In the debating club n °18 ext.elcam

Spaced simple continued fractions

18.1

Analyze and discuss the transcendent numbers z represented by the infinite simple continued fraction type

$[a_0;a,a_1,a,a,a_2,a,a,a,a_3,a,a,a,a,a_4,a^{\underline{5}},a_5,a^{\underline{6}},a_6,\ldots]=$

$$[a_0;a^{\underline{i}},a_i,...]_{i=1}^{\infty} \qquad a,a_i \in \mathbb{N} \qquad (1)$$

$a^{\underline{i}} = a,a,a,a,...,a$ – sequence of i scf-elements a

esp.

a = 1; $a_i = i;$ $i=1,2,3,\ldots$ (2)

$a_i = 2i$ (3)

$a_i = 2i-1$ (4)

$a_i = fib(i)$ - Fibonacci sequence (5)

$a_i = p_i$ - prime sequence (6)

$a_i = i^2$ (7)

$a_i = 2^i$ (8)

more.

F.e. with $a_0 = 1$

** [1;1,1,1,1,2,1,1,1,1,3,1,1,1,1,1,4,$1^{\underline{5}}$,5,$1^{\underline{6}}$,6,…] =

1.6123431202373869274614487428752341440307013162076464706686373495020479566449013356271033206381256 0… (2.1)

** [1;1,2,1,1,4,1,1,1,1,6,1,1,1,1,1,8,$1^{\underline{5}}$,10,$1^{\underline{6}}$,12,…] =
1.71819321776910799750140696382074086698464417329
03638591455827881286416486003737037915562606364790
7… (3.1)
** [1;1,1,1,1,3,1,1,1,1,5,1,1,1,1,1,7,$1^{\underline{5}}$,9,$1^{\underline{6}}$,11,…] =
1.60941409071333653060188965553205139559947383280 0
71420839654154695876433873085652762412486556728768
5… (4.1)
** [1;1,1,1,1,2,1,1,1,1,3,1,1,1,1,1,5,$1^{\underline{5}}$,8,$1^{\underline{6}}$,13,…] =
1.61234314877723042035579675242069559107828177672
89271205670199903916035242199111076160957718972449
5… (5.1)
** [1;1,2,1,1,3,1,1,1,1,5,1,1,1,1,1,7,$1^{\underline{5}}$,11,$1^{\underline{6}}$,13,…] =
1.71912133822328112250698483057449586761720164873 4
12313587692653829183001883938494384656704356976662
0… (6.1)
** [1;1,1,1,1,4,1,1,1,1,9,1,1,1,1,1,16,$1^{\underline{5}}$,25,$1^{\underline{6}}$,36,…] =
1.60761057219113500264317649478198521794154062409 4
92607639200862124214048343082608542636432436441882
7… (7.1)
** [1;1,2,1,1,4,1,1,1,1,8,1,1,1,1,1,16,$1^{\underline{5}}$,32,$1^{\underline{6}}$,64,…] =
1.71819075488409302539358218009337079943930561150 5
33354312136588177185501574367031137026362114332140
2… (8.1)

18.2

Analyze and discuss the transcendent numbers s_k represented by the infinite simple continued fractions

$s_0 = [fib(0);fib(1),fib(2),fib(3),fib(4), \ldots]$ (10)

$s_1 = [fib(0);1,fib(1),1,fib(2),1,fib(3),1,fib(4),\ldots]$ (11)

$s_{2} = [fib(0);1,1,fib(1),1,1,fib(2),1,1,fib(3),1,1,fib(4),\ldots]$ (12)

$s_{k} = [fib(0);1^{\underline{k}},fib(1),1^{\underline{k}},fib(2),1^{\underline{k}},fib(3),1^{\underline{k}},fib(4),\ldots]$ (13)

s_0 = [1;1,2,3,5,8,13,21,34,55,89,144,233,…] =
1.6981562788972284466931540436689336838294864493643766226750196575685068035717024085832623433947448 20… (10.1)
s_1 = [1;1,1,1,2,1,3,1,5,1,8,1,13,1,21,1,34,1,…] =
1.6345600864418520915071916466994457447530392296651255355666606277502368225262423136287724667335710 26… (11.1)
s_2 = [1;1,1,1,1,1,2,1,1,3,1,1,5,1,1,8,1,1,13,…] =
1.6200970526320895787798476370956507107823919321878044239061329227681647777800311026646954229656584 89… (12.1)
s_3 = [1;1,1,1,1,1,1,1,2,1,1,1,3,1,1,1,5,1,1,1,8,…] =
1.6183520376437176112178734843232062267816295172423133342453528028096414956445145787753726840307038 97… (13.1)
s4 =
1.6180793986784528581900440830403652451939180635742981302192055548203819956837761774419149903930329 64… (13.2)
s5 =
1.6180406683380980574819810412196856469032021438354875505615024715234478308929670420591334384727453 01… (13.3)
s6 =
1.6180349602388819692122892045073844766169942281168653212140867814806041520664386497401190627258007 30… (13.4)
s7 =

