^* = \cfrac{1}{3+\cfrac{5}{7+\cfrac{9}{11+\cfrac{13}{15+\cfrac{17}{19+\cfrac{21}{23+\cfrac{25}{27+\cfrac{29}{31+\cfrac{33}{35+\ldots}}}}}}}}}$$

$$= 0.27440778476725757131383250682537$$
$$70269288850299139 1...$$

$$= [0; 3, 1, 1, 1, 4, 3, 1, 1, 4, 4, 2, 2, 1, 1, 2, 1, 5,$$
1, 1, 1, 3, 1, 2, 3, 2, 3, 182, 1, 4, 1, 7, 3, 5, 10,
1, 6, 29, 1, 2, 8, 1, 1, 1, 1, 6, 25, 1, 9, 1, 3, 1,
2, 2, 2, 2, 1, 1, 3, 2, 7, 1, 6, 1, 2, 8, 1, 3, 5, 1,
15, 14, 1, 1, 1, 3, 3, 2, 1, 1, 1, 9, 1, 2, 72, 1, 4,
2, 2, 1, 1, 1, 4, 1, 33, 1, 11, 1, 1, 5, 5, 2, 2, 1,
3, 1, 1, 4, 2, 1, 1, 1, 3, 2, 1, 18, 1, 1, 2, 2, 1,
121, 1, 6, 1, 1, 9, 1, 2, 1, 3, 2, 4, 1, 3, 2, 1, 2,
1, 5, 2, 1, 3, 11, 2, 5, 2, 3, 1, 2, 1, 3, 1, 2, 4, 1,
2, 5, 86, 4, 1, 2, 2, 2, 1, 3, 45, 2, 1, 12, 1, 13,
2, 16, 28, 3, 1, 2, 298, 2, 5, 1, 3, 1, 58, 25, 1,
1, 2, 2, 7, 1, 3, 2, 1, 1, 3, 1, 3, 5, 1, 1, 1, 3, 1,
5, 26, 1, 2, 3, 20, 8, 4, 2, 1, 1, 3, 1, 14, 1, ...]

$$\text{cfBo**} = \cfrac{3}{5+\cfrac{7}{9+\cfrac{11}{13+\cfrac{15}{17+\cfrac{19}{21+\cfrac{23}{25+\cfrac{27}{29+\cfrac{31}{33+\cfrac{35}{37+\ldots}}}}}}}}}}$$

$$= 0.52496534441789491282122309406255623246846042999947\ldots$$

$$= [0; 1, 1, 9, 1, 1, 17, 1, 1, 25, 1, 1, 33, 1, 1, 41, 1, 1, 49, 1, 1, 57, 1, 1, 65, 1, 1, 73, 1, 1, 81, 1, 1, 89, 1, 1, 97, 1, 1, 105, 1, 1, 113, 1, 1, 121, 1, 1, 129, 1, 1, 137, 1, 1, 145, 1, 1, 153, 1, 1, 161, 1, 1, 169, 1, 1, 177, 1, 1, 185, 1, 1, 193, 1, 1, 201, 1, 1, 209, 1, 1, 217, 1, 1, 225, 1, 1, 233, 1, 1, 241, 1, 1, 249, 1, 1, 257, 1, 1, 265, 1, 1, 273, 1, 1, 281, 1, 1, 289, 1, 1, 297, 1, 1, 305, 1, 1, 313, 1, 1, 321, 1, 1, 329, 1, 1, 337, 1, 1, 345, 1, 1, 353, 1, 1, 361, 1, 1, 369, 1, 1, 377, 1, 1, 385, 1, 1, ...]$$

$$\mathrm{cfBu} = \cfrac{3}{1+\cfrac{5}{3+\cfrac{7}{5+\cfrac{9}{7+\cfrac{11}{9+\cfrac{13}{11+\cfrac{15}{13+\cfrac{17}{15+\cfrac{19}{17+\ldots}}}}}}}}}$$

$= 1.359802...$

$= [1;2,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,...]$

$$\mathrm{cfBu}^* = \cfrac{3}{1+\cfrac{7}{5+\cfrac{11}{9+\cfrac{15}{13+\cfrac{19}{17+\cfrac{23}{21+\cfrac{27}{25+\cfrac{31}{29+\cfrac{35}{33+\ldots}}}}}}}}}$$

