Supplement 2 to Deb 20

Even more about scfs

The simple continued fraction (scf)

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}} \qquad \ldots + \cfrac{1}{a_n} = [a_0; a_1, a_2, a_3, \ldots, a_n]$$

$n \to \infty$: infinite scf

is a very efficient tool to analyze the "inner discrete (wave) structure" of mathematical constants and functions.

Algorithm: transformation $r \in \mathbb{R}_+ \rightarrow$ scf(r)

$r \in \mathbb{R}_+$; length(approx.scf(r))	
x:=r ; i:=0	
a_0:=int(x)	
repeat until	$x:=(x - int(x))^{-1}$
	i:=i+1
	a_i:=int(x)
x=int(x) or i=length	
n:=i	
approx.scf(r)=$[a_0;a_1,a_2,\ldots a_n]$	

f.e.

$e = 2.7182818284\ldots = [2;\ 1,\ 2,\ 1,\ 1,\ 4,\ 1,\ 1,\ 6,\ 1,\ 1,\ 8,\ 1,\ldots]$

$\Phi = \frac{1}{2}(\sqrt{5}+1) = 1.618033989... = [1;\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ldots]$

$\sqrt{2} = [1;\ 2,\ 2,\ 2,\ 2,\ 2,\ 2,\ 2,\ 2,\ 2,\ldots]$

$5\sqrt{2} = [7;\ 14,\ 14,\ 14,\ 14,\ldots]$

$\sqrt{55} = [7;\ 2,\ 2,\ 2,\ 14,\ 2,\ 2,\ 2,\ 14,\ 2,\ 2,\ 2,\ 14,\ \ldots]$

$29\sqrt{2} = [41;\ 82,\ 82,\ 82,\ 82,\ldots]$

$½(\sqrt{2}+1) = [1;\ 4,\ 1,\ 4,\ 1,\ 4,\ 1,\ 4,\ 1,\ 4,\ldots]$

$\frac{5}{2}\sqrt{2} = [3;\ 1,\ 1,\ 6,\ 1,\ 1,\ 6,\ 1,\ 1,\ 6,\ 1,\ 1,\ 6,...]$

$\sqrt{2}+½ = [1;\ 1,\ 10,\ 1,\ 1,\ 1,\ 10,\ 1,\ 1,\ 1,\ 10,\ldots]$

$\frac{1}{4}\sqrt{22*23} = [5;\ 1,\ 1,\ 1,\ 1,\ 1,\ 10,\ 1,\ 1,\ 1,\ 1,\ 1,\ 10,\ ...]$

$\frac{278+197\sqrt{2}}{334} = [1;\ 1,\ 1,\ 1,\ 556,\ 1,\ 1,\ 1,\ 1,\ 556,\ ...]$

$\sqrt{3} = [1;\ 1,\ 2,\ 1,\ 2,\ 1,\ 2,\ 1,\ 2,\ 1,\ldots]$

$\sqrt{5} = [2;\ 4,\ 4,\ 4,\ 4,\ 4,\ 4,\ 4,\ 4,\ 4,\ldots]$

$\frac{515+233\sqrt{5}}{622} = [1;\ 1,\ 1,\ 1,\ 103,\ 1,\ 1,\ 1,\ 1,\ 103,\ ...]$

$\sqrt{7} = [2;\ 1,\ 1,\ 1,\ 4,\ 1,\ 1,\ 1,\ 4,\ 1,\ 1,\ 1,\ 4,\ldots]$

$\sqrt{21} = [4;\ 1,\ 1,\ 2,\ 1,\ 1,\ 8,\ 1,\ 1,\ 2,\ 1,\ 1,\ 8,\ \ldots]$

$\sqrt{112} = [10;\ 1,\ 1,\ 2,\ 1,\ 1,\ 20,\ 1,\ 1,\ 2,\ 1,\ 1,\ 20,\ \ldots]$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

An application field

$$\alpha = \frac{2393+144*\Phi}{1479+89*\Phi} = 1.617984451...$$

$= [1;\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 17,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ldots]$

For comparision

$$\Phi = \frac{1}{2}(\sqrt{5}+1) = \quad 1.618033989...$$
$$= [1; 1, 1, 1, 1, 1, 1, 1, 1, \ldots]$$

Obwohl die numerischen Werte von Φ und α sehr nahe beieinander liegen, sind ihre scf Darstellungen sehr unterschiedlich:

Φ: „small noise 1 / calm sea 1“,

α: „large signal 17 / massive energy burst 17 / monster wave 17“ in a background of small noise 1 / calm sea 1”.

Question 1

We change α to $\alpha' = 1.6179k4451\ldots$ (small change), k=0,1,2,…,9 .
What is the scf reaction? Extension?

Question 2

How we can design a positive real number with an α type scf?