1.6180341306577596843209340381809017873605018520318065805009908911552902914050406461070568104660607776 (13.5)

s8 = 1.6180340094445165445786845696624526178134467554516136183798107417812842359693971197499816458451495961… (13.6)

s9 = 1.6180339917697262765428514298947534190063845182911148025383294686369437892231558166461717234126328681… (13.7)

s10 = 1.6180339891904529534249151799353550702631645903560492216329159393783523000176504296086535289084233761… (13.8)

s11 = 1.6180339888141730457336383515633161015686075305001394649735867464122236884708577745864296708710183021 (13.9)

s12 = 1.6180339887592728196515514114689288406590057438342054335746122537445132737632768289517828916865414341… (13.10)

s13 = 1.6180339887512630808808217725997428980661077406564927917175417680668704617854390066340882778896924781… (13.11)

s14 = 1.6180339887500944703785761774821044531841608177460839913086957240152607599378846730601761192780195281… (13.12)

$$s_{\infty} = \frac{1}{2}(1+\sqrt{5}) = \Phi = 1.618033988... \quad (13.13)$$

Der goldene Schnitt überrollt hier alle Fibonacci-Stolpersteine auf dem scf-Weg in die Unendlichkeit.

$$ds_\infty = \lim_{k\to\infty} \frac{s_k - s_{k+1}}{s_{k+1} - s_{k+2}} = 6.854101966249684544...$$

$$= \frac{1}{2}(7 + 3\sqrt{5}) = 2 + 3\Phi \qquad (13.14)$$

18.3

Analyze and discuss the transcendent numbers r_k represented by the infinite simple continued fractions

$r_k = [0;1^{\underline{k}},2,1^{\underline{k}},4,1^{\underline{k}},6,1^{\underline{k}},8,1^{\underline{k}},10,\ldots]$. (14)

$$r_2 = \frac{1}{e-1} \qquad (14.1)$$

18.4

$$\left.\begin{array}{c} \tanh\left(\frac{1}{n}\right) \\ \tan\left(\frac{1}{n}\right) = j\tanh\left(\frac{1}{jn}\right) \end{array}\right\} =$$

$$[0; n, \pm 3n, 5n, \pm 7n, 9n, \pm 11n, 13n, \pm 15n, ...] \qquad (15)$$

Analyze and discuss the transcendent numbers t_k and u_k resp., represented by the infinite simple continued fractions

$$[0;1^{\underline{1}},n,1^{\underline{2}},\pm 3n,1^{\underline{3}},5n,1^{\underline{4}},\pm 7n,1^{\underline{5}},9n,1^{\underline{6}}\pm 11n,...]$$

$$[0;1^{\underline{k}},n,1^{\underline{k}},\pm 3n,1^{\underline{k}},5n,1^{\underline{k}},\pm 7n,1^{\underline{k}},9n,1^{\underline{k}}\pm 11n,...]\,.$$

(16,17)

Lateral

$$\left.\begin{matrix}\tanh(x)\\ \tan(x)=-j\tanh(jx)\end{matrix}\right\}=$$

$$-\begin{vmatrix} 0 & \frac{1}{1!} & \frac{1}{3!} & \frac{1}{5!} & \frac{1}{7!} & \frac{1}{9!} & \rightarrow \\ x^1 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \frac{1}{8!} & \\ \pm x^3 & 0 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & \\ x^5 & 0 & 0 & \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \\ \pm x^7 & 0 & 0 & 0 & \frac{1}{0!} & \frac{1}{2!} & \\ x^9 & 0 & 0 & 0 & 0 & \frac{1}{0!} & \\ \downarrow & & & & & & \searrow \end{vmatrix}$$

(18)

18.5 Sandbox

18.5.1

$$\frac{I_1\left(\frac{2}{n}\right)}{I_0\left(\frac{2}{n}\right)} = [0;n,2n,3n,4n,5n,6n,7n,\ldots] =$$

$$2*[0;2*n,n,2*3n,2n,2*5n,3n,2*7n,4n,2*9n,5n,\ldots]=$$

$$2*[0;\ 2*n,\ \frac{1}{2}*2n,\ 2*3n,\ \frac{1}{2}*4n,\ 2*5n,\ \frac{1}{2}*6n,\ \ldots]\ ;$$

$n=1,2,3,\ldots$ (19)

$$I_p\left(\frac{2}{n}\right) = \sum_{k=0}^{\infty} \frac{1}{k!*(k+p)!*n^{2k+p}} \qquad (19.1)$$

[]→2*[] convergence acceleration!

How about $n \in \mathbb{R}$?

18.5.2

$$\tanh\left(\frac{1}{n}\right) = [0;n,3n,5n,7n,9n,11n,\ldots] =$$

$$\frac{1}{2}*[0;\ \frac{1}{2}*n,\ 2*3n,\ \frac{1}{2}*5n,\ 2*7n,\ \frac{1}{2}*9n,\ 2*11n,\ \ldots] =$$

$$2*[0;\ 2*n,\ \frac{1}{2}*3n,\ 2*5n,\ \frac{1}{2}*7n,\ 2*9n,\ \frac{1}{2}*11n,\ \ldots]\ ;$$

$n=1,2,3,\ldots$ (20)

18.5.3

$$\left.\begin{matrix}\dfrac{I_1\left(\frac{2}{n}\right)}{I_0\left(\frac{2}{n}\right)}\\ \dfrac{J_1\left(\frac{2}{n}\right)}{J_0\left(\frac{2}{n}\right)}\end{matrix}\right\} = [0;\ n,\ \pm 2n,\ 3n,\ \pm 4n,\ 5n,\ \pm 6n,\ 7n,\ ...] =$$

$$2*[0;\ 2*n,\ \pm\frac{1}{2}*2n,\ 2*3n,\ \pm\frac{1}{2}*4n,\ 2*5n,\ ...]\ ;$$

$$n=1,2,3,...$$

$$\left.\begin{matrix}I_p\left(\frac{2}{n}\right)\\ J_p\left(\frac{2}{n}\right)\end{matrix}\right\} = \sum_{k=0}^{\infty}\frac{(\pm 1)^k}{k!*(k+p)!*n^{2k+p}} \qquad (21)$$