$= 1.39604504...$

$= [1;2,1,1,9,1,1,17,1,1,25,1,1,...]$

$$\text{cfBu**} = \cfrac{5}{3+\cfrac{9}{7+\cfrac{13}{11+\cfrac{17}{15+\cfrac{21}{19+\cfrac{25}{23+\cfrac{29}{27+\cfrac{33}{31+\cfrac{37}{35+\ldots}}}}}}}}}}$$

$$= 1.215321805...$$

$$= [1;4,1,1,14,3,1,1,4,4,2,...]$$

$$\frac{1}{\text{cfBu}-1} - \frac{1}{\text{cfB}} = \frac{1}{\text{cfB}} - \frac{1}{\text{cfBo}+1} = 1 \rightarrow$$

$$\text{cfB} = \frac{\text{cfBo}+1}{2+\text{cfBo}} = \frac{\text{cfBu}-1}{2-\text{cfBu}} \ ;$$

$$\text{cfBo} = \frac{2\text{cfB}-1}{1-\text{cfB}} \ ; \ \text{cfBu} = \frac{2\text{cfB}+1}{1+\text{cfB}}$$

$$\frac{1}{\text{cfBu}-1} + \frac{1}{\text{cfBo}+1} = \frac{2}{\text{cfB}} \ ; \ \frac{1}{\text{cfBu}-1} - \frac{1}{\text{cfBo}+1} = 2$$

$$(\text{cfBo**} + 2)(\text{cfBu*} - 1) = 1$$

- C -

$$cfC = \cfrac{2}{2+\cfrac{4}{4+\cfrac{6}{6+\cfrac{8}{8+\cfrac{10}{10+\cfrac{12}{12+\cfrac{14}{14+\cfrac{16}{16+\cfrac{18}{18+\ldots}}}}}}}}}$$

$= 0.708874905...$

$= [0;1,1,3,1,1,5,1,1,7,1,1,9,1,1,11,1,1,13,1,1,...]$

$$cfCo = \cfrac{2}{4+\cfrac{4}{6+\cfrac{6}{8+\cfrac{8}{10+\cfrac{10}{12+\cfrac{12}{14+\cfrac{14}{16+\cfrac{16}{18+\cfrac{18}{20+\ldots}}}}}}}}}$$

$= 0.434949564...$

$= [0;...]$

$$\mathrm{cfCo}^* = \cfrac{2}{4+\cfrac{6}{8+\cfrac{10}{12+\cfrac{14}{16+\cfrac{18}{20+\cfrac{22}{24+\cfrac{26}{28+\cfrac{30}{32+\cfrac{34}{36+\ldots}}}}}}}}}$$

$= 0.427038590216724870861404731039564014614151970686 52...$

$= [0; 2, 2, 1, 12, 1, 1, 2, 126, 7, 4, 1, 122, 1, 2, 1, 34, 2, 3, 1, 7, 3, 1, 1, 3, 1, 2, 13, 1, 2, 3, 2, 1, 1, 3, 1, 1, 5, 2, 9, 1, 36, 1, 1, 20, 1, 17, 45, 1, 18, 1, 2, 1, 46, 1, 4, 4, 4, 2, 3, 1, 6, 2, 2, 1, 9, 1, 1, 5, 7, 2, 1, 2, 1, 39, 59, 4, 2, 7, 1, 1, 1, 5, 7, 1, 1, 2, 1, 1, 23, 1, 11, 1, 2, 1, 1, 45, 1, 3, 10, 1, 1, 3, 20, 5, 3, 1, 1, 8, 1, 3, 1, 18, 3, 1, 11, 4, 5, 3, 4, 5, 8, 1, 1, 1, 2, 1, 9, 3, 4, 3, 2, 3, 1, 6, 1, 32, 1, 1, 1, 3, 2, 9, 1, 1, 2, 1, 4, 1, 11, 1, 2, 5, 110, 1, 1, 1, 9, 37, 7, 4, 1, 2, 4, 1, 9, 107, 1, 9, 2, 1, 3, 1, 1, 6, 8, 6, 1, 2, 2, 1, 2, 1, 2, 1, 23, 8, 2, 85, 1, 9, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 10, 3, 1, 12, 1, 2, 5, 5, 3, 1, 21, 2, 1, 10, 1, 16, 1, 2, 2, 1, 2, 6, 1, 1, 3, 18, 2, 2, 1, 3, 1, ...]$