Answer 1

$$\frac{1}{2[afib(n-1)+fib(n-2)]} * \left[afib(n)+\sqrt{[afib(n)]^2+4afib(n-1)fib(n)+4fib(n-2)fib(n)} \right] =$$

$$[1; \underbrace{1, 1, ...,1}_{n \text{ times}}, a, 1^{\underline{n}}, a, 1^{\underline{n}}, a, 1^{\underline{n}}, ...]$$

$fib(0) = fib(1) = 1$; $fib(i+2) = fib(i+1)+fib(i)$

Problem:

Given n. Find a with

$$\sqrt{[afib(n)]^2+4afib(n-1)fib(n)+4fib(n-2)fib(n)} = A\sqrt{2} \ \underline{or} \ B\sqrt{5} \ ;$$
$$A,B \in \mathbb{N} .$$

Example

n=4 $\rightarrow$ a=556, A=1970

Answer 2

$$\frac{afib(m)+Tfib(m)+fib(m-1)}{afib(m-1)+Tfib(m-1)+fib(m-2)} =$$

$[1; \underbrace{1, 1, ...,1}_{m \text{ times}}, a, 1, 1, 1, 1, ...] = [1^{\underline{m}}, a, 1^{\underline{\infty}}]$

$fib(0) = fib(1) = 1 ; \; fib(i+2) = fib(i+1)+fib(i)$

$$T = \Phi^{-1} = \frac{1}{2}(\sqrt{5} - 1)$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

The above example and its interpretation shows that the "scf coding" can be used as a tool to model

** monster waves in a calm sea,

the α example shows a **Rogue Wave** of the high 17 within normal sea waves of the high 1. The next example models another form of monster waves, the **Three Sisters** 17, 17, 17 within normal sea waves 1

$$\beta = \frac{2405 + 41026 * \Phi}{1501 + 25605 * \Phi} = 1.602265182...$$

= [1; 1, 1, 1, 1, 17, 17, 17, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,…] ,

http://en.wikipedia.org/wiki/Monster_waves

see also

M. Farhat, S. Guenneau, S. Enoch, G. Tayeb. Analytical and numerical analysis of lensing effect for linear surface water waves through a square array of nearly touching rigid square cylinders. Physcal Review E 77 (2008), 046308

** a large sharp signal within a background of low noise,

** local-temporal closely limited massive energy impact/burst (Fiktivtransmitter, …),

f.e.

$$\text{FibonacciBomb}(m,n) = \text{scf}\left[\left(\frac{\text{fib}(n+1)}{\text{fib}(n)}\right)^{m}\right] ;\ m,n \in \mathbb{N}$$

$$\text{fib}(0) = 0 ;\ \text{fib}(1) = \text{fib}(2) = 1$$

$$\text{fib}(n+2) = \text{fib}(n+1) + \text{fib}(n)$$

$$\frac{\text{fib}(n+1)}{\text{fib}(n)} = [1;\ \underbrace{1,\ 1,\ \ldots\ 1,\ 1}_{\text{n times}}]$$

$$\lim_{n\to\infty} \frac{\text{fib}(n+1)}{\text{fib}(n)} = \frac{1}{2}(1+\sqrt{5}) = \Phi$$

$$\Phi^{m} = \text{fib}(m-1) + \text{fib}(m)*\Phi$$

$$\Phi^{1} = [1;1,1,1,1,1,1,1,1,1,1,1,1,1,1,\ldots]$$

$$\Phi^{2} = [2;1,1,1,1,1,1,1,1,1,,1,1,1,1,1,\ldots]$$

$$\Phi^{3} = [4;4,4,4,4,4,4,4,4,4,4,4,\ldots]$$

$$\Phi^{4} = [6;1,5,1,5,1,5,1,5,1,5,1,5,\ldots]$$

$$\Phi^{5} = [11,11,11,11,11,11,11,11,\ldots]$$

$$\Phi^{6} = [17;1,16,1,16,1,16,1,16,1,\ldots]$$

FibonacciBomb(5,50) =

[11, 11, 11, 11, 11, 11, 11, 11, 11, 13, 437845991669110338 04, 1, 5, 1, 10, 11, 11, 11, 11, 11, 11, 11, 11, 4, 1, 10, 11, 11, 11, 11, 11, 11, 11, 11, ...]

FibonacciBomb(5,55) =

[11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 12, 1, 538514970164685621 5201, 6, 1, 10, 11, 11, 11, 11, 11, 11, 11, 11, 11, 4, 1, 10, 11, 11, 11, 11, 11, 11, 11, 11, 11, ...]

FibonacciBomb(5,99) =

[11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 8, 132467738706170972563516284311410413798 15, 1, 1, 5, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 1, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, ...]

http://research.microsoft.com/en-us/um/people/cohn/contfrac.html
http://research.microsoft.com/en-us/um/people/cohn/Papers/monthly.pdf