18.5.4

$$\left\{\frac{I_p\left(\frac{2}{n}\right)}{I_{p-1}\left(\frac{2}{n}\right)},\ \frac{J_p\left(\frac{2}{n}\right)}{J_{p-1}\left(\frac{2}{n}\right)}\right\} = pn,\ \pm(p+1)n,\ (p+2)n,\ \pm(p+3)n,\ (p+4)n,\ \pm(p+5)n,\ (p+6)n,\ ...] =$$

$$2*[0;\ 2*pn,\ \pm\frac{1}{2}*(p+1)n,\ 2*(p+2)n,\ \pm\frac{1}{2}*(p+3)n,\ 2*(p+4)n,\ \pm\frac{1}{2}*(p+5)n,\ ...]\ ;\ p,n=1,2,3,...$$

(22)

Your turn is to verify/falsify 18.5.4 .

18.5.5

$$\left\{\tanh\left(\frac{1}{n}\right),\ \tan\left(\frac{1}{n}\right)\right\} = [0;\ n,\ \pm 3n,\ 5n,\ \pm 7n,\ 9n,\ \pm 11n,\ ...] =$$

$$\left\{\begin{array}{c} ? \\ [0;\ n-1,\ 1,\ 3n-2,\ 1,\ 5n-2,\ 1,\ 7n-2,\ 1,\ 9n-2,\ 1,\ ...] \end{array}\right. =$$

18.5.6

$$\tanh(1) = [0;1,3,5,7,9,\ldots$$

$$\frac{1}{\tan(1)} = [0;1,1,1,3,1,5,1,7,1,9,\ldots]$$

Your turn:

$$?_2 = [0;1,1,1,1,1,3,1,1,5,1,1,7,1,1,9,\ldots]$$

$$?_k = [0;1^{\underline{k}},1,1^{\underline{k}},3,1^{\underline{k}},5,1^{\underline{k}},7,1^{\underline{k}},9,\ldots]$$

18.5.7

$$\frac{\sqrt{e}}{1+\sqrt{e}} = 0.622459331\ldots$$

$$= \frac{1|}{|1} + \frac{1|}{|1} + \frac{1|}{|1} + \frac{2|}{|3} + \frac{4|}{|5} + \frac{6|}{|7} + \frac{8|}{|9} + \frac{10|}{|11} + \frac{12|}{|13} +$$

$$= [0;1,1,1,1,1,5,1,1,9,1,1,13,1,1,17,1,1,21,\ldots]$$

$$= \frac{1}{2}[1;4,12,20,28,36,44,52,\ldots]$$

$$? = 0.56201675...$$

$$= \frac{1|}{|1} + \frac{1|}{|1} + \frac{1|}{|3} + \frac{3|}{|5} + \frac{5|}{|7} + \frac{7|}{|9} + \frac{9|}{|11} + \frac{11|}{|13} + \frac{13|}{|15} + ...$$

$$= \frac{1|}{|1} + \frac{3|}{|3} + \frac{5|}{|5} + \frac{7|}{|7} + \frac{9|}{|9} + \frac{11|}{|11} + \frac{13|}{|13} + \frac{15|}{|15} + ...$$

$$= [0;1,1,3,1,1,7,1,1,11,1,1,15,1,1,19,1,1,23,...]$$

$$= \frac{1}{2}[1;8,16,24,32,40,48,56,...]$$

$$? = 0.621249806...$$

$$= \frac{|}{|} + \frac{|}{|} + ...$$

$$= [0;1,1,1,1,1,3,1,1,5,1,1,7,1,1,9,1,1,11,1,1,13,...]$$

$$= \frac{1}{2}[1;4,8,12,16,20,24,28,...]$$

$$? = 0.549724342...$$

$$= \frac{|}{|} + \frac{|}{|} +$$

$$= [0;1,1,4,1,1,8,1,1,12,1,1,16,1,1,20,1,1,24,...]$$

$$= \frac{1}{2}[1;10,18,26,34,42,50,58,...]$$

$$? = 0.582356079...$$

$$= \frac{|}{|} + \frac{|}{|} +$$

$$= [0;1,1,2,1,1,6,1,1,10,1,1,14,1,1,18,1,1,22,...]$$

$$= \frac{1}{2}[1;6,14,22,30,38,46,54,...]$$

$$\frac{1}{e-1} = 0.581976706...$$

$$= \frac{1|}{|1} + \frac{1|}{|1} + \frac{1|}{|2} + \frac{2|}{|3} + \frac{3|}{|4} + \frac{4|}{|5} + \frac{5|}{|6} + \frac{6|}{|7} + \frac{7|}{|8} + ...$$