$$\text{cfCo**} = \cfrac{4}{6+\cfrac{8}{10+\cfrac{12}{14+\cfrac{16}{18+\cfrac{20}{22+\cfrac{24}{26+\cfrac{28}{30+\cfrac{32}{34+\cfrac{36}{38+\ldots}}}}}}}}}}$$

$$= 0.59345609320911987645213140550843012415090977140811...$$

$$= [0; 1, 1, 2, 5, 1, 2, 2, 11, 2, 1, 2, 9, 1, 1, 4, 1, 3, 3, 129, 1, 2, 1, 4, 9, 1, 1, 16, 2, 1, 1, 7, 2, 2, 1, 1, 5, 1, 3, 1, 2, 4, 3, 23, 1, 2, 1, 3, 4, 10, 1, 1, 1, 3, 1, 8, 2, 1, 2, 12, 2, 2, 4, 1, 1, 1, 1, 1, 1, 1, 6, 1, 18, 3, 1, 3, 1, 5, 2, 1, 1, 6, 1, 3, 1, 2, 2, 1, 3, 1, 1, 3, 11, 3, 2, 10, 3, 5, 1, 1, 3, 5, 162, 1, 2, 1, 9, 4, 1, 6, 1, 1, 2, 1, 1, 1, 9, 3, 1, 2, 1, 1, 3, 2, 5, 1, 172, 3, 1, 5, 1, 4, 3, 1, 7, 1, 6, 1, 1, 2, 3, 1, 1, 6, 7, 2, 1, 11, 1, 3, 9, 3, 1, 1, 4, 5, 3, 9, 45, 4, 2, 1, 1, 1, 1, 71, 1, 3, 4, 1, 1, 1, 1, 3, 1, 1, 4, ...]$$

$$\mathrm{cfCu} = \cfrac{4}{2+\cfrac{6}{4+\cfrac{8}{6+\cfrac{10}{8+\cfrac{12}{10+\cfrac{14}{12+\cfrac{16}{14+\cfrac{18}{16+\cfrac{20}{18+\ldots}}}}}}}}}}$$

$$= 1.26168609848292345958173047652448556$$
$$4819024968680\ldots$$

$$= [1; 3, 1, 4, 1, 1, 2, 22, 3, 1, 4, 1, 1, 21, 2, 1, 1, 1,$$
$$2, 1, 8, 3, 5, 1, 3, 145, 2, 75, 1, 2, 3, 18, 3, 1, 3,$$
$$36, 7, 1, 2, 5, 5, 1, 4, 2, 1, 10, 2, 2, 4, 1, 3, 12, 8,$$
$$1, 6, 1, 1, 1, 4, 4, 2, 1, 1, 6, 4, 24, 1, 1, 4, 1, 1, 1,$$
$$9, 10, 3, 2, 2, 1, 1, 2, 4, 2, 2, 1, 210, 1, 45, 4, 1,$$
$$1, 6, 14, 1, 4, 1, 1, 1, 1, 1, 3, 5, 1, 4, 2, 4, 1, 2, 2,$$
$$2, 1, 9, 1, 1, 2, 3, 1, 1, 6, 3, 1, 114, 2, 1, 3, 1, 2, 1,$$
$$7, 5, 5, 1, 6, 1, 10, 2, 36, 6, 1, 6, 28, 2, 1, 5, 1, 1,$$
$$7, 13, 29, 1, 10, 5, 1, 1, 10, 1, 1, 1, 90, 2, 27, ...]$$

$$\mathrm{cfCu}^* = \cfrac{4}{2+\cfrac{8}{6+\cfrac{12}{10+\cfrac{16}{14+\cfrac{20}{18+\cfrac{24}{22+\cfrac{28}{26+\cfrac{32}{30+\cfrac{36}{34+\ldots}}}}}}}}}$$