$$= [0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,10,...]$$

$$= \frac{1}{2}[1;6,10,14,18,22,26,30,...]$$

$$? = 0.620096861...$$

$$= \frac{|}{|} + \frac{|}{|} +$$

$$= [0;1,1,1,1,1,1,2,1,1,3,1,1,4,1,1,5,1,1,6,1,1,7,...]$$

$$= \frac{1}{2}[1;4,6,8,10,12,14,16,...]$$

$$[0;1,1,a,1,1,a+b,1,1,a+2b,1,1,a+3b,1,1,a+4b,1,1,\ldots] =$$

$$\frac{1}{2}[1;2(a+1),2(a+1+b),2(a+1+2b),2(a+1+3b),\ldots]$$

18.5.8

$$\frac{I_1(2)}{I_0(2)} = 0.697774658\ldots$$

$$= [0;1,2,3,4,5,6,7,\ldots$$

$$= 2[0;2,1,6,2,10,3,14,4,18,5,22,6,26,7,\ldots]$$

$$\frac{I_2(2)}{I_1(2)} = 0.433127426\ldots$$

$$= [0;2,3,4,5,6,7,\ldots$$

$$= \frac{1}{2}[0;1,6,2,10,3,14,4,18,5,22,6,26,7,\ldots]$$

$$\frac{I_3(2)}{I_2(2)} = 0.308789373\ldots$$

$$= [0;3,4,5,6,7,\ldots$$

$$= 2[0;6,2,10,3,14,4,18,5,22,6,26,7,\ldots]$$

18.5.9

$$\frac{I_1(2)}{I_0(2)} = 0.697774658...$$

$$= [0;1,2,3,4,5,6,7,...$$

$$= 2[0;2,1,6,2,10,3,14,4,18,5,22,6,26,7,...]$$

$[0;2,4,6,8,10,12,\ldots] = 0.446389965\ldots =$
$2[0;4,2,12,4,20,6,28,8,36,\ldots] =$
$4[0;8,1,24,2,40,3,56,4,72,\ldots]$

$[0;4,8,12,16,20,24,\ldots] = 0.242499612\ldots =$
$2[0;8,4,24,8,40,12,56,16,72,\ldots] =$
$4[0;16,2,48,4,80,6,112,8,144,\ldots]$

18.5.10

$\tanh(1) = [0;1,3,5,7,9,11,13,15,17,\ldots]$
$= 0.761594156\ldots$
$= ?$
$\tan(1) = [0;1,-3,5,-7,9,-11,13,-15,17,\ldots]$
$= 1.557407725\ldots$
$= [1;1,1,3,1,5,1,7,1,9,1,11,1,13,\ldots]$

18.5.11

$$\frac{J_0(2)}{J_1(2) - 2J_0(2)} = [1;1,2,1,3,1,4,1,5,1,6,1,7,1,8,\ldots] =$$

$1.736351302 =$
$[0;-2,1,-2,3,-4,5,-6,7,-8,9,-10,\ldots] =$

$$1+\frac{1|}{|1}+\frac{1|}{|2}+\frac{2|}{|2}+\frac{2|}{|3}+\frac{3|}{|3}+\frac{3|}{|4}+\frac{4|}{|4}+\frac{4|}{|5}+\frac{5|}{|5}+\ldots$$

[0;1,a,1,a+b,1,a+2b,1,a+3b,1,a+4b,1,…] = ?

18.5.12

$$\tanh(1)=\frac{e^2-1}{e^2+1}=0.761594156\ldots$$

$$=[0;1,3,5,7,9,11,13,\ldots]$$

$$=\frac{1}{2}[0;\frac{1}{2},6,\frac{5}{2},14,\frac{9}{2},22,\frac{13}{2},30,\frac{17}{2},38,\ldots]$$

$$=\frac{1}{2}[1;1,1,10,3,1,1,4,22,6,1,1,7,34,9,1,1,10,46,12,\ldots]$$

$$=\frac{1}{2}[1,1,\overline{a_i=3i\text{-}2,b_i=12i\text{-}2,c_i=3i,}]_{i=1}^{\infty}$$

$$=2[0;2,\frac{3}{2},10,\frac{7}{2},18,\frac{11}{2},26,\frac{15}{2},34,\frac{19}{2},\ldots]$$

For comparison see

$$\frac{1}{e^2-1} = 0.156517642...$$

$$= [0;6,2,1,1,3,18,5,1,1,6,30,8,1,1,9,42,11,1,1,...]$$

$$= [\overline{a_i=3i, b_i=12i+6, c_i=3i+2, 1, 1}]_{i=0}^{\infty}$$

$$= \frac{1}{2}[0;3,5,7,9,11,13,...]$$

$$= 2[0,\ 12,\ \ 1,\ 3,\ 1,\ \ 1,\ \ 36,\ \ 2,\ 1,\ 3,\ \ 3,$$
$$60,\ \ 4,\ 3,\ 1,\ \ 4,\ \ 84,\ \ 5,\ 1,\ 3,\ \ 6,$$
$$108,\ \ 7,\ 3,\ 1,\ \ 7,\ 132,\ \ 8,\ 1,\ 3,\ \ 9,$$
$$156,\ 10,\ 3,\ 1,\ 10,\ 180,\ 11,\ 1,\ 3,\ 12,$$
$$204,\ 13,\ 3,\ 1,\ 13,\ 228,\ 14,\ 1,\ 3,\ 15,$$
$$252,\ 16,\ 3,\ 1,\ 16,\ 276,\ 17,\ 1,\ 3,\ 18,$$
$$300,\ 19,\ 3,\ 1,\ 19,\ 324,\ 20,\ 1,\ 3,\ 21,$$
$$348,\ 22,\ 3,\ 1,\ 22,\ 372,\ 23,\ 1,\ 3,\ 24,$$
$$396,\ 25,\ 3,\ 1,\ 25,\ 420,\ 26,\ 1,\ 3,\ 27,$$
$$444,\ 28,\ 3,\ 1,\ 28,\ 468,\ 29,\ 1,\ 3,\ 30,$$
$$\ldots]$$

18.5.13

$$\tanh\left(\frac{1}{2}\right) = \frac{e-1}{e+1} = 0.462117157...$$

$$= [0;2,6,10,14,18,22,...]$$

$$= \frac{1}{2}[0;1,12,5,28,9,44,13,60,17,76,...]$$

$$= 2[0;4,3,20,7,36,11,52,15,68,19,...]$$

For comparison see

$$\frac{1}{e-1} = 0.581976706...$$

$$= [0;1,1,2,1,1,4,1,1,6,1,1,8,1,1,...]$$

$$= \frac{1}{2}[1;6,10,14,18,22,26,...]$$

$$= 2[0,\ 3,\ 2,\ 3,\ 2,\ 3,\ 1,\ \ 2,\ 1,\ 3,\ \ 4,$$
$$3,\ 1,\ \ 4,\ 1,\ 3,\ \ 6,$$
$$3,\ 1,\ \ 6,\ 1,\ 3,\ \ 8,$$
$$3,\ 1,\ \ 8,\ 1,\ 3,\ 10,$$
$$3,\ 1,\ 10,\ 1,\ 3,\ 12,$$
$$3,\ 1,\ 12,\ 1,\ 3,\ 14,$$
$$3,\ 1,\ 14,\ 1,\ 3,\ 16,$$
$$3,\ 1,\ 16,\ 1,\ 3,\ 18,$$
$$3,\ 1,\ 18,\ 1,\ 3,\ 20,$$
$$3,\ 1,\ 20,\ 1,\ 3,\ 22,$$
$$3,\ 1,\ 22,\ 1,\ 3,\ 24,$$
$$3,\ 1,\ 24,\ 1,\ 3,\ 26,$$
$$3,\ 1,\ 26,\ 1,\ 3,\ 28,\ \ldots]$$

18.5.14

Serenade

We consider three infinite sequences $\langle a_0, a_1, a_2, a_3, a_4, .. \rangle$ of odd numbers.

Sequence 1

1,5,7,11,13,17,19,23,25,29,31,35,37,41,43,47,49,
53,55,59,61,65,67,71,73,77,…

** the distances between successive elements are
4,2,4,2,4,2,…

** most elements are primes

Sequence 2

1,7,13,19,25,31,37,43,49,55,61,67,73,…

** the distance between successive elements is 6
** most elements are primes
Sequence 3
5,11,17,23,29,35,41,47,53,59,65,71,77,…
** the distance between successive elements is 6
** most elements are primes

** Find the decimal representations of the infinite continued fractions

$$A = [a_0; a_1, a_2, a_3, a_4, ..] \qquad \text{scf}$$

$$B = a_0 + \frac{a_1|}{|a_1} + \frac{a_2|}{|a_2} + \frac{a_3|}{|a_3} + \frac{a_4|}{|a_4} + ... \quad \text{gcf}$$

** Find the infinite scf of $\frac{1}{2}$A, 2A, $\frac{1}{2}$B, and 2B .
** Discuss your results.
** Lateral for the sequence 1:
Calculate, display and discuss the prime density function of sequ. 1

$$p(i) ::= \frac{1}{i}(\text{number of primes in } \langle a_1, a_2, ..., a_i, \rangle) .$$

Results for sequence 1, A
A = [1; 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37,
41, 43, 47, 49, 53, 55, 59, 61, 65, 67, 71, 73,
77, 79, 83, 85, 89, 91, 95, 97, 101, 103, 107,
109, 113, 115, 119, 121, 125, 127, 131, 133,
137, 139, 143, 145, 149, 151, 155, 157, 161,
163, 167, 169, 173, 175, 179, 181, 185, 187,
191, 193, …]
= 1.19451324273619330935556806471336869
49827781913989090929126591669011931774
16574472202856440809334768…

A/2 = 0.597256621368096654677784032356684347491389095699454546456329583450596588708287236101428220404 6…

scf(A/2) = [0; 1, 1, 2, 14, 5, 1, 1, 6, 34, 9,
1, 1, 11, 50, 14, 1, 1, 15, 70, 18,
1, 1, 20, 86, 23, 1, 1, 24, 106, 27,
1, 1, 29, 122, 32, 1, 1, 33, 142, 36,
1, 1, 38, 158, 41, 1, 1, 42, 178, 45,
1, 1, 47, 194, 50, 1, 1, 51, 214, 54,
1, 1, 56, 230, 59, 1, 1, 60, 250, 63,
1, 1, 65, 266, 68, 1, 1, 69, 286, 72,
1, 1, 74, 302, 77, 1, 1, 78, 322, 81,
1, 1, 83, 338, 86, 1, 1, 87, 358, 90,
1, 1, 92, 374, 95, 1, 1, 96, 394, 99,
1, 1, …]

$$= \left[0; \overline{1,1,a_i, b_i, c_i,}\right]_{i=1}^{\infty}$$

$$a_i = \begin{cases} 9k-7, & \text{für } i=2k-1 \\ 9k-3, & \text{für } i=2k \end{cases}$$

$$b_i = \begin{cases} 36k-22, & \text{für } i=2k-1 \\ 36k-2, & \text{für } i=2k \end{cases}; \; c_i = a_i + 3$$