$$= 1.27828362836818592568705530830352189293325794865 49\ldots$$

$$= [1; 3, 1, 1, 2, 5, 1, 2, 2, 11, 2, 1, 2, 9, 1, 1, 4, 1, 3, 3, 129, 1, 2, 1, 4, 9, 1, 1, 16, 2, 1, 1, 7, 2, 2, 1, 1, 5, 1, 3, 1, 2, 4, 3, 23, 1, 2, 1, 3, 4, 10, 1, 1, 1, 3, 1, 8, 2, 1, 2, 12, 2, 2, 4, 1, 1, 1, 1, 1, 1, 1, 6, 1, 18, 3, 1, 3, 1, 5, 2, 1, 1, 6, 1, 3, 1, 2, 2, 1, 3, 1, 1, 3, 11, 3, 2, 10, 3, 5, 1, 1, 3, 5, 162, 1, 2, 1, 9, 4, 1, 6, 1, 1, 2, 1, 1, 1, 9, 3, 1, 2, 1, 1, 3, 2, 5, 1, 172, 3, 1, 5, 1, 4, 3, 1, 7, 1, 6, 1, 1, 2, 3, 1, 1, 6, 7, 2, 1, 11, 1, 3, 9, 3, 1, 1, 4, 5, 3, 9, 45, 4, 2, 1, 1, 1, 1, 71, 1, 3, 4, 1, 1, 1, 1, 3, 1, 1, 4, 74, 5, 1, 4, 85, 3, 2, 6, 1, 11, 1, 2, 3, 1, 3, 1, 44, 3, 6, ...]$$

$$\text{cfCu}^{**} = \cfrac{6}{4+\cfrac{10}{8+\cfrac{14}{12+\cfrac{18}{16+\cfrac{22}{20+\cfrac{26}{24+\cfrac{30}{28+\cfrac{34}{32+\cfrac{38}{36+\ldots}}}}}}}}}}$$

$$= 1.17595047393893808137553569625596552386399501281 88\ldots$$

$$= [1; 5, 1, 2, 6, 3, 2, 1, 62, 1, 1, 3, 9, 1, 60, 1, 6, 1, 16, 1, 2, 1, 1, 2, 3, 1, 1, 1, 3, 1, 1, 2, 1, 27, 2, 1, 1, 5, 8, 11, 1, 19, 1, 17, 1, 3, 10, 2, 8, 1, 1, 22, 2, 9, 2, 1, 2, 23, 2, 2, 8, 2, 4, 1, 1, 13, 1, 5, 1, 4, 3, 1, 2, 14, 1, 2, 1, 2, 19, 1, 1, 29, 8, 1, 15, 3, 2, 1, 1, 3, 3, 2, 3, 11, 1, 24, 2, 3, 22, 1, 7, 5, 3, 1, 1, 40, 2, 1, 1, 1, 3, 1, 3, 1, 8, 1, 8, 1, 1, 1, 2, 5, 1, 1, 1, 1, 1, 2, 6, 2, 10, 4, 3, 6, 1, 4, 6, 2, 6, 1, 7, 1, 2, 1, 66, 3, 1, 1, 2, 1, 4, 3, 2, 10, 1, 5, 2, 1, 10, 55, 3, 19, 18, 1, 1, 3, 9, 2, 1, 1, 1, ...]$$

$$\frac{1}{cfCu-1}-\frac{2}{cfC}=1$$

$$\rightarrow\ cfC=\frac{2cfCu-2}{2-cfCu}\ ;\ cfCu=\frac{2cfC+2}{cfC+2}$$

$$\frac{1}{cfC}-\frac{1}{cfCo+2}=1$$

$$\rightarrow\ cfC=\frac{cfCo+2}{cfCu+3}\ ;\ cfCo=\frac{3cfC-2}{1-cfC}$$

(cfCo** + 3)(cfCu* - 1) = 1

FERROFERROFERROFERROFERRO

$$\text{cfA} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{4+\cfrac{5}{5+\cfrac{6}{6+\cfrac{7}{7+\cfrac{8}{8+\cfrac{9}{9+\ldots}}}}}}}}}}$$