2A = 2.389026485472386618711136129426737389965556382797818185825318333802386354833148944405712881618669536838938 07…

scf(2A) = [2; 2, 1, 1, 3, 22, 6, 1, 1, 8, 38, 11,
1, 1, 12, 58, 15, 1, 1, 17, 74, 20,
1, 1, 21, 94, 24, 1, 1, 26, 110, 29,
1, 1, 30, 130, 33, 1, 1, 35, 146, 38,
1, 1, 39, 166, 42, 1, 1, 44, 182, 47,
1, 1, 48, 202, 51, 1, 1, 53, 218, 56,

1, 1, 57, 238, 60, 1, 1, 62, 254, 65,
1, 1, 66, 274, 69, 1, 1, 71, 290, 74,
1, 1, 75, 310, 78, 1, 1, 80, 326, 83,
1, 1, 84, 346, 87, 1, 1, 89, 362, 92,
1, 1, …]

$$= [2; 2, \overline{1,1, a_i, b_i, c_i,}]_{i=1}^{\infty}$$

$$a_i = \begin{cases} 9k-6, & \text{für } i=2k-1 \\ 9k-1, & \text{für } i=2k \end{cases}$$

$$b_i = \begin{cases} 36k-14, & \text{für } i=2k-1 \\ 36k+12, & \text{für } i=2k \end{cases}; \; c_i = a_i + 3$$

For comparison

e^2 - 1= 6.389056099…

[6;2,1,1, 3,18, 5,
1,1, 6,30, 8,
1,1, 9,42,11,
1,1,12,54,14,
1,1,15,66,17,...]

$$= [6; 2, \overline{1,1, a_i = 3i, b_i = 12i+6, c_i = a_i + 2}]_{i=1}^{\infty}$$

$$\frac{1}{2}(e^2 - 1) = 3.194528049...$$

$$= [3; 5, 7, 9, 11, 13, 15, 17, ...]$$

$$\frac{I_2(2)}{I_3(2)} = 3.238453416...$$

$$= [3; 4, 5, 6, 7, 8, 9, 10, ...]$$

Results for sequence 2, A

A = [1; 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73,
79, 85, 91, 97, 103, 109, 115, 121, 127, 133,
139, 145, 151, 157, 163, 169, 175, 181, 187,
193, 199, 205, 211, 217, 223, 229, 235, 241,
247, 253, 259, 265, 271, 277, 283, 289,295,
301, 307, 313, 319, 325, 331, 337, 343, 349,
355, 361, 367, 373, 379, 385, …]

= 1.14131052836745985712411989771832646
52622467435011345416987853986746402878930262590714797025654703 1…

A/2 = [0.57065526418372992856205994885916
32326311233717505672708493926993373
20143946651312953573985128273515713112554025441030001494250563872831194621892959102582663025536623512932
1159067611687867868756756314 5…

scf(A/2) =

[0; 1, 1, 3, 26, 9, 1, 1, 12, 62, 18,
1, 1, 21, 98, 27, 1, 1, 30, 134, 36,
1, 1, 39, 170, 45, 1, 1, 48, 206, 54,
1, 1, 57, 242, 63, 1, 1, 66, 278, 72,
1, 1, 75, 314, 81, 1, 1, 84, 350, 90,
1, 1, 93, 386, 99, 1, 1, 102, 422, 108,
1, 1, 111, 458, 117, 1, 1, 120, 494, 126,
1, 1, 129, 530, 135, 1, 1, 138, 566, 144,
1, 1, 147, 602, 153, 1, 1, 156, 638, 162,
1, 1, …] =

$$\overline{[0;1,1,a_i=9i-6,b_i=36i-10,c_i=9i]}_{i=1}^{\infty}$$

2A = 2.828621056734919714248239795436652930524493487002269083397570797349280575786605251814295940513094062852450216101764120005977002255491324778487571836410330652102146494051728463627044675147147502702525 8…

scf(2A) =

[2; 3, 1, 1, 6, 38, 12, 1, 1, 15, 74, 21,
1, 1, 24, 110, 30, 1, 1, 33, 146, 39,
1, 1, 42, 182, 48, 1, 1, 51, 218, 57,
1, 1, 60, 254, 66, 1, 1, 69, 290, 75,
1, 1, 78, 326, 84, 1, 1, 87, 362, 93,
1, 1, 96, 398, 102, 1, 1, 105, 434, 111,
1, 1, 114, 470, 120, 1, 1, 123, 506, 129,
1, 1, 132, 542, 138, 1, 1, 141, 578, 147,
1, 1, …] =

$$[2;\overline{3,1,1,a_i=9i-3,b_i=36i+2,c_i=9i+3}]_{i=1}^{\infty}$$

$$B = 1+\frac{|7}{7}+\frac{|13}{13}+\frac{|19}{19}+\frac{|25}{25}+\frac{|31}{31}+\frac{|37}{37}+\frac{|43}{43}+\frac{|49}{49}+\ldots$$

= 1.882525630158049366026754188263357938764386588225024054231007445321878249098422203082831594357142949…

= [1;1,7,1,1,19,1,1,31,1,1,43,1,1,55,1,1,67,…]

B/2 = 0.941262815079024683013377094131678969382193294112512027115503722660939124549211101541415797178571474391266547631553063789838563398474614369717672322707552968176600571704806136062380341120468847489224 0…

scf(B/2) = [0; 1, 16, 40, 64, 88, 112, 136, 160, 184,
208, 232, 256, 280, 304, 328, 352, 376,
400, 424, 448, 472, 496, 520, 544, 568,
592, 616, 640, 664, 688, 712, 736, 760,
784, 808, 832, 856, 880, 904, 928, …]

2B = 3.765051260316098732053508376526715875287731764500481084620148906437564981968444061656631887142858975650661905262122551593542535938984574788706892908302118727064022868192245442495213644819475389956…

scf(2B) = [3; 1, 3, 3, 1, 9, 3, 1, 15, 3, 1, 21,
3, 1, 27, 3, 1, 33, 3, 1, 39, 3, 1, 45,
3, 1, 51, 3, 1, 57, 3, 1, 63, 3, 1, 69,
3, 1, 75, 3, 1, 81, 3, 1, 87, 3, 1, 93,
3, 1, 99, 3, 1, 105, 3, 1, 111, 3, 1, 117,
3, 1, 123, 3, 1, 129, 3, 1, 135, 3, 1, 141,
3, 1, 147, 3, 1, 153, 3, 1, 159, 3, 1, 165,
3, 1, 171, 3, 1, 177, 3, 1, 183, …

$$= [3;1,\overline{a_i=6i-3,3,1,}]_{i=1}^{\infty}$$

Results for sequence 3, A

A = [5; 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71,
77, 83, 89, 95, 101, 107, 113, 119, 125, 131,
137, 143, 149, 155, 161, 167, 173, 179, 185,
191, 197, 203, 209, 215, 221, 227, 233, 239,
245, 251, 257, 263, 269, 275, 281, 287, 293,
299, 305, 311, 317, 323, 329, 335, 341, 347,
353, 359, 365, 371, 377, 383, 389, …]

= 5.090426757108460889792371417963452972653083547336526808796595147926475378989073478921447517747043342…

A/2 = 2.545213378554230444896185708981726486326541773668263404398297573963237689494536739460723758873521670848088767556597826297441856917068625475884265797716536888225221223…

scf(A/2) =

[2; 1, 1, 5, 34, 11, 1, 1, 14, 70, 20,
1, 1, 23, 106, 29, 1, 1, 32, 142, 38,
1, 1, 41, 178, 47, 1, 1, 50, 214, 56,
1, 1, 59, 250, 65, 1, 1, 68, 286, 74,
1, 1, 77, 322, 83, 1, 1, 86, 358, 92,
1, 1, 95, 394, 101, 1, 1, 104, 430, 110,
1, 1, 113, 466, 119, 1, 1, 122, 502, 128,
1, 1, 131, 538, 137, 1, 1, 140, 574, 146,
1, 1, 149, 610, 155, 1, 1, 158, 646, 164,
1, 1, 167, 682, 173, 1, 1, 176, 718, 182,
1, 1, 185, 754, 191, 1, 1, 194, 790, 200,
1, 1, 203, 826, 209, 1, 1, 212, 862, 218,
1, 1, …] =

$$[2;1,1,\overline{a_i=9i-4,b_i=36i-2,c_i=9i+2,}]_{i=1}^{\infty}$$

2A = 10.180853514216921779584742835926905945306167094673053617593190295852950757978146957842895035494086683392355070226391305189767427668274501903537063190866147552900884895673387465038230 5…

scf(2A) =
[10; 5, 1, 1, 8, 46, 14, 1, 1, 17, 82, 23,
1, 1, 26, 118, 32, 1, 1, 35, 154, 41,
1, 1, 44, 190, 50, 1, 1, 53, 226, 59,
1, 1, 62, 262, 68, 1, 1, 71, 298, 77,
1, 1, 80, 334, 86, 1, 1, 89, 370, 95,
1, 1, 98, 406, 104, 1, 1, 107, 442, 113,
1, 1, 116, 478, 122, 1, 1, 125, 514, 131,
1, 1, 134, 550, 140, 1, 1, 143, 586, 149,
1, 1, 152, 622, 158, 1, 1, 161, 658, 167,
1, 1, …] =

$$[10;\overline{5,1,1,a_i=9i-1,b_i=36i+10,c_i=9i+5,}]_{i=1}^{\infty}$$

18.5.15

Freerunning 1

Given

$$D = \frac{1|}{|3} + \frac{5|}{7} + \frac{9|}{|11} + \frac{13|}{|15} + \ldots$$

Find

** d = decimal representation of D

** scf(d)

** scf(d/2)

** scf(d/3)

The same for

$$E = \frac{3|}{|1} + \frac{7|}{5} + \frac{11|}{|9} + \frac{15|}{|13} + \ldots$$

Result for E

$$E = \frac{3|}{|1} + \frac{7|}{|5} + \frac{11}{|9} + \frac{15|}{|13} + \frac{19|}{|17} + \frac{23|}{|21} + \frac{27|}{|25} + \ldots$$