$$= 0.581976706\ldots$$

$$= [0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,\ldots]$$

$$= \frac{1}{e-1}$$

$$\text{cfAo} = \cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7+\cfrac{7}{8+\cfrac{8}{9+\cfrac{9}{10+\ldots}}}}}}}}}}$$

$$= 0.392211191\ldots$$

$$= [0;2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,\ldots]$$

$$= \frac{3-e}{e-2}$$

$$\text{cfa.1,6} = \cfrac{1}{6+\cfrac{2}{7+\cfrac{3}{8+\cfrac{4}{9+\ldots}}}} =$$

0.153193128536240689855310492315471782291570396755 61… =
[0, 6,1,1,8,1,1,10,1,1,12,1,1,14,1,1,16,1,1, 18,1,1,…]

$$\text{cfB} = \cfrac{1}{1+\cfrac{3}{3+\cfrac{5}{5+\cfrac{7}{7+\cfrac{9}{9+\cfrac{11}{11+\cfrac{13}{13+\cfrac{15}{15+\cfrac{17}{17+\ldots}}}}}}}}}$$

= 0.562016750958962357308282181493428281335215578656 94...

= [0;1,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,...]

$$\text{cfC} = \cfrac{2}{2+\cfrac{4}{4+\cfrac{6}{6+\cfrac{8}{8+\cfrac{10}{10+\cfrac{12}{12+\cfrac{14}{14+\cfrac{16}{16+\cfrac{18}{18+\ldots}}}}}}}}}}$$

$$= 0.708874905\ldots$$

$$= [0;1,1,3,1,1,5,1,1,7,1,1,9,1,1,11,1,1,13,1,1,\ldots]$$

$$\text{cfAo**} = \cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\cfrac{10}{11+\cfrac{12}{13+\cfrac{14}{15+\cfrac{16}{17+\cfrac{18}{19+\ldots}}}}}}}}}}$$

$$= 0.541494082\ldots$$

$$= [0;1,1,5,1,1,9,1,1,13,1,1,17,1,1,21,1,1,\ldots]$$

$$= \frac{2-\sqrt{e}}{\sqrt{e}-1}$$

$$\mathrm{cfBo} = \cfrac{1}{3+\cfrac{3}{5+\cfrac{5}{7+\cfrac{7}{9+\cfrac{9}{11+\cfrac{11}{13+\cfrac{13}{15+\cfrac{15}{17+\cfrac{17}{19+\ldots}}}}}}}}}}$$

$$= 0.28319234169228049286689887344036139$$
$$57353949653724 7...$$

$$= [0;3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,27,1,1,...]$$

$$\mathrm{cfBo}^{**} = \cfrac{3}{5+\cfrac{7}{9+\cfrac{11}{13+\cfrac{15}{17+\cfrac{19}{21+\cfrac{23}{25+\cfrac{27}{29+\cfrac{31}{33+\cfrac{35}{37+\ldots}}}}}}}}}}$$

$$= 0.52496534441789491282122309406255 6$$
$$2324684604299994 7...$$

$$= [0;1,1,9,1,1,17,1,1,25,1,1,33,1,1,41,1,1,49,1,1,...]$$

$$\mathrm{cfBu} = \cfrac{3}{1+\cfrac{5}{3+\cfrac{7}{5+\cfrac{9}{7+\cfrac{11}{9+\cfrac{13}{11+\cfrac{15}{13+\cfrac{17}{15+\cfrac{19}{17+\ldots}}}}}}}}}$$

$= 1.359802...$

$= [1;2,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,1,1,...]$

$$\mathrm{cfBu}^* = \cfrac{3}{1+\cfrac{7}{5+\cfrac{11}{9+\cfrac{15}{13+\cfrac{19}{17+\cfrac{23}{21+\cfrac{27}{25+\cfrac{31}{29+\cfrac{35}{33+\ldots}}}}}}}}}$$

$= 1.39604504...$

$= [1;2,1,1,9,1,1,17,1,1,25,1,1,33,1,1,41,1,1,...]$