= 1.396045039671837489058645721659497782
916225005279628092257529484556130032 9
168218773001137620921377162493031546 1
287952025828745606814297820873061614 3
944066127128165606495049725183968422 2
719153720801896…

scf(E) =

[1; 2, 1, 1, 9, 1, 1, 17, 1, 1, 25, 1, 1, 33,
1, 1, 41, 1, 1, 49, 1, 1, 57, 1, 1, 65,
1, 1, 73, 1, 1, 81, 1, 1, 89, 1, 1, 97,
1, 1, 105, 1, 1, 113, 1, 1, 121, 1, 1, 129,
1, 1, 137, 1, 1, 145, 1, 1, 153, 1, 1, 161,
1, 1, 169, 1, 1, 177, 1, 1, 185, 1, 1, 193,
1, 1, 201, 1, 1, 209, 1, 1, 217, 1, 1, 225,
1, 1, 233, 1, 1, 241, 1, 1, 249, 1, 1, 257,
1, 1, 265, 1, 1, 273, 1, 1, 281, 1, 1, 289,
1, 1, …]

E/2 = 0.698022519835918744529322860829748 8
914581125026398140461287647422780 65
016458410938650056881046068858124 65
157730643976012914372803407148910 43
653080719720330635640828 03…

scf(E/2) =

[0; 1, 2, 3, 4, 1, 3, 8, 1, 3, 12, 1, 3, 16,
1, 3, 20, 1, 3, 24, 1, 3, 28,
1, 3, 32, 1, 3, 36, 1, 3, 40,
1, 3, 44, 1, 3, 48, 1, 3, 52,
1, 3, 56, 1, 3, 60, 1, 3, 64,
1, 3, 68, 1, 3, 72, 1, 3, 76,

1, 3, 80, 1, 3, 84, 1, 3, 88,
1, 3, 92, 1, 3, 96, 1, 3, …]

E/3 =
0.465348346557279163019548573886499260972075001759
876030752509828185376677638940625766704587364045905416434384870959840008…scf(E/3) =
[0; 2, 6, 1, 2, 1, 1, 53, 1, 1, 8, 5, 1, 10,
1, 1, 125, 1, 1, 16, 5, 1, 18,
1, 1, 197, 1, 1, 24, 5, 1, 26,
1, 1, 269, 1, 1, 32, 5, 1, 34,
1, 1, 341, 1, 1, 40, 5, 1, 42,
1, 1, 413, 1, 1, 48, 5, 1, 50,
1, 1, 485, 1, 1, 56, 5, 1, 58,
1, 1, 557, 1, 1, 64, 5, 1, 66,
1, 1, 629, 1, 1, 72, 5, 1, 74,
1, 1, 701, 1, 1, 80, 5, 1, 82,
1, 1, 773, 1, 1, 88, 5, 1, 90,
1, 1, 845, 1, 1, 96, 5, 1, 98, …]

Freerunning 2

$$G = \frac{7|}{|1} + \frac{19|}{|13} + \frac{31}{|25} + \frac{43|}{|37} + \frac{55|}{|49} + \frac{67|}{|61} + \frac{79|}{|73} + \ldots$$

= 2.992338632346752124135842209134035997564645799557426916562073175665821635487470631891681047009354699490393585595768811123555193573561932580357848458968821623001882618…
scf(G) =
[2; 1, 129, 1, 1, 9, 1, 1, 369, 1, 1, 19, 9, 1, 23, 1, 1,
729, 1, 1, 33, 1, 1, 969, 1, 1, 43, 9, 1, 47, 1, 1,
1329, 1, 1, 57, 1, 1, 1569, 1, 1, 67, 9, 1, 71, 1, 1,

1929, 1, 1, 81, 1, 1, 2169, 1, 1, 91, 9, 1, 95, 1, 1,
2529, 1, 1, 105, 1, 1, 2769, 1, 1, 115, 9, 1, 119, 1, 1,
3129, 1, 1, 129, 1, 1, 3369, 1, 1, 139, 9, 1, 143, 1, 1,
3729, 1, 1, 153, 1, 1, 3969, 1, 1, 163, 9, 1, 167, 1, 1,
4329, 1, 1, 177, 1, 1, 4569, 1, 1, 187, 9, 1, 191, 1, 1, …]

G/2 =
1.4961693161733760620679211045670179963782322899778713458281036587832910817743735315945840523504677349745196792797884405561777596786780966290178924229484410811500941306579507393975870062061496947348201…

scf(G/2)
= [1, 2, 64, 1, 3, 4, 1, 3, 184, 1, 3, 9, 1, 1, 4, 2, 11, 1, 3, 364, 1, 3, 16, 1, 3, 484, 1, 3, 21, 1, 1, 4, 2, 23, 1, 3, 664, 1, 3, 28, 1, 3, 784, 1, 3, 33, 1, 1, 4, 2, 35, 1, 3, 964, 1, 3, 40, 1, 3, 1084, 1, 3, 45, 1, 1, 4, 2, 47, 1, 3, 1264, 1, 3, 52, 1, 3, 1384, 1, 3, 57, 1, 1, 4, 2, 59, 1, 3, 1564, 1, 3, 64, 1, 3, 1684, 1, 3, 69, 1, 1, 4, 2, 71, 1, 3, 1864, 1, 3, 76, 1, 3, 1984, 1, 3, 81, 1, 1, 4, 2, 83, 1, 3, 2164, 1, 3, 88, 1, 3, 2284, 1, 3, 93, 1, 1, 4, 2, 95, 1, 3, 2464, 1, 3, …]

References

[1] http://isc.carma.newcastle.edu.au/index

Links

Unit 26
Unit 30.1
Unit 193

To be continued.